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Data Modeling Using Transmuted Distributions

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Abstract

In this paper, we discussed two distributions (the Lindley Distribution and the Transmuted Lindley Distribution) , in addition to some functions such as the PDF, CDF function, and we discussed some statistical properties such as Moments, the Moment Generating Function, the Coefficient of Variation, Skewness, Kurtosis, and Ordered Statistics. The parameters are estimated via the maximum likelihood method. This paper concluded the importance of the Transmuted Lindley Distribution on the Lindley distribution and on all sample sizes in the simulation side.

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1. Introduction

Not long ago, many new distributions were proposed, which are called extended (transmuted) distributions, in which one or more parameters are added to the original distributions. The rank transmutation method (RTM) was used by (Shaw and Buckley), which results in new families, and the quadratic rank transmutation map (QRTM) was used. By many researchers to produce more flexible and accurate distributions in modeling and analyzing data in several fields, especially applied ones, and thus there are many transmuted distributions for modelling data of some biological phenomena that the original distributions could not model. In this research, the quadratic transform map method was used to find the transmuted lindley distribution (TLind). the Cumulative Distribution Function (CDF) satisfy the relationship:

$$F1(x)=(1+\lambda)F2(x)-\lambda(F2(x))2 \qquad (1)$$

which on differentiation yields,

$$f1(x)=f2(x)[(1+\lambda)-2\lambda F2(x)] \qquad (2)$$

where $F2(x)$ is the CDF of the base distribution

2. Transmuted Lindley Distribution (TLind)

To model life times in various fields, especially in engineering sciences and medicine, the transformed Lindley distribution is used and has been shown to be particularly efficient in analyzing and modeling data related to mortality studies. [3].

The probability density function of the Lindley distribution is as in the following formula [2]:

$$f(x;\ \gamma) = \frac{\gamma^2}{\gamma+1}(1+x)e^{-\gamma x} \qquad x\,,\gamma > 0 \tag{3}$$

Where γ is shape parameter

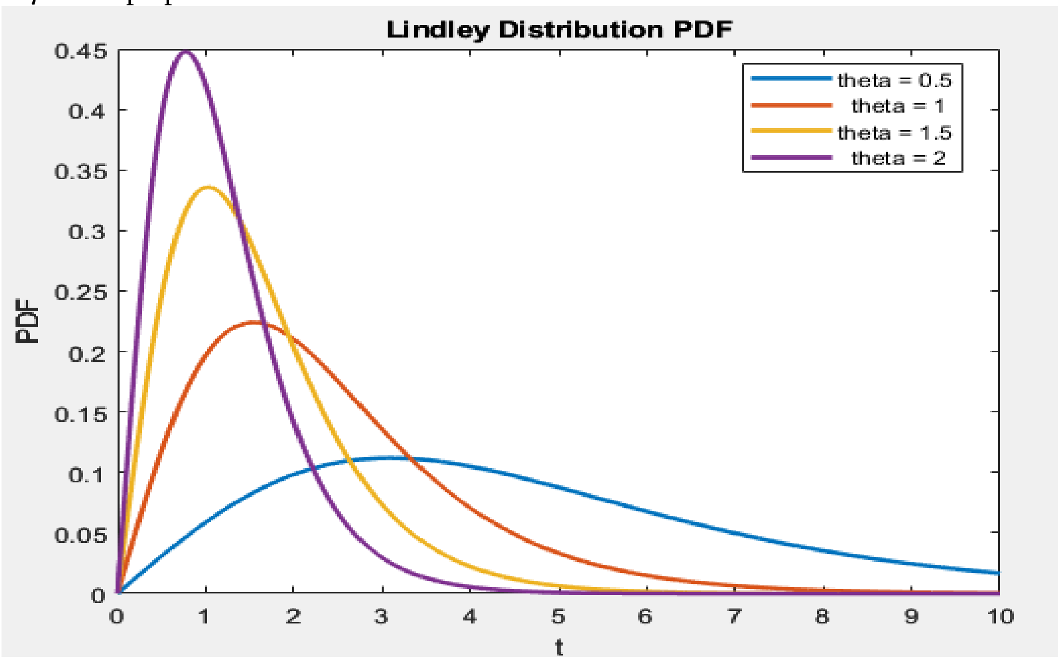

Figure (1): The PDF of Lindley Distribution

The (CDF) of Lindley Distribution is given by[2]:

$$F(x;\gamma) = 1 - \frac{(1+\gamma+\gamma x)}{1+\gamma}e^{-\gamma x} \qquad ,\gamma > 0\,, x > 0 \tag{4}$$

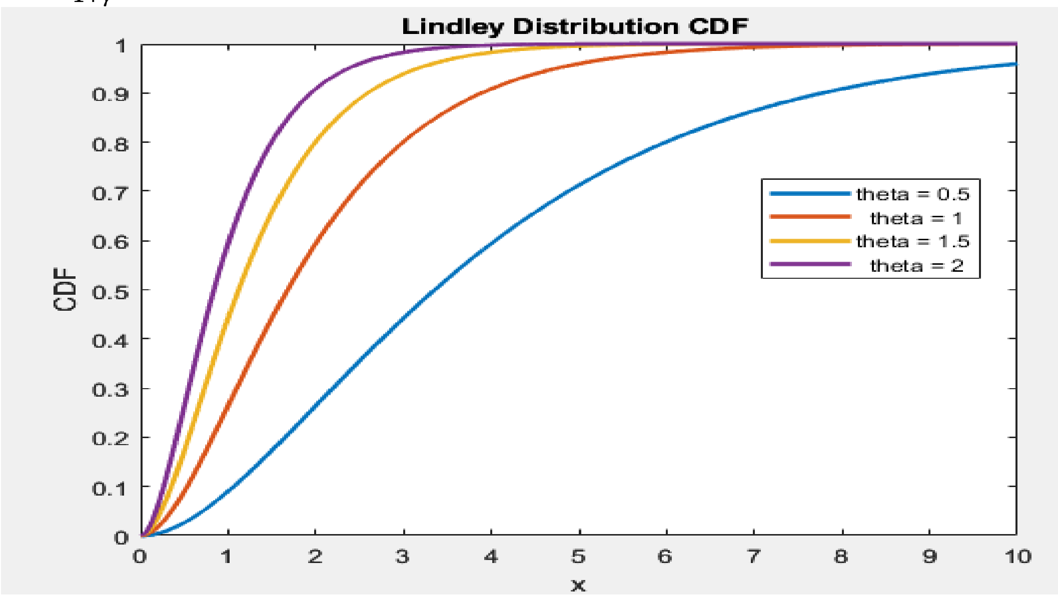

Figure (2): The CDF of Lindley Distribution

From equation (1) and (4) we conclude the (CDF) of the TLind [4]:

$$F(x) = (1 - \frac{(1+\gamma+\gamma x)}{1+\gamma} e^{-\gamma x})\,(1 + \lambda \frac{(1+\gamma+\gamma x)}{1+\gamma} e^{-\gamma x}) \quad (5)$$

Where $|\lambda| \leq 1$, $\gamma > 0, x > 0$

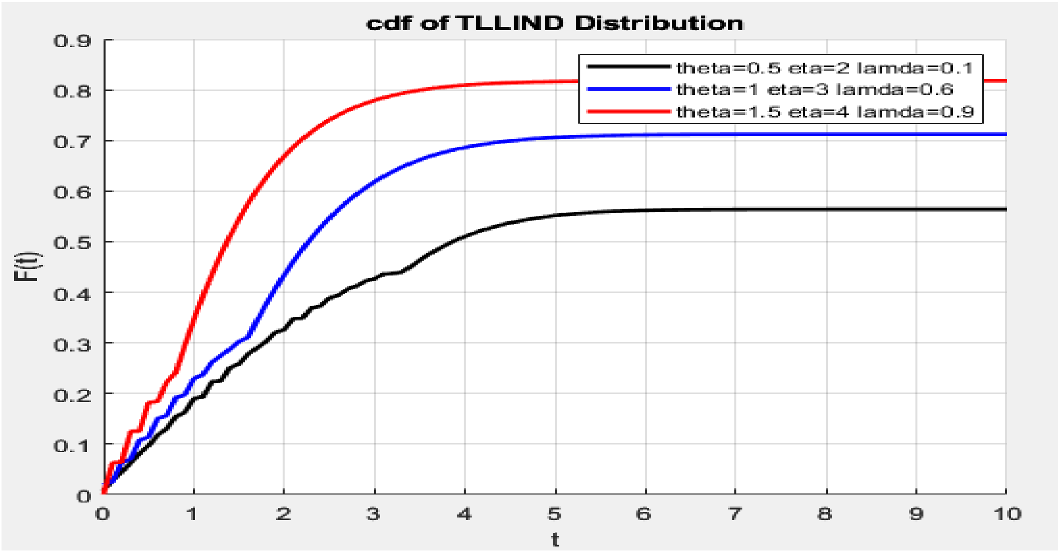

Figure (3): The CDF of Transmuted Lindley Distribution

The PDF of (TLind) random variable may by written as [4] :

$$f(x) = \frac{\gamma^2}{1+\gamma}(1+x)e^{-\gamma x}(1 - \lambda + 2\lambda \frac{(1+\gamma+\gamma x)}{1+\gamma} e^{-\gamma x}) \quad (6)$$

Since $|\boldsymbol{\lambda}| \leq \mathbf{1}$, $\quad \boldsymbol{x}, \boldsymbol{\gamma} > \mathbf{0}$

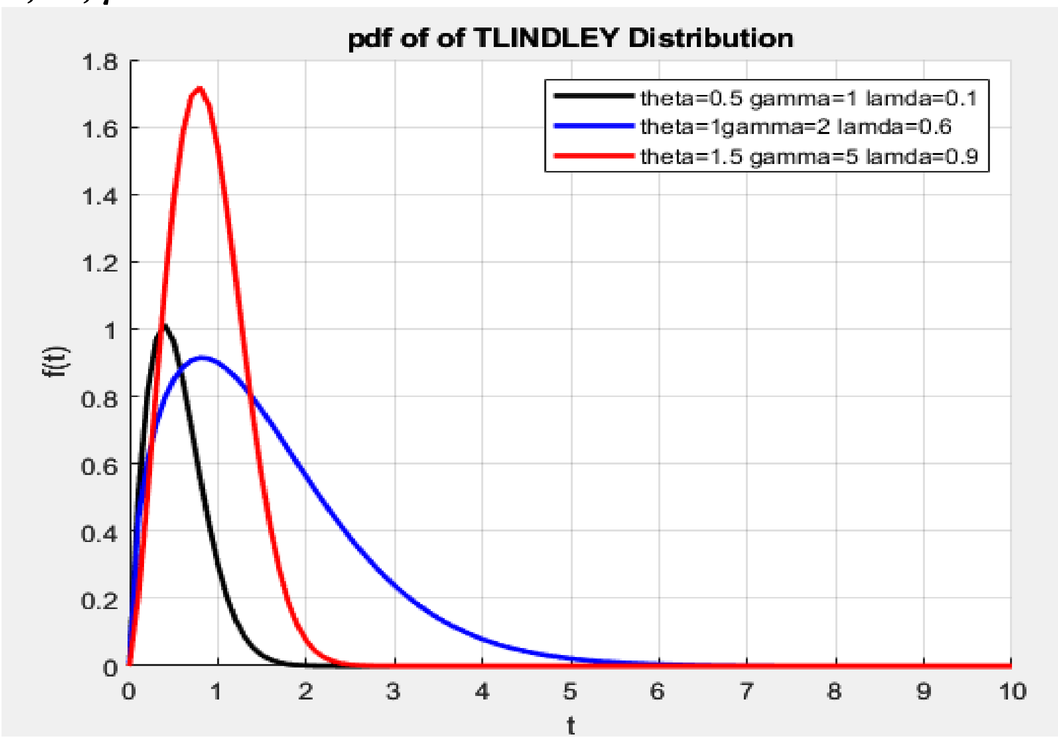

Figure (4): The PDF of Transmuted Lindley Distribution

3. Mathematical Properties of Transmuted Lindley Distribution:

3.1. Moments

Let x a random variable and distributes the transmuted Lindley distribution, it has the moment [7]:

$$\acute{\mu}_r = \int_{-\infty}^{\infty} x^r f(x)\, dx$$

$$\acute{\mu}_r = \int_{0}^{\infty} x^r \frac{\gamma^2}{1+\gamma}(1+x)e^{-\gamma x}(1-\lambda+2\lambda\frac{(1+\gamma+\gamma x)}{1+\gamma}e^{-\gamma x})dx$$

$$\acute{\mu}_r = \frac{r!}{\gamma^r(\gamma+1)}\left[(1-\lambda)(\gamma+r+1)+\frac{\lambda\gamma}{2^{r-1}(\gamma+1)}(2\gamma+3+r)\right] \quad (7)$$

When r =1 , the first moment of the TLind we get the following :

$$\mu = \frac{1}{\gamma(\gamma+1)}[(1-\lambda)(\gamma+2)+\frac{\lambda\gamma}{(\gamma+1)}(2\gamma+4)]$$

$$\mu = \frac{1}{\gamma(\gamma+1)}\left[(1-\lambda)(\gamma+2)+\frac{2\lambda\gamma}{(\gamma+1)}(\gamma+2)\right] \quad (8)$$

When r = 2, the second moment of the TLind we get the following:

$$\acute{\mu}_2 = \frac{2}{\gamma^2(\gamma+1)}[(1-\lambda)(\gamma+3)+\frac{\gamma\lambda(2\gamma+5)}{2(\gamma+1)} \quad (9)$$

So the variance is given by :

$$Var(x) = E(X^2)-(E(X))^2 = \frac{2}{\gamma^2(\gamma+1)}\left[(1-\lambda)(\gamma+3)+\frac{\lambda\gamma(2\gamma\ +5)}{2(\gamma\ +1)}\right]-[\frac{1}{\gamma(\gamma+1)}((1-\lambda)(\gamma+2)+\frac{2\lambda\gamma}{\gamma+1}(\gamma+2))]^2 \quad (10)$$

When r = 3, the third moment of the TLind we get the following formula:

$$\acute{\mu}_3 = \frac{6}{\gamma^3(\gamma+1)}[(1-\lambda)(\gamma+4)+\frac{\gamma\lambda(\gamma+3)}{2(\gamma+1)}] \quad (11)$$

We can get the coefficient of variation of TLind by the following formula:

$$CV = \frac{\sqrt{\boldsymbol{Var(x)}}}{\boldsymbol{E(X)}} = \frac{\sqrt{\frac{\mathbf{2}}{\boldsymbol{\gamma^2(\gamma+1)}}\left[(\mathbf{1}-\boldsymbol{\lambda})(\boldsymbol{\gamma}+\mathbf{3})+\frac{\boldsymbol{\lambda\gamma(2\gamma+5)}}{\mathbf{2}(\boldsymbol{\gamma}+\mathbf{1})}\right]-[\frac{\mathbf{1}}{\boldsymbol{\gamma(\gamma+1)}}((\mathbf{1}-\boldsymbol{\lambda})(\boldsymbol{\gamma}-\mathbf{2})+\frac{\mathbf{2}\boldsymbol{\gamma\lambda(\gamma+2)}}{(\boldsymbol{\gamma}+\mathbf{1})})\ ]^{\mathbf{2}}}}{\frac{\mathbf{1}}{\boldsymbol{\gamma(\gamma+1)}}[(\mathbf{1}-\boldsymbol{\lambda})(\boldsymbol{\gamma}+\mathbf{2})+\frac{\mathbf{2}\boldsymbol{\lambda\gamma}}{(\boldsymbol{\gamma}+\mathbf{1})}(\boldsymbol{\gamma}+\mathbf{2})]} \quad (12)$$

As for when r = 4, the fourth moment of TLind is given:

$$E(X^4) = \frac{24}{\gamma^4(\gamma+1)}\left[(1-\lambda)(\gamma+5)+\frac{\gamma\lambda(2\gamma+7)}{8(\gamma+1)}\right] \quad (13)$$

The Coefficient of Skewness (CS) can also be obtained by the following formula:

$$CS\ = \frac{E(X-\mu)^3}{(Var(x))^{3/2}}$$

$$E[(X-\mu)^3] = E[\sum_{i=o}^{3}\binom{3}{i}(-\mu)^i X^{3-i}]$$

$$= \mathrm{E}\left[\binom{3}{0}(-\mu)^0X^3+\binom{3}{1}(-\mu)X^2+\binom{3}{2}(-\mu)^2X+\binom{3}{3}(-\mu)^3X^0\right]$$

$$= \acute{\mu}_3 - 3\mu\,\acute{\mu}_2 + 2\mu^3$$

$$CS = \frac{E(X-\mu)^3}{(Var(x))^{3/2}} \quad (14)$$

As for the kurtosis coefficient, it is as in the following formula :

$$CK = \frac{E(X-\mu)^4}{(Var(x))^2}$$

$$E[(X-\mu)^4] = E[\textstyle\sum_{i=o}^{4}\binom{4}{i}(-\mu)^i X^{4-i}] = \acute{\mu}_4 - 4\mu\,\acute{\mu}_3 + 6\mu^2\mu_2 - 3\mu^4$$

$$CK \quad = \frac{\acute{\mu}_4 - 4\mu\, \acute{\mu}_3 + 6\mu^2\mu_2 - 3\mu^4}{(Var(x))^2} \tag{15}$$

3.2. Moment Generating Function

The transmuted Lindley distribution has the moment generating function which is as follows[6] :

$MX(t) = E[ext]$

$$M_X(t) = \int_0^\infty e^{tx} f(x)dx$$

$$M_X(t) = \int_0^\infty e^{tx}\ \frac{\gamma^2}{1+\gamma}(1\ +\ x)\, e^{-\gamma x}(1 - \lambda\ +\ 2\lambda\ \frac{(1+\gamma+\gamma x)}{1+\gamma}\ e^{-\gamma x})\, dx$$

$$M_X(t) = \frac{\gamma^2}{1+\gamma}\int_0^\infty e^{(t-\gamma)x}(1+x)(1-\lambda\ \ +2\lambda\frac{(1+\gamma+\gamma x)}{1+\gamma}e^{-\gamma x})\, dx \tag{16}$$

3.3. Survival Function

We find the survival function S(x) of the TLind given by the following formula:

$S(x) = 1 - F(x)$

$$S(x) = 1 - [(1 - \frac{(1+\gamma+\gamma x)}{1+\gamma}e^{-\gamma x})\ (1 + \lambda\frac{(1+\gamma+\gamma x)}{1+\gamma}e^{-\gamma x})]$$

$$S(x) = \frac{1+\gamma+\gamma x}{1+\gamma}e^{-\gamma x}\left(1-\lambda+\lambda\frac{1+\gamma+\gamma x}{1+\gamma}e^{-\gamma x}\right) \qquad x>0, \gamma>0\,, |\lambda| \le 1 \tag{17}$$

3.4. Order Statistics

Let $X_1, X_2, \ldots, X_n$ are a sample random variables with distribution function (CDF)$F_X(x)$ and (PDF)$f_X(x)$.The corresponding order statistics are the X_i arranged in ascending order. The PDF of $X_{(i)}$ is given by[1]:

$$f_{X(i)}(x) = \frac{n\,!}{(i-1)!\,(n-i)!}f_X(x)[F_X(x)]^{i-1}[1-F_X(x)]^{n-i} \tag{18}$$

The PDF of order statistics for the TLind is given by[7] :

$$f_{X(i)}(x) = \frac{n\,!}{(i-1)!\,(n-i)!}(\frac{\gamma^2}{\gamma+1}(1+x)e^{-\gamma x}(1-\lambda+2\lambda\frac{1+\gamma+\gamma x}{\gamma+1}e^{-\gamma x}))\ [(1 - \frac{1+\gamma+\gamma x}{1+\gamma}e^{-\gamma x})\ (1+\lambda\frac{1+\gamma+\gamma x}{1+\gamma}e^{-x})]^{i-1}[\frac{1+\gamma+\gamma x}{1+\gamma}e^{-\gamma x}(1-\lambda + \lambda\frac{1+\gamma+\gamma x}{1+\gamma}e^{-\gamma x})]^{n-i} \tag{19}$$

As the PDF of the largest and smallest order statistic ($X_{(n)}, X_{(1)}$) respectively, is given by:

$$f_{X(n)}(x) = \frac{n\gamma^2}{\gamma+1}(1+x)e^{-\gamma x}\left(1-\lambda+2\lambda\frac{(1+\gamma+\gamma x)}{1+\gamma}e^{-\gamma x}\right)[\left(1 - \frac{1+\gamma+\gamma x}{1+\gamma}e^{-\gamma x}\right)\left(+\lambda\frac{1+\gamma+\gamma x}{1+\gamma}e^{-\gamma x}\right)]^{\,n-1} \tag{20}$$

$$f_{X(1)}(x) = \frac{n\gamma^2}{1+\gamma}(1+x)e^{-\gamma x}(1-\lambda+2\lambda\frac{1+\gamma+\gamma x}{1+\gamma}e^{-\gamma x})[1-(1-\frac{1+\gamma+\gamma x}{1+\gamma}e^{-x}\gamma)\,(1 + \lambda\frac{1+\gamma+\gamma x}{1+\gamma}e^{-\gamma x}))]^{\,n-1} \quad (21)$$

4. Estimation Method of Transmuted Lindley Distribution:

In this paper will be used the Maximum Likelihood Method for estimating the parameters of TLind. Let $x_1, x_2, \dots, x_n$ is the random sample of size n from the TLind .Then the likelihood function write as follows [7]:

$$L = (x_i, \gamma, \lambda) = \prod_{i=1}^{n} f(x_i, \gamma, \lambda)$$

$$= \prod_{i=1}^{n} \left[\frac{\gamma^2}{\gamma+1}(1+x)e^{-\gamma x}\left(1-\lambda+2\lambda\frac{(1+\gamma+\gamma x)}{1+\gamma}e^{-\gamma x}\right)\right]$$

$$= \frac{\gamma^{2n}}{(\gamma+1)^n}e^{-\sum_{i=1}^{n}\gamma x_i}\prod_{i=1}^{n}(1+x_i)(1-\lambda+2\lambda\frac{(1+\gamma+\gamma xi)}{1+\gamma}e^{-\gamma x}i)$$

$$lnL = 2n\,ln\gamma - nln(\gamma+1) - \gamma\sum_{i=1}^{\gamma n} x_i + \sum_{=1}^{n} ln(1+x_i) + \sum_{i=1}^{n} ln(1-\lambda+2\lambda\frac{1+\gamma+\gamma x_i}{\gamma+1}\,e^{-\gamma x_i}$$

$$\frac{\partial L}{\partial \gamma} = \frac{2n}{\gamma} - \frac{n}{\gamma+1} - \sum_{i=1}^{n} x_i + \sum_{i=1}^{n}\frac{2\lambda x_i\frac{1+\gamma+\gamma x_i}{1+\gamma}\,e^{-\gamma x_i}e^{-\gamma x_i}\left[\frac{1}{(\gamma+1)^2}-\frac{1+\gamma+\gamma x_i}{1+\gamma}\right]}{1-\lambda+2\lambda\frac{1+\gamma+\gamma x_i}{1+\gamma}e^{-\gamma x_i}} = 0 \quad (22)$$

$$\frac{\partial L}{\partial \lambda} = \sum_{i=1}^{n}\frac{2\frac{1+\gamma+\gamma x_i}{\gamma+1}\,e^{-\gamma x_i}-1}{1-\lambda+2\lambda\frac{1+\gamma+\gamma x_i}{\gamma+1}e^{-\gamma x_i}} = 0 \quad (23)$$

By using numerical methods, we find the solution of nonlinear equations (22), (23). The Newton-Raphson method is employed to evaluate the estimated values of the parameters γ, λ

5. simulation Side

We conducted Monte Carlo simulation studies to assess on the finite sample behavior of the maximum likelihood and the simulations were carried out using the statistical software Matlab 2022b program. This study is repeated 1000 times each with sample sizes (25 , 50, 75, 100, 150). Default values can be specified for parameters of the Lind is (γ = 2) and TLind is (γ= 2 , λ =0.1) . Table (1) shows the estimation results as follows:

Table (1): The values of the real density function and estimated of the distribution of Lind and TLind

n	$f(x)_{Lind}$	$\hat{f}(x)_{Lind}$	MSE	$f(x)_{TLind}$	$\hat{f}(x)_{TLind}$	MSE
Parameter	$\hat{\gamma}$ =2.56335			$\hat{\gamma}$=2.23822, $\hat{\lambda}$=0.25633		
25	0.92474	0.65981	0.07019	0.89851	0.86029	0.00146
	0.73460	0.58934	0.02110	0.88173	0.84563	0.00130
	0.72927	0.58717	0.02019	0.84543	0.81357	0.00102
	0.70275	0.57618	0.01602	0.71974	0.70012	0.00038
	0.65865	0.55723	0.01029	0.69649	0.67883	0.00031
	0.56662	0.51478	0.00839	0.69580	0.67820	0.00031
	0.49406	0.47817	0.00813	0.62453	0.61245	0.00015
	0.42376	0.43956	0.00671	0.59224	0.58244	0.00010
	0.41764	0.43604	0.00269	0.53233	0.52639	0.00004
	0.17255	0.26271	0.00172	0.44151	0.44057	0.00004
IMSE			**0.23936**			**0.00021**
AIC			**-12.38310**			**-54.27134**
Best			$\hat{f}(x)_{TLind}$			
Parameter	$\hat{\gamma}$=2.46564			$\hat{\gamma}$=2.23113, $\hat{\lambda}$=0.21672		
50	0.85914	0.57241	0.08221	0.62546	0.58535	0.00161
	0.82167	0.56285	0.06699	0.54986	0.52551	0.00059
	0.81347	0.56069	0.06390	0.44910	0.44260	0.00044
	0.61330	0.50001	0.01283	0.44640	0.44033	0.00044
	0.61005	0.49889	0.01236	0.44190	0.43654	0.00044
	0.60179	0.49600	0.01196	0.44188	0.43652	0.00043
	0.54679	0.47593	0.01194	0.43398	0.42984	0.00043
	0.47636	0.44779	0.01181	0.41345	0.41238	0.00042
	0.46208	0.44171	0.01174	0.41073	0.41006	0.00042
	0.44592	0.43466	0.01146	0.39884	0.39986	0.00042
IMSE			**0.12295**			**0.00021**
AIC			**-26.75951**			**-75.61761**
Best			$\hat{f}(x)_{TLind}$			
Parameter	$\hat{\gamma}$=2.43578			$\hat{\gamma}$=2.11314, $\hat{\lambda}$=0.21146		
75	0.73124	0.56055	0.02913	0.64437	0.67575	0.00098
	0.70032	0.54975	0.02267	0.63848	0.66871	0.00091
	0.68559	0.54446	0.01992	0.63165	0.66056	0.00084
	0.67408	0.54026	0.01791	0.62700	0.65502	0.00079
	0.63865	0.52697	0.01247	0.60661	0.63084	0.00059
	0.62623	0.52218	0.01083	0.59247	0.61415	0.00047
	0.60947	0.51559	0.01044	0.56376	0.58055	0.00029
	0.60792	0.51498	0.01042	0.54947	0.56393	0.00029
	0.59529	0.50991	0.01004	0.51868	0.52840	0.00029
	0.54711	0.48985	0.00988	0.51583	0.52513	0.00028
IMSE			**0.09471**			**0.00017**
AIC			**-41.15213**			**-90.78444**
Best			$\hat{f}(x)_{TLind}$			
Parameter	$\hat{\gamma}$=2.35779			$\hat{\gamma}$=2.10694, $\hat{\lambda}$=0.15667		
100	0.70475	0.57777	0.01612	0.53566	0.55402	0.00086
	0.69550	0.57386	0.01480	0.53110	0.54827	0.00086
	0.68512	0.56942	0.01339	0.50868	0.52018	0.00085
	0.62437	0.54249	0.00834	0.49633	0.50486	0.00084
	0.62263	0.54170	0.00818	0.47764	0.48188	0.00084
	0.61222	0.53690	0.00781	0.47103	0.47380	0.00083
	0.58319	0.52325	0.00776	0.46026	0.46071	0.00083

	0.56516	0.51454	0.00707	0.45940	0.45968	0.00082
	0.55674	0.51042	0.00670	0.45799	0.45797	0.00078
	0.55403	0.50909	0.00655	0.44495	0.44226	0.00076
IMSE			**0.07909**			**0.00017**
AIC			**-55.51464**			**-141.11887**
Best			$\hat{f}(x)_{TLind}$			
Parameter	$\hat{\gamma}$=2.21157			$\hat{\gamma}$=2.10189, $\hat{\lambda}$=0.10138		
150	0.57378	0.52586	0.00746	0.47742	0.47760	**0.00000**
	0.57299	0.52545	0.00745	0.47556	0.47574	**0.00000**
	0.56240	0.51998	0.00744	0.47547	0.47565	**0.00000**
	0.55617	0.51672	0.00743	0.46693	0.46707	**0.00000**
	0.53235	0.50411	0.00738	0.45769	0.45780	**0.00000**
	0.52597	0.50067	0.00728	0.45197	0.45206	**0.00000**
	0.48959	0.48064	0.00726	0.44701	0.44708	**0.00000**
	0.48049	0.47551	0.00722	0.44577	0.44583	**0.00000**
	0.46973	0.46937	0.00722	0.42570	0.42568	**0.00000**
	0.57378	**0.52586**	**0.00746**	**0.41600**	**0.41595**	**0.00000**
IMSE			**0.06023**			**0.00001**
AIC			**-84.30144**			**-212.54533**
Best			$\hat{f}(x)_{TLind}$			

Table (1) showed that under sample sizes (25, 50,75, 100, 150) the (TLind) is the best of the Lindley distribution (Lind), this is because the IMSE and AIC for $\hat{f}(x)_{TLind}$ less than the $\hat{f}(x)_{Lind}$ for all sample sizes.

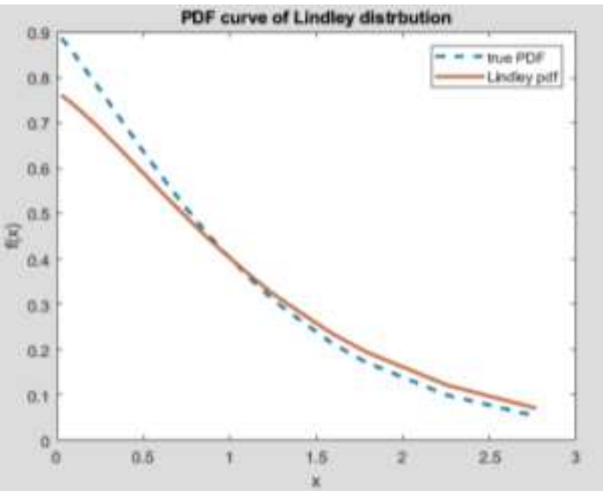

Figure (1.1) Real and Maximum Likelihood curve Lind under n=25

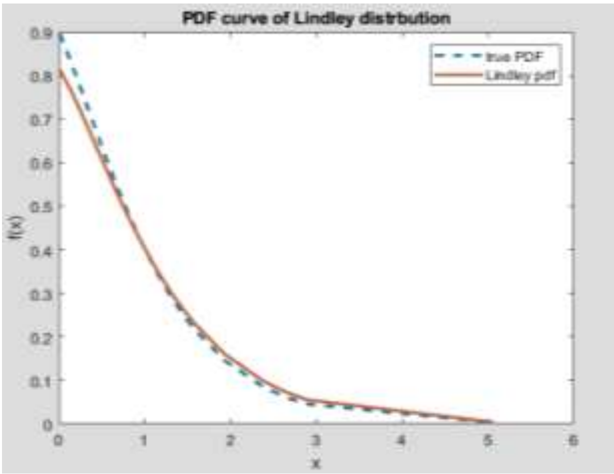

Figure (1.2) Real and Maximum Likelihood curve for Lind under n=50

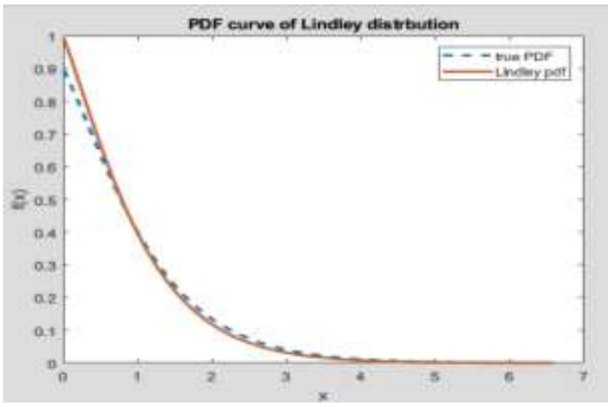

Figure (1.3) Real and Maximum Likelihood curve for Lind under n=75

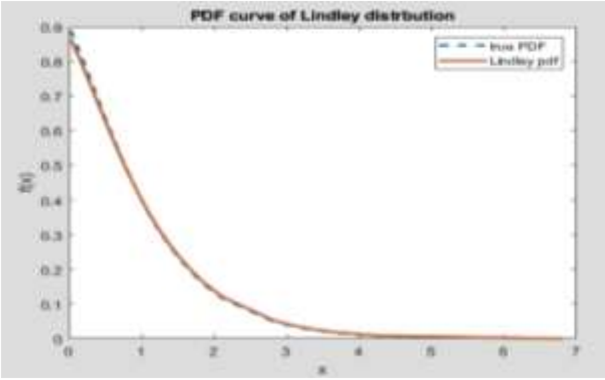

Figure (1.4) Real and Maximum Likelihood curve for Lind under

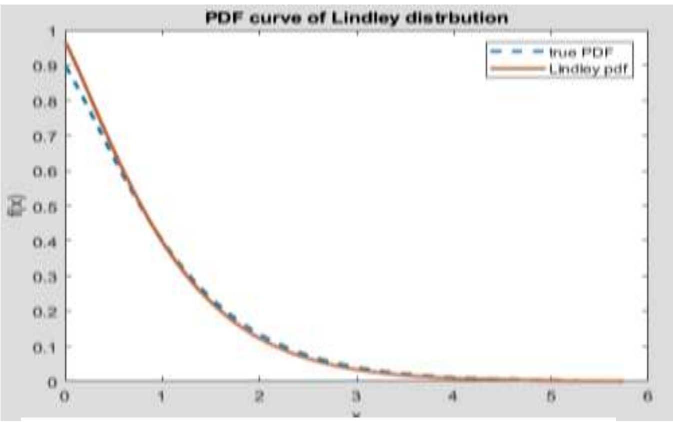

Figure (1.5) Real and Maximum Likelihood curve for Lind under n=150

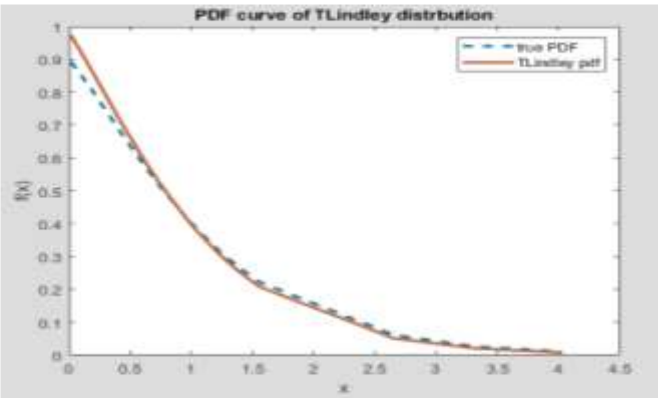

Figure (1.6) Real and Maximum Likelihood curve for TLind under n=25

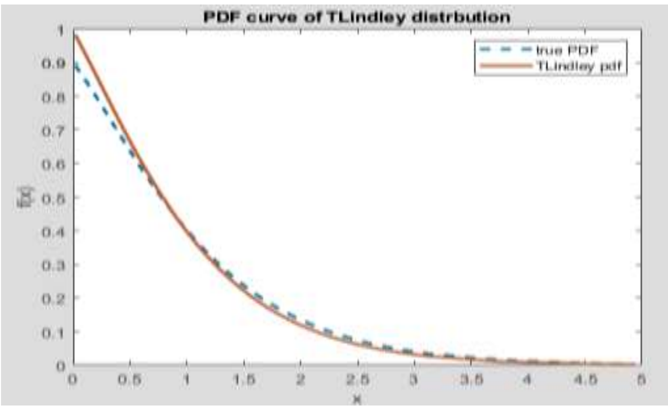

Figure (1.7) Real and Maximum Likelihood curve for TLind under

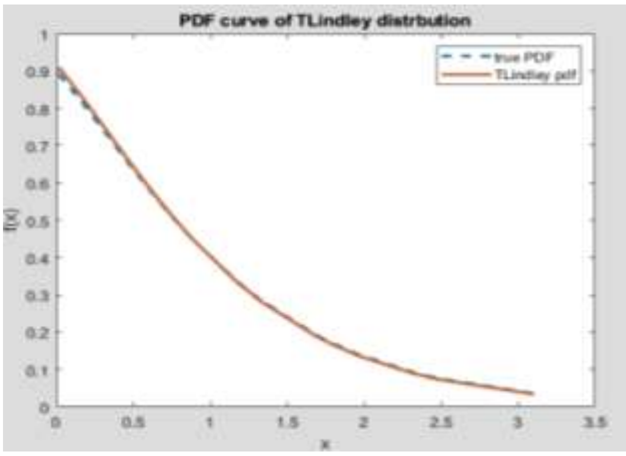

Figure (1.8) Real and Maximum Likelihood curve for TLind under

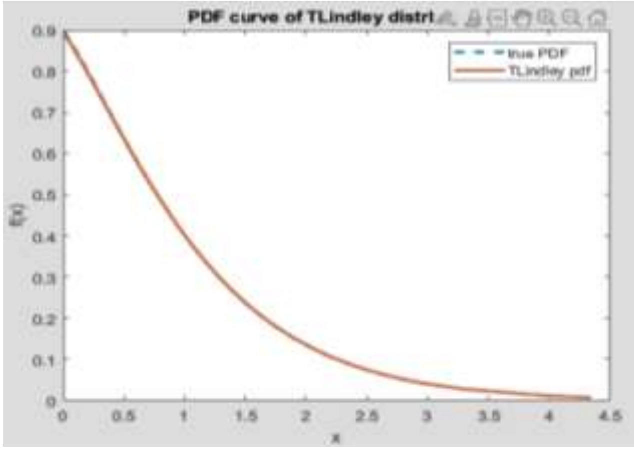

Figure (1.9) Real and Maximum Likelihood curve for TLind under

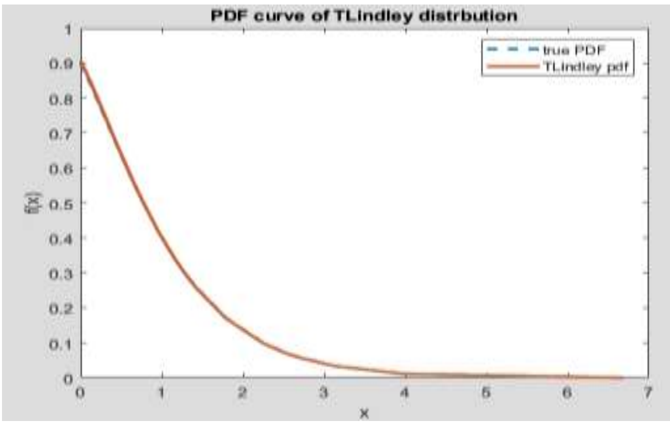

Figure (1.10) Real and Maximum Likelihood curve for TLind under n=150

6. Conclusions

In this paper , we dealt with the generating extended models from classical models was addressed through the use of the quadratic transmutation map , by adding new parameters that make these distributions more flexible and appropriate , which have proven their importance in many researches and studies . Thus, the Transmuted Lindley Distribution was used and presented mathematical properties , and the behavior of the probability density function and the cumulative distribution function of the transmuted distribution was demonstrated . We conducted a simulation experiment and the obtained results display that the Transmuted Lindley Distribution offers better appropriate than Lindley Distribution .

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نمذجة البيانات باستخدام التوزيعات المحولة

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المستخلص

تناولنا في هذا البحث توزيعتين (توزيع ليندلي و توزيع ليندلي المحول) بالإضافة إلى بعض الدوال مثل دالة PDF و CDF وناقشنا بعض الخصائص الإحصائية مثل العزوم و دالة توليد العزوم و معامل التباين والتواء والتفرطح والإحصائيات المرتبة. يتم تقدير المعلمات عبر طريقة الاحتمالية القصوى. خلص هذا البحث إلى أهمية توزيع ليندلي المتحول على توزيع ليندلي وعلى جميع أحجام العينات في جانب المحاكاة.

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