Investigation of Nuclear Energy Levels in ^{90}Sr Nucleus

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Abstract

The nuclear structure of energy levels ,electromagnetic transitions and the potential energy surface of ^{90}Sr have been investigated in terms of the interacting boson model .

In this research and from the IBM test examine closely ,nucleus has a transition behavior between the SU(5) and O(6) limits.

IBM calculations are expected many new energy levels values set with spin and parity as (3.039 MeV) for 3+ , (3.383 MeV) for 6+ ,(3.449 MeV) for 4_2+ ,(4.646 MeV) for 8+ and (4.685 MeV) for 4_3+.plus to many B(E2) transitions values

الخلاصة

تم بحث التركيب النووي لمستويات الطاقة والانتقالات الكهرومغناطيسية وجهد طاقة السطح لنواة السترونتيوم 90 من خلال نموذج البوزونات المتفاعلة .

في هذا البحث ومن اختبار نموذج البوزونات المتفاعلة, تمتلك النواة سلوك انتقالي بين التحديدين SU(5) و O(6).

حسابات نموذج البوزونات المتفاعلة توقعت قيم عدد من مستويات الطاقة الجديدة حددت بزخم وتماثل كالتالي (3.039 MeV) لـ(+ 3) و(3.383 MeV) لـ(+ 6) و (3.449 MeV) لـ (+4_2) و (4.646 MeV) لـ (+ 8) و (4.685 MeV) لـ (+4_3) بالاضافة لعدد من قيم الانتقالات الكهرومغناطيسية B(E2).

Introduction:

The quadrupole collectivity is a prominent aspect in the nuclear structure for both stable and exotic nuclei and has been extensively studied in terms of the interacting boson model (IBM) While the IBM has been successful in describing the experimental data, the parameters of its Hamiltonian are in many cases determined phenomenologically **(**K. Nomura , 2009)**.** The radionuclide ^{90}Sr is a common fission product with a half-life of 28.78 years. Strontium-90 decays via beta-decay and emits an electron with a maximum energy of 546 keV. The daughter nucleus, ^{90}Y, has a half-life of only 64.10 hours and also undergoes beta-decay with a maximum electron energy of 2282 keV. The decay of ^{90}Y produces the stable isotope^{90} Zr (C. Runkle,2005) **.** In (1997) M. Sugita , was investigated the shape transitions from A = 80 to 90 of the Sr and Zr isotopes. by applied the Interacting Boson Model with Isospin (IBM-3) to these nuclei. Using a complete diagonalization program for IBM-3 Hamiltonians,(M. Sugita,1997) **.** in our research we study the nuclear structure for ^{90}Sr isotopes in term of the interacting boson model due to experimental and theoretical data fewness in the same field .

Theory

The Interacting Boson Model (IBM):

The Interacting Boson Model (IBM) is a model for describing collective excitations in atomic nuclei. It has been introduced by (Iachello ,1987)and has been used to model a wide variety of nuclear properties and phenomena. One of the advantages of the model is its use of the symmetries of the boson operators introduced in the model, which allows for analytic expressions of the states and expectation values for three different ideal limits of nuclei.

In the IBM-1, the number of bosons is given by the number of pairs of protons and pairs of neutrons outside of closed shells. No distinction is made between proton type and neutron type bosons(Casten,1988).

Collective excitations are a common phenomena in atomic nuclei. These excitations arise from the coherent movement of many particles in the nucleus. A special class of collective excitations, called mixed-symmetry states, which are defined in the Interacting Boson Model-2, have been found in atomic nuclei and are interpreted geometrically as an out of phase motion of protons and neutrons. Together with collective excitations in which the protons and neutrons move in phase, these states can be used as building blocks for a general description of collective phenomena in nuclei. Mixed symmetry states are also sensitive to the strength of the residual proton-neutron quadrupole interaction in the valence shell and thus their properties are important in constraining the strength of this interaction(Wood,1992)

The basic foundation of the Interacting Boson Model is that collective excitations can be described with bosons. These bosons can be of two types, s and d having angular momentum of either L = 0 or L = 2 respectively. Both bosons have positive parity. The number of bosons is determined by the number of nucleon pairs or hole pairs that are outside of a closed shell. The reason for this comes from the interpretation of the bosons as correlated nucleon pairs. The total number of bosons N in the IBM is a conserved quantity. In the IBM-1, the nucleon or hole pairs must be the same type of nucleon, meaning pairs consisting of a proton and neutron are not included. The IBM-1 is applicable only to even-even nuclei. The nuclear states are represented in the framework of second quantization. The boson creation operators are given by ($s^\dagger$) and($d^\dagger_\mu$) and the boson annihilation operators by(s) and (d_μ)where ($\mu = -2,-1, 0, 1, 2$) They satisfy the following commutation relations (Ahn,2008).

$$[s, s] = [s^\dagger, s^\dagger] = 0,$$
$$[s, d_\mu] = [s^\dagger, d] = [s, d^\dagger] = [s^\dagger, d^\dagger] = 0, \qquad \ldots\ldots\ldots\ldots\ldots.. \qquad (1)$$
$$[d_\mu, d_{\mu'}] = [d^\dagger_\mu, d^\dagger_{\mu'}] = 0,$$
$$[d_\mu, d_{\mu'}] = \delta_{\mu\mu'} .$$

The multipole expansion Hamiltonian operator is given by the expression(Pfeifer ,1998)

$$H = \varepsilon\ n_d + a_0 P^\dagger P + a_1 L^\dagger L + a_2 Q^\dagger Q + a_3 T_3^\dagger T + a_4 T_4^\dagger\ T_4^\dagger \ldots\ldots\ldots\ldots(2)$$

Where ε is the boson energy, the parameters a_i's designate the strengths of the, pairing, angular momentum, quadrupole, octupole, and hexadecapole interaction between bosons respectively. With such a Hamiltonian, one is able to see more easily what the effect each multipole degree of freedom has on the nuclear states and determine which ones are the most important for a given set of nuclei.

In general not solvable analytically. Three solvable cases with SO(3) symmetry (Abrahams,1981):

$$U(6) \supset U(5) \supset SO(5) \supset SO(3)$$
$$U(6) \supset SU(3) \supset SO(3)$$
$$U(6) \supset SO(6) \supset SO(5) \supset SO(3) \ldots\ldots\ldots\ldots(3)$$

A successful nuclear model must yield a good description not only of the energy spectrum of the nucleus but also of its electromagnetic properties.

Where the operator of the quadrupole transition B(E2) can be written as(Ahn,2008).

$$T(E2)=\alpha_2[d^\dagger \times s+s^\dagger \times d]^{(2)} +\beta_2[d^\dagger \times d]^{(2)} \ldots\ldots\ \ldots\ldots\ldots\ldots\ldots\ldots(4)$$

α_2 and β_2 are two parameters, and $s^\dagger$, $d^\dagger$, s ,and d are the creation and annihilation operators of s and d bosons.

In the IBM-1, geometrical shapes can be assigned to the algebras of the three possible chains, which correspond directly to the description of nuclear shapes by Bohr and Mottlesohn's shape variables (Bohr,1952),(Bohr and Mottelson,1953) In the IBM-2, the mixed-symmetry states correspond to a quadrupole vibration where the protons and neutrons oscillate out of phase as shown in part (a) of Fig. (1) (Ahn,2008) .For deformed nuclei, the protons and neutrons oscillate with respect to one another as the nucleus as a whole rotates as shown in part (b) of Fig. (1) while the Phase diagram of IBM is shown in Fig.(2)(Jolie,2001).

For large boson number N the minimum of $V(\gamma,\beta)=\langle N;\gamma\beta | H | N;\gamma\beta\rangle$ approaches the exact ground-state energy(Casten,1988):

$$V(\beta,\gamma)\propto\begin{cases}U(5): & \dfrac{\beta^2}{1+\beta^2}\\ SU(3): & \dfrac{\beta^4-4\sqrt{2}\beta^3\cos3\gamma+8\beta^2}{8\left(1+\beta^2\right)^2}\\ SO(6): & \left(\dfrac{1-\beta^2}{1+\beta^2}\right)^2\end{cases} \qquad \ldots\ldots\ldots\ldots(5)$$

Calculation

The ^{90}Sr isotope has 38 protons and 52 neutrons are adjacent to the closed neutrons shell of N=50 .For ^{90}Sr there are six active bosons formed by five protons(particles) pairs and one neutron (hole) pair outside of closed shell (28,50) respectively .

Calculations of energy levels for even-even 90Sr isotope were performed with the complete Hamiltonian form (equation 1) using IBM-1 computer code "PHINT"(Scholten ,1990).

The parameters of equation (1) were calculated from the empirical decay schemes of this nucleus (Browen,1997)and the analytical solutions for the three dynamical systems , see (Pfeifer ,1998) These parameters were tabulated in table (1).

Table (1) Parameters of Hamiltonian equation for the even even 90Sr isotope

Nucleus	Paramrters					
	Eps	P.P	L.L	Q.Q	T3.T3	T4.T4
90Sr	1.895	0.430	0.0005	0.0	0.0051	0.0

Then the output energy levels were drawn in figure (3) with a comparison with the corresponding empirical levels taken from reference(Browen,1997).

The parameters of equation (4) were calculated using the empirical B(E2) value(RAMAN,1993) of the transition $2_1^+\rightarrow0_1^+$ and table (2) shows these values for the three isotopes. Where

$$E2SD=\alpha_2, E2DD=\sqrt{5}\beta_2 \text{ And in SU(5)}, \quad \beta_2=\frac{-0.7}{5}\alpha_2, \beta_2=-\sqrt{7/2}\,\alpha_2 \, and \beta=0$$

SU(3) and O(6) respectively. see table(3)

Table (2) Parameters of B(E2) equation for the even- even ^{90} Sr isotopes .

Nucleus	B(E2)	E2SD	E2DD
^{90}Sr	0.06	0.1	-0.03

Table (3) Comparison between present values (Theo) of B(E2) for even-even ^{90}Sr and empirical one (Exp.) (Browen,1997) isotope

Transition	^{90}Sr	
	Exp.	Theo.
$2_1^+\rightarrow0_1^+$	0.6	0.06
$2_2^+\rightarrow0_1^+$	-	0.0
$0_2^+\rightarrow2_1^+$	-	0.1
$2_2^+\rightarrow2_1^+$	-	0.1
$4_1^+\rightarrow2_1^+$	-	0.1
$0_3^+\rightarrow2_1^+$	-	0.0015
$2_3^+\rightarrow2_2^+$	-	0.022
$2_3^+\rightarrow0_1^+$	-	0.0
$2_3^+\rightarrow2_1^+$	-	0.0
$4_2^+\rightarrow2_3^+$	-	0.0008
$Q2_1^+$	-	-0.0671

The parameters of the energy surface were calculated by transforming the parameters of Hamiltonian of equation 1 by several equations see (Iachello ,1987), and they found to be as in table (4)

Table (4) Parameters of energy surface for the even- even ^{90}Sr isotopes

	ES	ED	A_1	A_2	A_3	A_4
^{90}Sr	0.0	1.905	0.108	0.0	0.0	0.0

Then the contour plots in the γ-β plane for even- even ^{90}Sr ,see fig.(4) and it is sufficient to take γ between 0^o and 60^o.

Discussion and conclusion

The nuclear structure of energy levels ,electromagnetic transitions and the potential energy surface of ^{90}Sr have been investigated in terms of the interacting boson model .

The information of this nucleus was very insufficient due to the intensified use to it as a radiation source ,then no many theoretical and experimental studies about it .In our research and from the IBM test examine closely ,nucleus has a transition behavior between the SU(5) and O(6) limits. the experimental and calculated ratio values $E^+_{2\gamma}/E^+_2$, E^+_4/E^+_2 & E^+_6/E^+_2 occur between them, see table (5).

IBM calculations are expected many new energy levels values explain in fig.(3) as (3.039 MeV) for 3+ , (3.383 MeV) for 6+ ,(3.449 MeV) for 4_2+ ,(4.646 MeV) for 8+ and (4.685 MeV) for 4_3+.plus to many B(E2) transitions values. Figs.(4) was drawn to show isometric potential lines E(β,γ) for ^{90} Sr isotope to study the potential energy surface which represented nuclear potential energy as a function of collective coordination's for coefficients nuclear shape β,γ are helpful to realizing principal conceptions of nuclear structure for that nucleus.

In ^{90}Sr the potential surface is slightly different from that of a spherical vibrator which minimum at β=0 and have circular contours centered at this point .Neither do the contours resemble those of a SU(5)&O(6) models potential since the minimum potential occurs approximately at β=0.2 which lei between β=0 for SU(5) and β=1 for O(6) see fig.(2) ,which make clear the similarity with transitions region as will as the $Q2_1$+ value

Table (5): the ratios of the energy of the level 2γ, 4_1 and 6_1 to the level 2_1 for ^{90}Sr isotope

		$E^+_{2\gamma}/E^+_2$	E^+_4/E^+_2	E^+_6/E^+_2
^{90}Sr	Exp.	2.27	2	-
	Theo.	2.26	2.26	3.7

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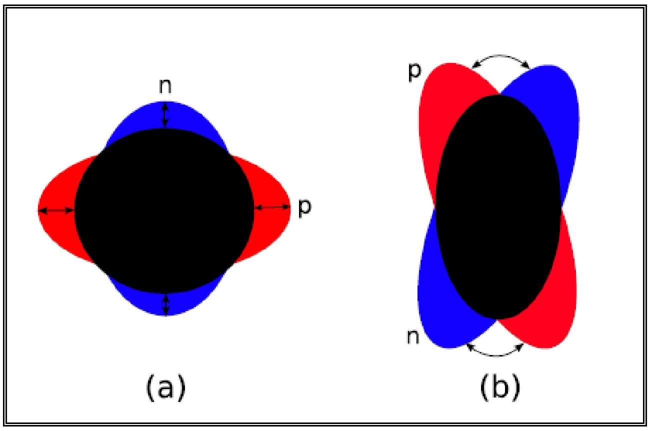

Fig.1: Geometric interpretation of mixed-symmetry states are shown.The figure represents a snapshot of the nucleus in time where the red indicates the proton fluid and the blue represents the neutron fluid. Part (a) represents the out of phase vibration for spherical nuclei and (b) represents the vibration of protons and neutrons with respect to each other for prolate or oblate deformed nuclei (Ahn,2008).

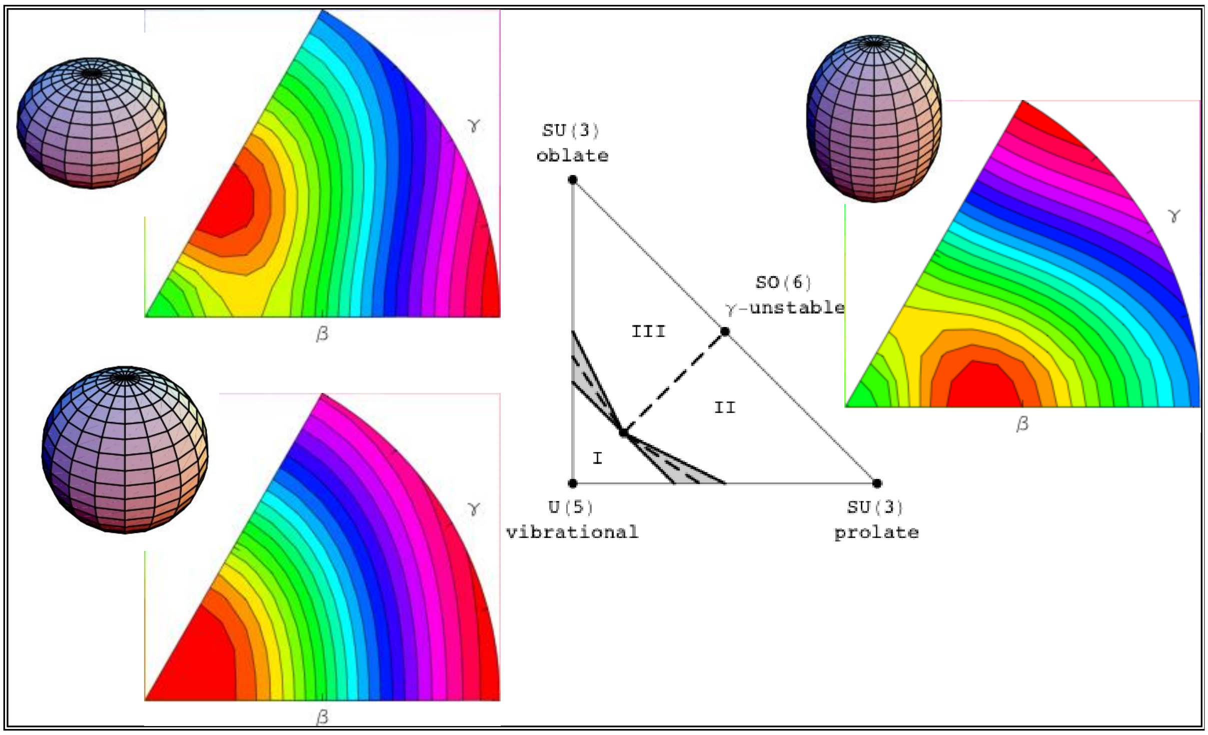

Fig.(2) :Phase diagram of IBM (Jolie,2001).

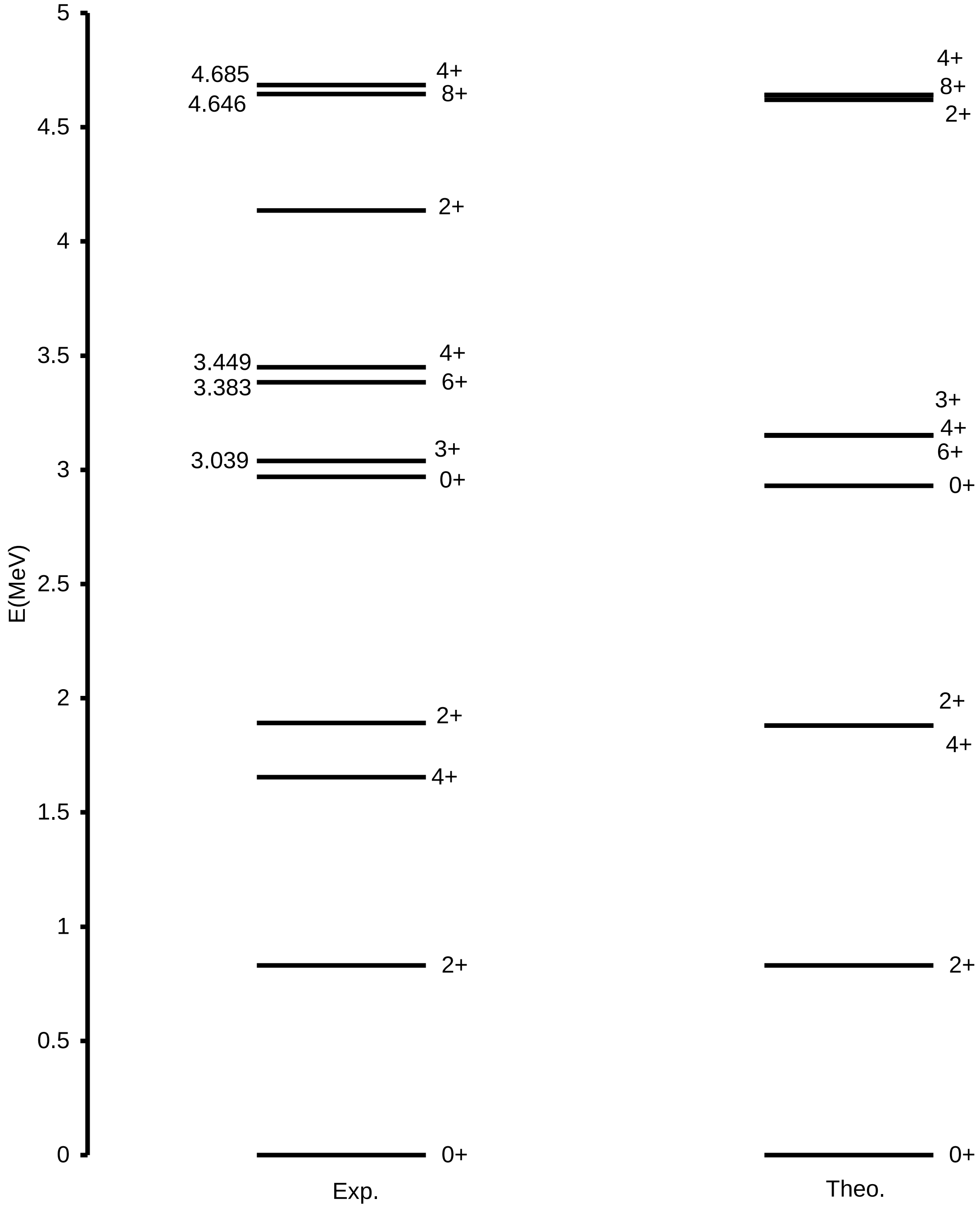

Fig. (3): A comparison between theoretical values of energy levels and the corresponding experimental one for ^{90}Sr (Browen,1997).

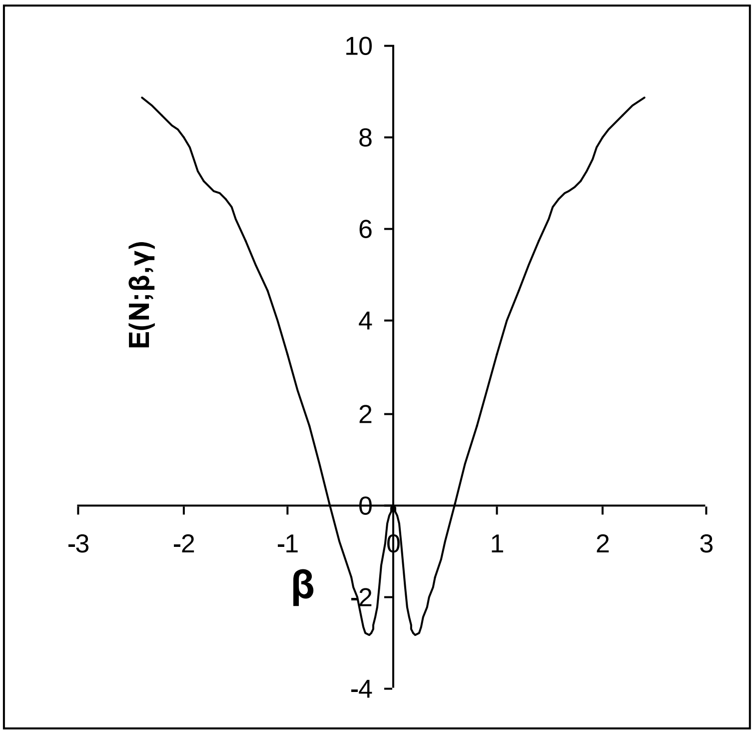

Fig .(4-a):The corresponding β-γ plot for [90] Sr isotope.

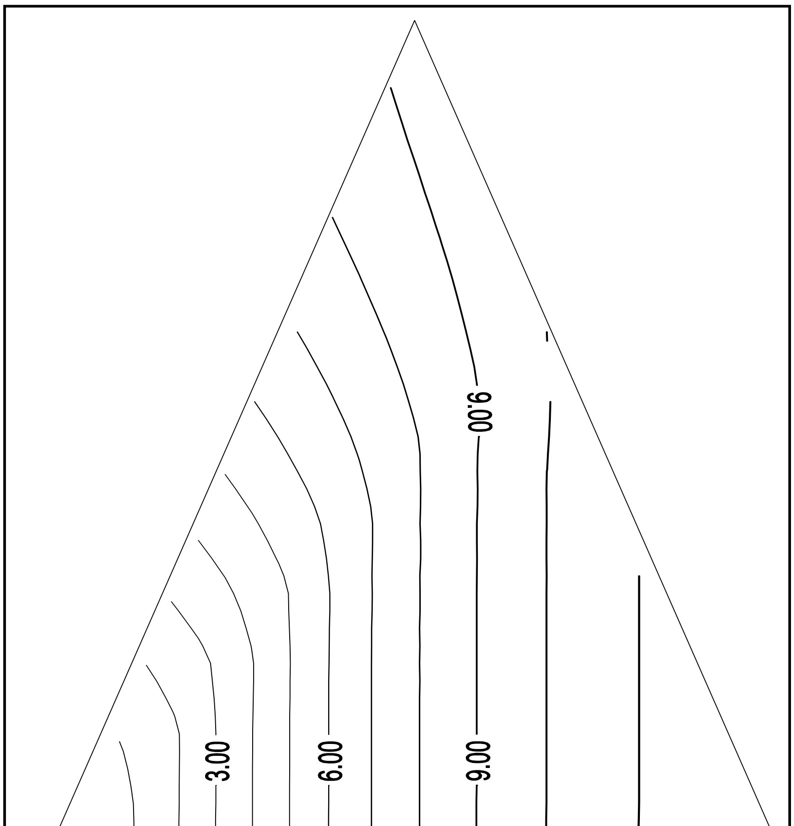

Fig .(4-b):The energy functional E(N; β,γ) as a function of β for [90] Sr isotope.