Long-Term Deflections for Normal and High Strength Reinforced Concrete Beams

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Abstract

The study aims to present new form to calculate the long-term deflections (time dependent) for reinforced concrete beams (normal & high strength).The presented form taking into account the effect of several factors such compressive strength of concrete, the reinforcement at compressive zone, cross section dimensions, and span length. The results of the presented form were compared with experimental results of other researchers, and a good agreement was obtained. Among the conclusions drawn, the long-term deflections are highly reduced by increasing the compressive strength of concrete (the long term deflection reduced about 50% for compressive strength increased from 20 to 100 MPa). Additionally, the reinforcement at compressive zone is less benefit in decrease the long-term deflection when use the high strength concrete.

Key ward: creep; shrinkage; reinforced concrete; long term; deflection.

الخلاصة

تهدف هذه الدراسة إلى تقديم صيغة جديدة لحساب الانحرافات طويلة الأمد (المعتمدة على الزمن) للعتبات الخرسانية المسلحة(ذات المقاومة الاعتيادية وذات المقاومة العالية).الصيغة المقدمة تأخذ بنظر الاعتبار تأثير عدة عوامل مثل مقاومة الانضغاط للخرسانة ، حديد التسليح في منطقة الانضغاط ، إبعاد المقطع وكذلك طول الفضاء. نتائج الصيغة المقدمة قورنت مع نتائج عملية لباحثين آخرين، ووجد تقارب جيد معها. من بين نتائج هذه الدراسة أن الانحرافات طويلة الأمد تقل بزيادة مقاومة الانضغاط للخرسانة (تقل بنسبة حوالي 50% بزيادة مقاومة الانضغاط من 20-100 نت/مم^2) كما أظهرت النتائج أيضا إن حديد التسليح في منطقة الانضغاط يكون اقل فائدة في التقليل من الانحرافات طويلة الأمد عند استعمال خرسانة عالية المقاومة.

Introductions

In recent years, high strength concrete (HSC) has found many applications not only in columns of high-rise buildings but also in abroad range of long span flexural members**(ACI Committee 363, 1984)**. The use of high strength concrete leads to slender members with a reduction in dead load. However, the decrease in member dimensions may create the serviceability problem of excessive deflection due to the reduced stiffness. It is, therefore, important that while maintaining the strength requirements and self- weight reduction, apropos measures are taken in the design to check and control deflections of high strength concrete members under service loads. For reinforced flexural members, immediate (short –term) and time-dependent (long-term) deflections are of significance **(ACI Committee 435, 1990)**.

Time-dependent deflection of reinforced concrete members involves a complicated interaction of many factors including cracking, creep, shrinkage, and loading history. Uncertainties in material properties and loading exacerbate the problem further and make prediction of deflection a difficult task at the design stage. Nevertheless, engineers need to design structures that perform under service loads in a manner that satisfies serviceability requirements of the structure by providing an acceptable level of deflection control. To aid the engineer, a methodology or basis for the calculation is usually provided. It is not necessary for the calculated deflection to precisely match the deflection that actually occurs in the field because it is recognized that this is not possible due to the uncertainties involved. Nevertheless, the calculation procedure should take into account the most important factors affecting deflection to give reasonable results and make the deflection calculation meaningful **(Scanlon *et al*., 2008).**

For computation of long-term deflections, **ACI 318-08** recommends a simplified procedure, by multiplying the short-term deflections by a 'multiplier':

$$\lambda = \frac{\xi}{1 + 50\rho'} \tag{1}$$

where, ξ ≡ time dependent factor for sustained loads having values equal to 1.4, 1.2, and 1 for 12 months, 6 months and 3 months respectively and 2 for a period of 5 years or more.

This method of predicting deflection are generally applicable to normal strength concrete of 21 to 42 MPa .The aptness of these methods to high strength concrete flexure members has not yet been fully established.(**ACI Committee 363, 1987**)

Paulson *et al*. (1991) showed that creep coefficient and deflection of high strength concrete beams are less than for those of similar beams of normal strength concrete. The influence of compression reinforcement on time dependent deflections is also noted to be less significant for high strength concrete beams than for normal strength concrete beams.

Based on experimental information, it appears that with its characteristic low creep coefficient, high strength concrete unfairly penalized by current ACI building code provisions that greatly over predict time dependent deflections of high strength concrete beams. The research herein was directed toward development a practical equation for predicting long-term deflections for reinforced concrete beams with any concrete strength.

The model proposed in the present study for the long-term deflection has the following form:

$$\Delta_t = \Delta_i + \Delta_l \tag{2}$$

$$\Delta_l = \lambda . \Delta i \tag{3}$$

$$\lambda_P = 2.7 \times \alpha_1 \times \alpha_2 \times ccu \times \frac{T^{0.3}}{10 + T^{0.3}} \tag{4}$$

$$\alpha_1 = 0.7 + \left(\frac{L}{13 * P} \right)^2 \geq 1.0 \tag{5}$$

$$\alpha_2 = \frac{1}{1 + \left(\frac{16}{fc'} \times \frac{\rho'}{\rho} \right)} \tag{6}$$

Where

b= Cross section width.

ccu :Ultimate creep coefficient and can be found From Table(1) for different compressive strength of concrete

fc' : Compressive strength of concrete

H= Cross section depth.

L: Span length

P: Perimeter of the cross section
T= Time of loading in months
α_1 = Factor to take the effect of (span to perimeter ratio)
α_2 = Factor to take the effect of (reinforcement at compressive zone)
Δi :Instantaneous deflection
Δ_l :Long-term deflection
Δ_t : Total deflection
λ = Long-term multiplication factor(dimensionless)
λ_P = Proposed long-term multiplication factor(dimensionless)
ρ' = Ratio of steel reinforcement at compression zone.
ρ = Ratio of steel reinforcement at tension zone.

Table (1): Typical creep parameters(Nilson *et al*, 2004)

Compressive strength fc' (MPa)	Creep coefficient *ccu*
21	**3.1**
28	**2.9**
41	**2.4**
55	**2.0**
69	**1.6**
83	**1.4**

From table (1) and by using the curve fitting, the ultimate creep coefficient can be represented as a function of the concrete compressive strength as follows:-

$$ccu = 4.1 - 0.005 \times fc' + 0.00022 \times fc'^2 \qquad \textbf{(7)}$$

Experimental Calibrations

The validity of the proposed model is demonstrated as follows by comparison with experimental results for reinforced concrete beams of different sizes, spans, compressive reinforcement, and compressive strength of concrete reported by **Washa and Fluck (1952), Corley and Sozen(1966)**, **Hajnal *et al*.(1963)**,and **Paulson *et al*,(1991)**.

1. Reinforced concrete simply supported beams tested by Washa and Fluck.

Washa and Fluck (1952) measured deflections during 2.5 years of sustained loading on 34 beams with different sizes, spans, and reinforcement. All the beams were simply supported and subjected to uniform load with concrete blocks and bricks. The dimensions and material properties of the tested beams are listed in Table. (2). Table (3) compare the measured long-term multiplication factors after 2.5 years of sustained loading with values determined by the present form and by ACI 318-08 Formula;

Table (2): Dimensions and material properties of reinforced concrete beam tested by **Washa & Fluck(1952)**

Beam No.	fc' MPa	b mm	h mm	As mm^2	d mm	As' mm^2	d' mm	L mm	w kN/m
A1/4	25.9	203	305	855	257	855	48	6096	5.52
A2/5	25.9	203	305	855	257	396	48	6096	5.63
A3/6	25.9	203	305	855	257	0	-----	6096	5.63
B1,B4	20.8	152	203	396	157	396	46	6096	1.59
B2,B5	20.8	152	203	396	157	198	46	6096	1.59
B3,B6	20.8	152	203	396	157	0	-----	6096	1.59
C1,C4	20.3	305	127	506	102	506	26	6340	1.2
C2,C5	20.3	305	127	506	102	253	26	6340	1.2
C3,C6	20.3	305	127	506	102	0	-----	6340	1.2
D1,D4	20.1	305	127	506	108	506	19	3810	1.2
D2,D5	20.1	305	127	506	108	253	19	3810	1.2
D3,D6	22.2	305	127	506	108	0	-----	3810	1.2
E1,E4	20.6	305	76	285	59	285	18	5334	1.2
E2,E5	20.6	305	76	285	59	143	18	5334	1.2
E3,E6	20.6	305	76	285	59	0	-----	5334	1.2

Table (3): Measured and predicted short &long-term deflections for experiments of **Washa & Fluck**.

Beam No.	experimental			λ ACI Eq.(1)	λ_P Present study Eq.(4)
	Δi (mm)	Δ_t (mm)	λ		
A1/4	13.5	23.6	0.75	0.97	1.07
A2/5	15.7	32.3	1.06	1.27	1.35
A3/6	17.0	44.7	1.63	1.76	1.73
B1,B4	23.4	51.1	1.18	0.96	1.19
B2,B5	24.9	65.0	1.61	1.24	1.52
B3,B6	26.4	86.4	2.27	1.76	2.10
C1,C4	40.1	80.0	1.00	0.97	1.06
C2,C5	43.4	100.6	1.32	1.25	1.36
C3,C6	47.8	140.7	1.94	1.76	1.90
D1,D4	11.9	27.7	1.33	0.99	1.06
D2,D5	14.2	33.8	1.38	1.27	1.34
D3,D6	17.8	48.5	1.72	1.76	1.82
E1,E4	59.4	124.0	1.09	0.98	1.05
E2,E5	55.9	128.8	1.30	1.26	1.34
E3,E6	63.0	184.9	1.93	1.76	1.86

2. Reinforced concrete simply supported beams tested by Corley &Sozen.

Corly & Sozen (1966) measured deflections during 700 days of sustained loading; the beams were simply supported and subjected to two point load. The dimensions and material properties of the tested beams are listed in Table. (4). Table

(5) shows a comparison with the experimental and the numerical results for the short and long-term deflection at mid span.

Table (4): Dimensions and material properties of reinforced concrete beam tested by **Corley &Sozen(1966)**

Beam No.	fc' MPa	b mm	h mm	As mm^2	d mm	As' mm^2	d' mm	L mm	P kN
C1	24	76	153	143	136	-------	-------	1830	5
C3	24	76	110	143	91.5	-------	-------	1830	5
C4	24	76	110	214	91.5	-------	-------	1830	5

Table (5): Measured and predicted short &long-term deflections for experiments of **Corley & Sozen**

Beam No.	experimental			λ ACI Eq.(1)	λ_P Present study Eq.(4)
	Δi (mm)	Δ_t (mm)	λ		
C1	3.0	7.4	1.47	1.67	1.68
C3	7.9	17.3	1.19	1.67	1.68
C4	6.1	15.5	1.54	1.67	1.68

Table (6): Dimensions and material properties of reinforced concrete beam tested **by Hajnal *et al*. (1963).**

3. Reinforced concrete simply supported beams tested by Hajnal etal.

Hajnal *et al*. (1963) measured deflections during (4.75) years of sustained loading; the beams were simply supported with different span and subjected to mid span concentrated load. The dimensions and material properties of the tested beams are listed in Table. (6). Table (7) shows a comparison with the experimental and the numerical results for the short and long-term deflection at mid span.

Beam No.	fc' MPa	b mm	h mm	As mm^2	d mm	As' mm^2	d' mm	L mm	P kN
L2,L8	24.5	130	191	143	153	-------	-------	6400	
L4,L10	24.5	130	191	143	153	-------	-------	4800	
L6,L12	24.5	130	191	143	153	-------	-------	3200	

4. Reinforced concrete simply supported beams tested by Paulson et al.

Paulson *et al*. (1991) measured deflections during 1 year years of sustained loading on simple supported beams with different compressive strength of concrete. The dimensions and material properties of the tested beams are listed in Table. (8).Table (9) shows a comparison with the experimental and the numerical results for the short and long-term deflection at mid span.

Table (7): Measured and predicted short &long-term deflections of **Paulson *et al*.**

Beam No.	experimental			λ_{ACI} Eq.(1)	λ_P Present study Eq.(4)
	Δi (mm)	Δ_t (mm)	λ		
A0	31	47.8	0.54	1.4	0.66
A1	29	44.5	0.53	1.02	0.66
A2	30	44	0.47	0.8	0.56
B1	36.1	61.2	0.70	1.4	0.83
B2	35.1	53.6	0.53	1.02	0.74
B3	31.5	48.3	0.53	0.8	0.67
C0	34	58.2	0.71	1.4	1.21
C1	32.5	50.8	0.56	1.02	1.0
C2	32	47.4	0.44	0.8	0.685

Table (8): Dimensions and material properties of reinforced concrete beam tested by **Paulson *et al*. (1991).**

Beam No.	fc' MPa	b mm	h mm	As mm^2	d mm	As' mm^2	d' mm	L mm	w^* kN/m
A0	90	127	254	400	210	0	44.5	5486	4.67
A1	90	127	254	400	210	200	44.5	5486	4.67
A2	90	127	254	400	210	400	44.5	5486	4.67
B1	66	127	254	400	210	0	44.5	5486	4.67
B2	66	127	254	400	210	200	44.5	5486	4.67
B3	66	127	254	400	210	400	44.5	5486	4.67
C0	37	127	254	400	210	0	44.5	5486	4.67
C1	37	127	254	400	210	200	44.5	5486	4.67
C2	37	127	254	400	210	400	44.5	5486	4.67

Table (7): Measured and predicted short &long-term deflections of **Hajnal *et al*.**

Beam No.	experimental			λ_{ACI} Eq.(1)	λ_P Present study Eq.(4)
	Δi (mm)	Δ_t (mm)	λ		
L2,L8	19.3	44.1	2.29	1.95	2.64
L4,L10	8.9	23.0	2.58	1.95	2.11
L6,L12	4.4	9.7	2.20	1.95	2.05

From these tables it can be noted that:

- The comparison includes the results of long term deflection for simply supported reinforced concrete beams with different (sizes, spans, compressive strength of concrete and compressive /tensile reinforcement).
- Good agreement between the results of the proposed formula and the results obtained from experimental tests by the other researchers.
- The result of ACI code model doesn't take into account several important factors such compressive strength of concrete and the (span / perimeter of cross section) which are very important factor affected on the long term deflection.
- The long term deflection greatly affected by the compressive strength of concrete (reduced by increasing the compressive strength of concrete), this can be attributed to low creep and coefficients of the high strength concrete.
- The compressive reinforcement reduced the long term deflection at all time of loading.
- The effect of compressive reinforcement in reducing the long term deflection is degreased with increased the compressive strength of concrete.
- The long term deflection also effected by the ratio of the span / perimeter of the cross section, the long term deflection increased by increase span / perimeter.

<u>Parametric Study</u>

1- Effect of compressive strength of concrete on the long-term deflection.

A simply supported rectangular cross section beam was analyzed with a range of compressive strength of concrete from (20-100 MPa). The properties of the beam are shown in figure 1

L=2500
b=200 mm
H=200 mm
f_y=413 **MPa**,

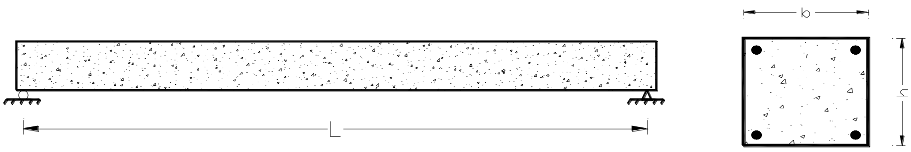
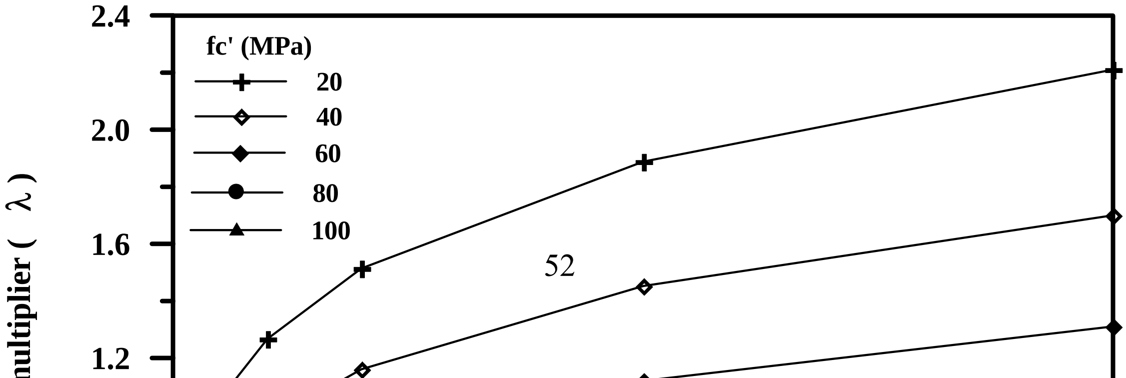


Fig. (1) Dimensions and material properties of the studied beam

Figures (2 & 3) show the effect of compressive strength of concrete on the long term multiplier.



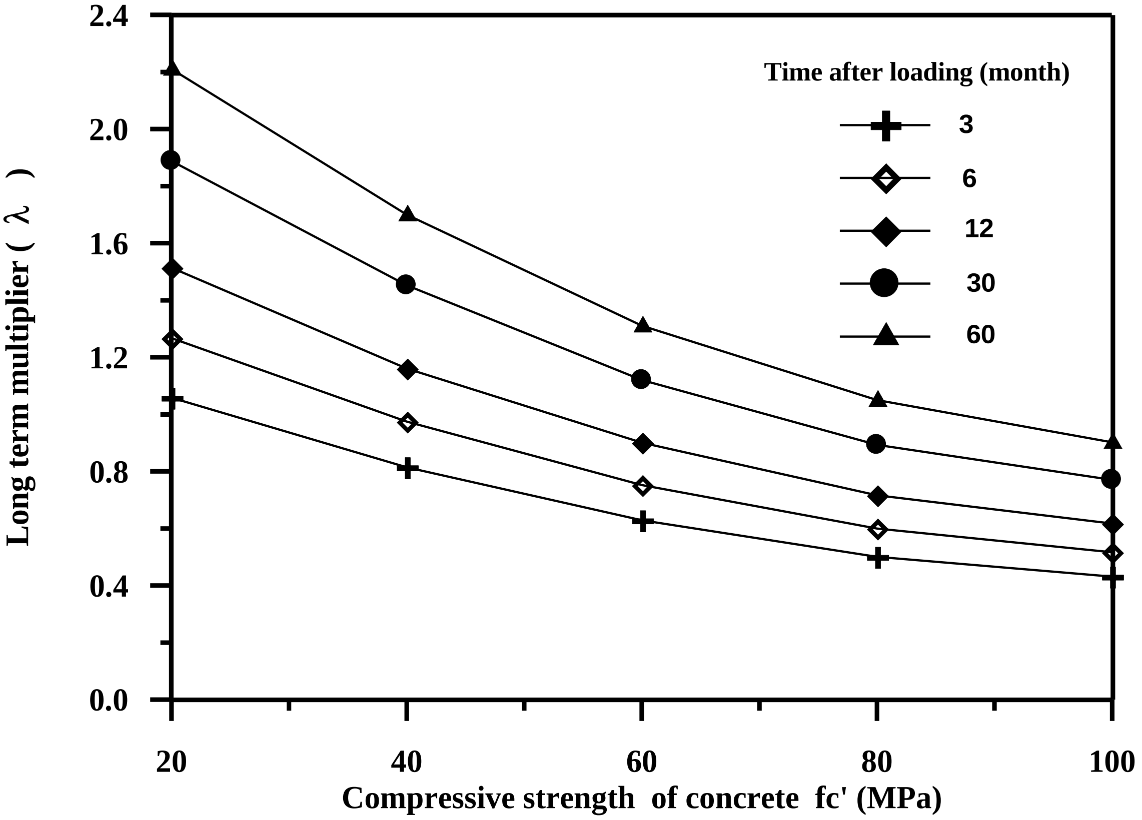


Fig.(3) The relation between long term multiplier with compressive strength of concrete for different time of loading (3months to 5years).

From these figures can be noted that, the increase of compressive strength of concrete lead to decrease in the long term deflection (the long term deflection is reduced by about 50% by increase the compressive strength of concrete from 20 to 100 MPa) (this can be attributed to the low creep coefficient for high strength concrete in compared for that of normal strength concrete) (**Paulson *et al*. 1991)**.

2- Effect the ratio of (compressive/tensile) reinforcement on the long-term deflection and its relation with compressive strength of concrete

A beam with same dimension and material properties as shown in figure 1 is used to study the effect of compressive reinforcement on the long term deflection for different compressive strength of concrete (20-100) MPa. The ratio of compressive /tensile reinforcement is ranged from (0 to 1).

Figures (4 to 13) show the effect of compressive/tensile reinforcement ratio on the long term deflection for different compressive strength of concrete and at different time of loading.

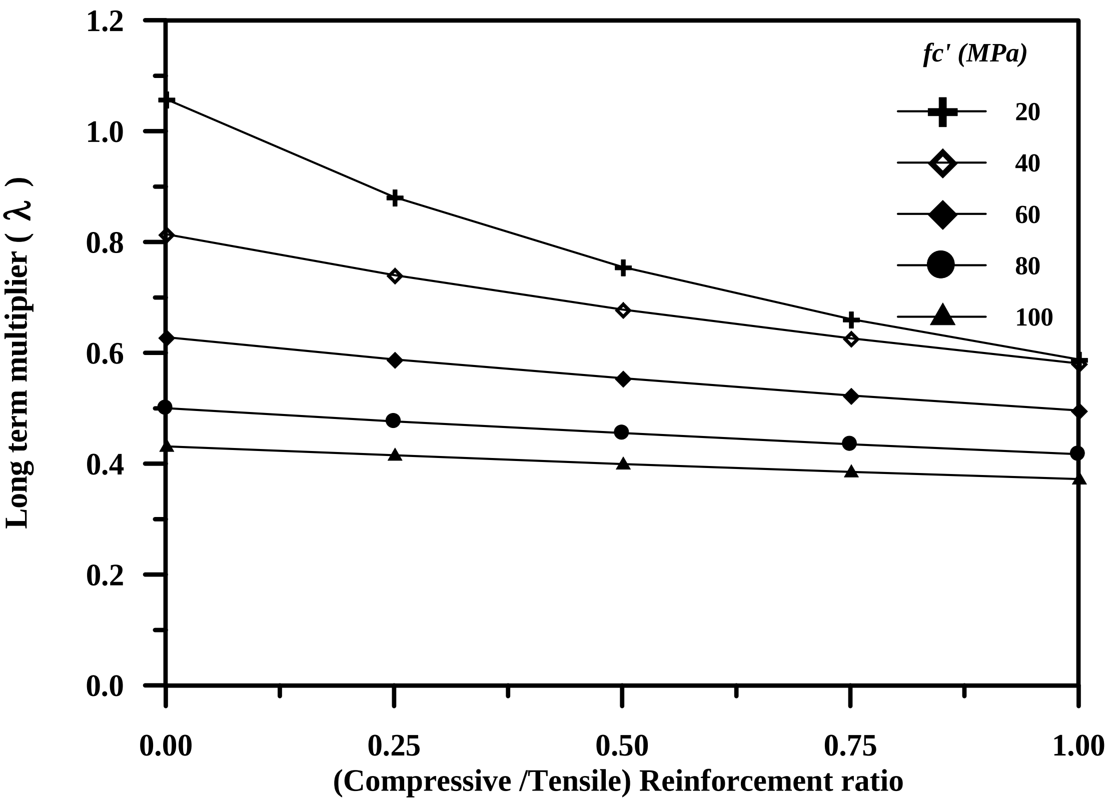


Fig. (4) The relation between long term multiplier with the ratio of compressive/tensile reinforcement for different compressive strength of concrete (after 3 months of loading).

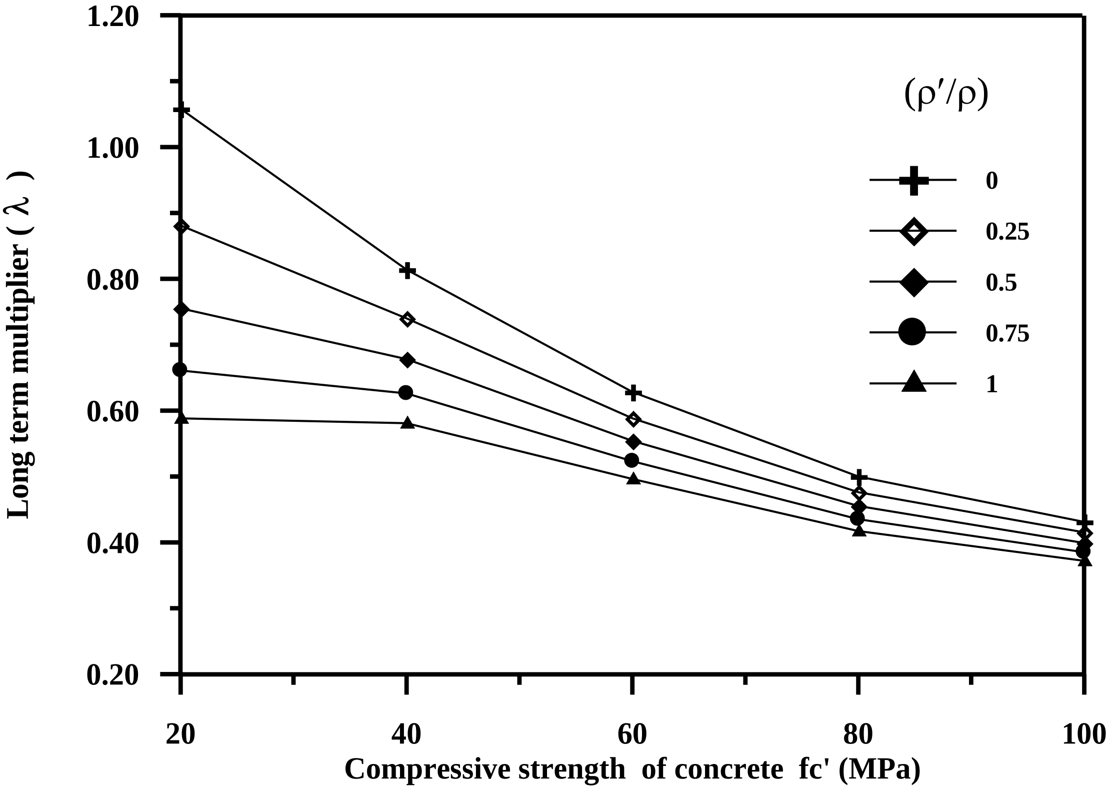


Fig. (5) The relation between long term multiplier with compressive strength of concrete for different ratio of compressive /tensile reinforcement (after 3 months of loading).

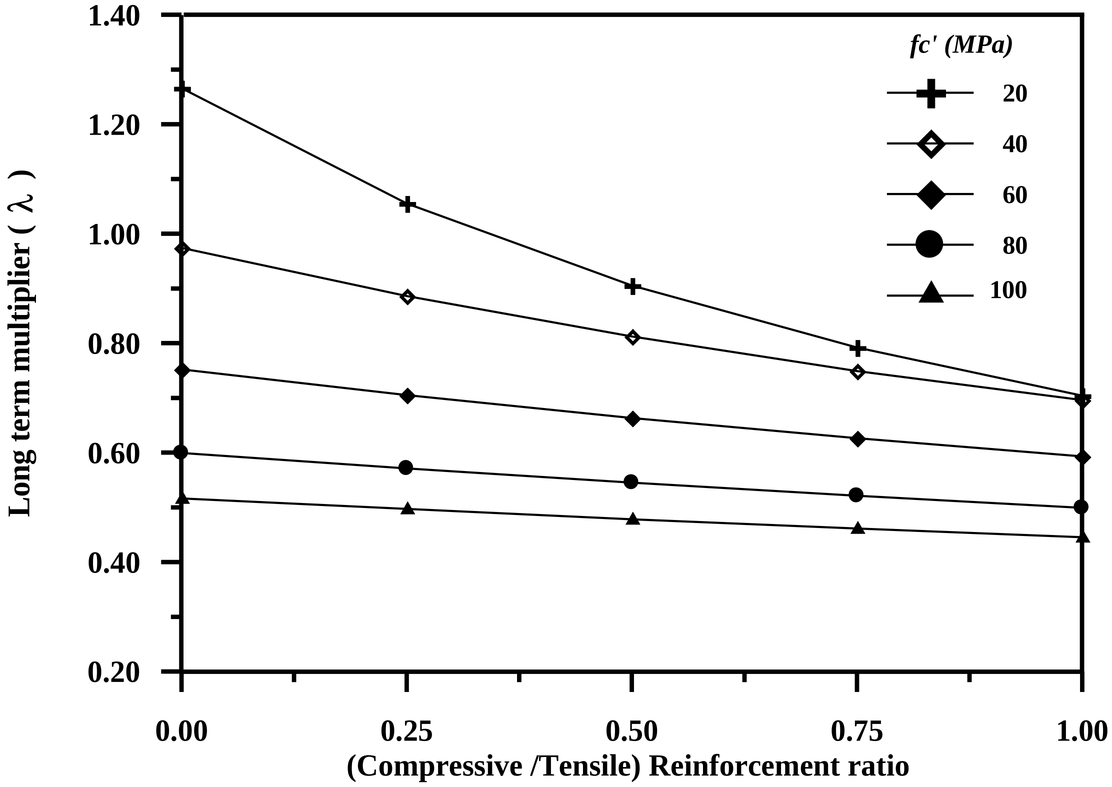


Fig. (6) The relation between long term multiplier with the ratio of compressive /tensile reinforcement for different compressive strength of concrete (after 6 months of loading)

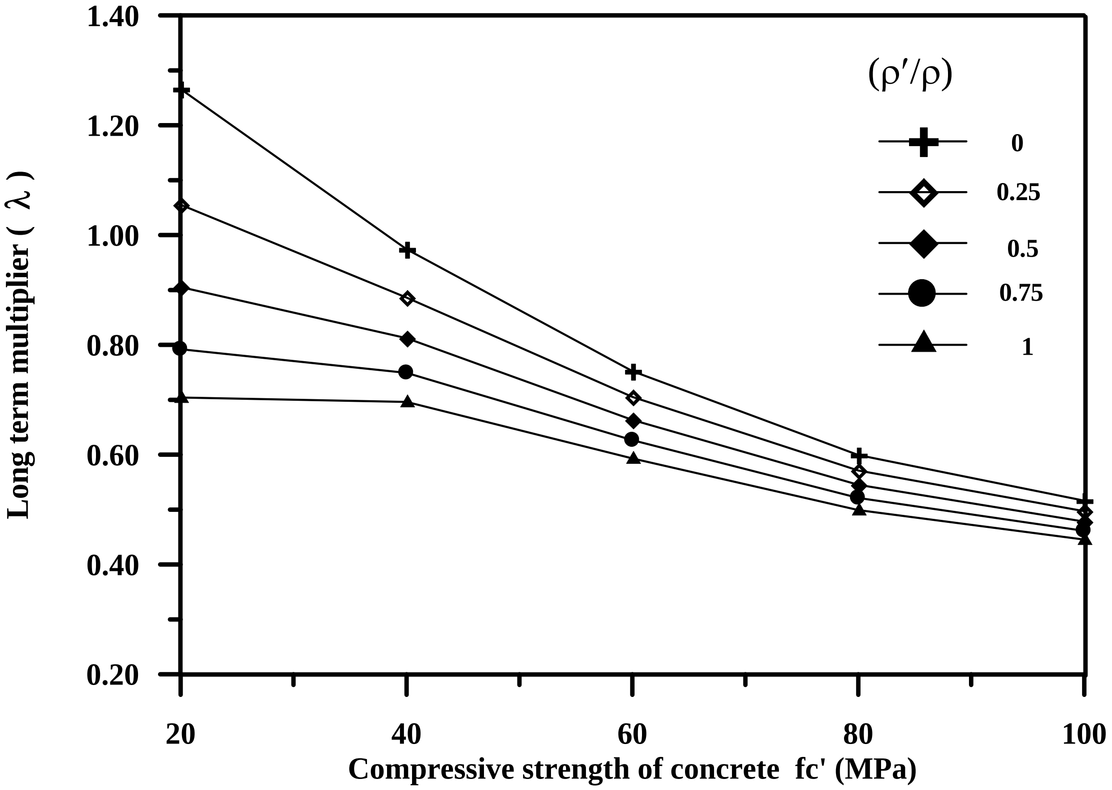


Fig. (7) The relation between long term multiplier with compressive strength of concrete for different ratio of compressive /tensile reinforcement (after 6 months of loading)

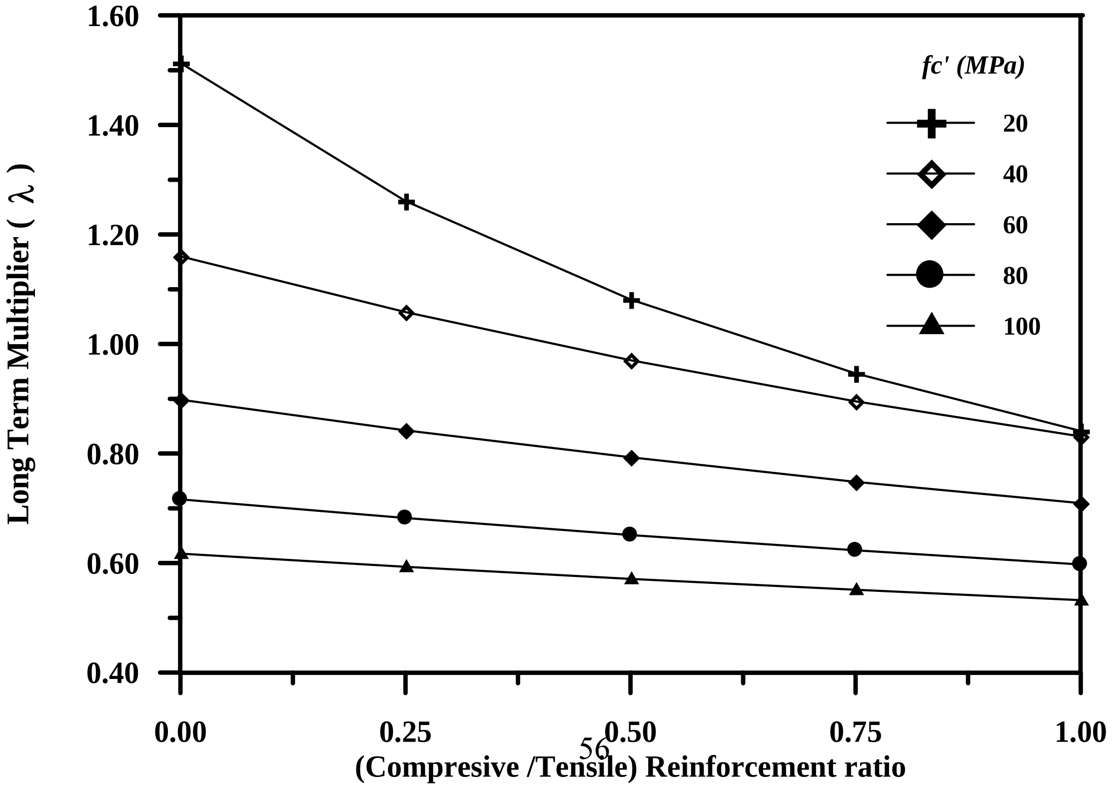


Fig. (8) The relation between long term multiplier with the ratio of compressive /tensile

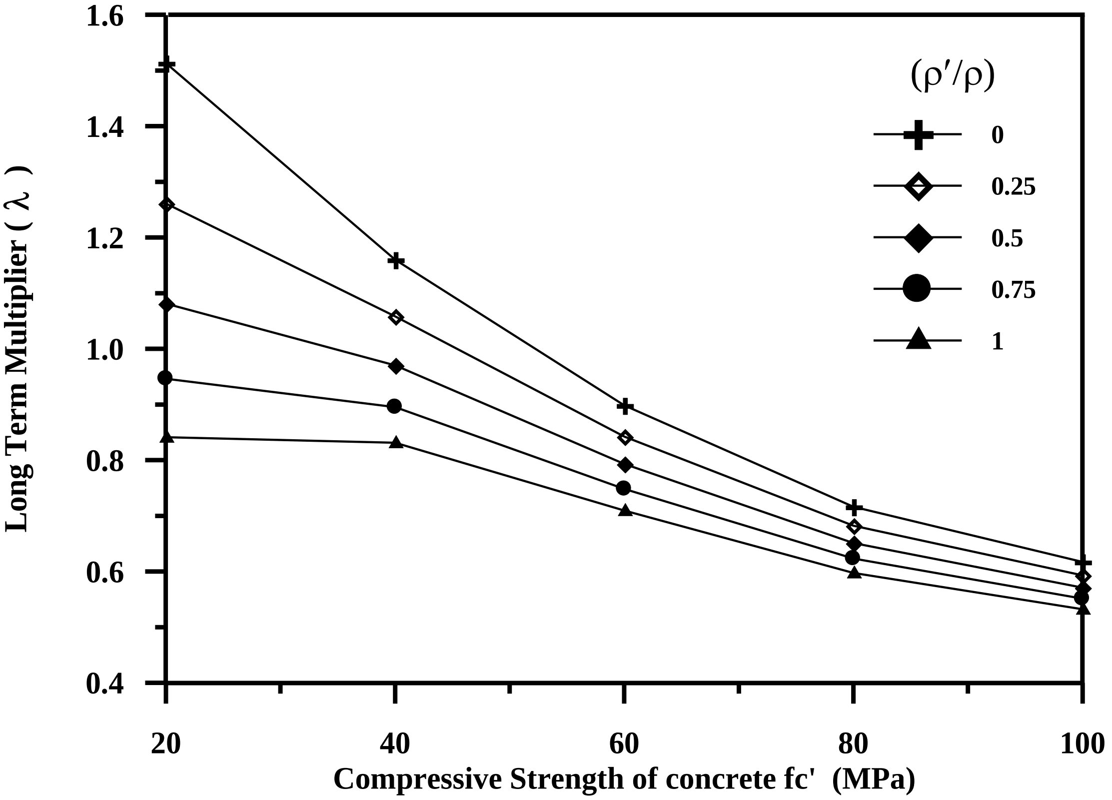
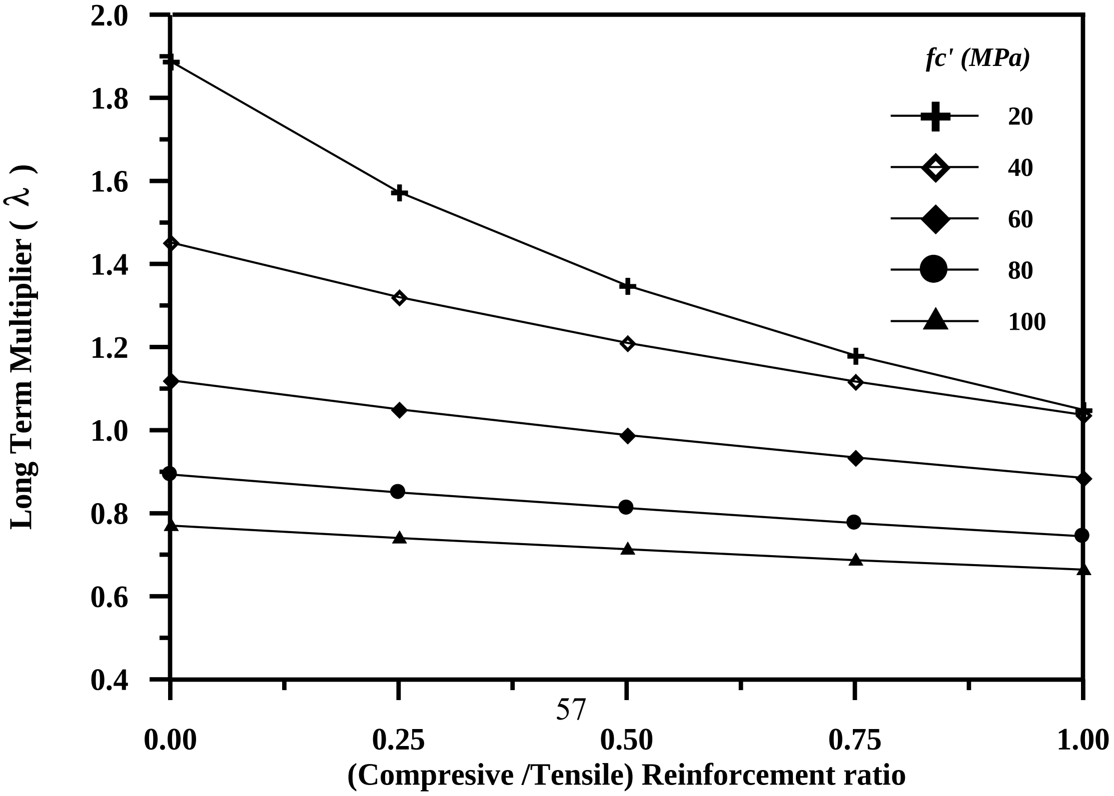


Fig. (9) The relation between long term multiplier with compressive strength of concrete for different ratio of compressive /tensile reinforcement (after 1 year of loading)



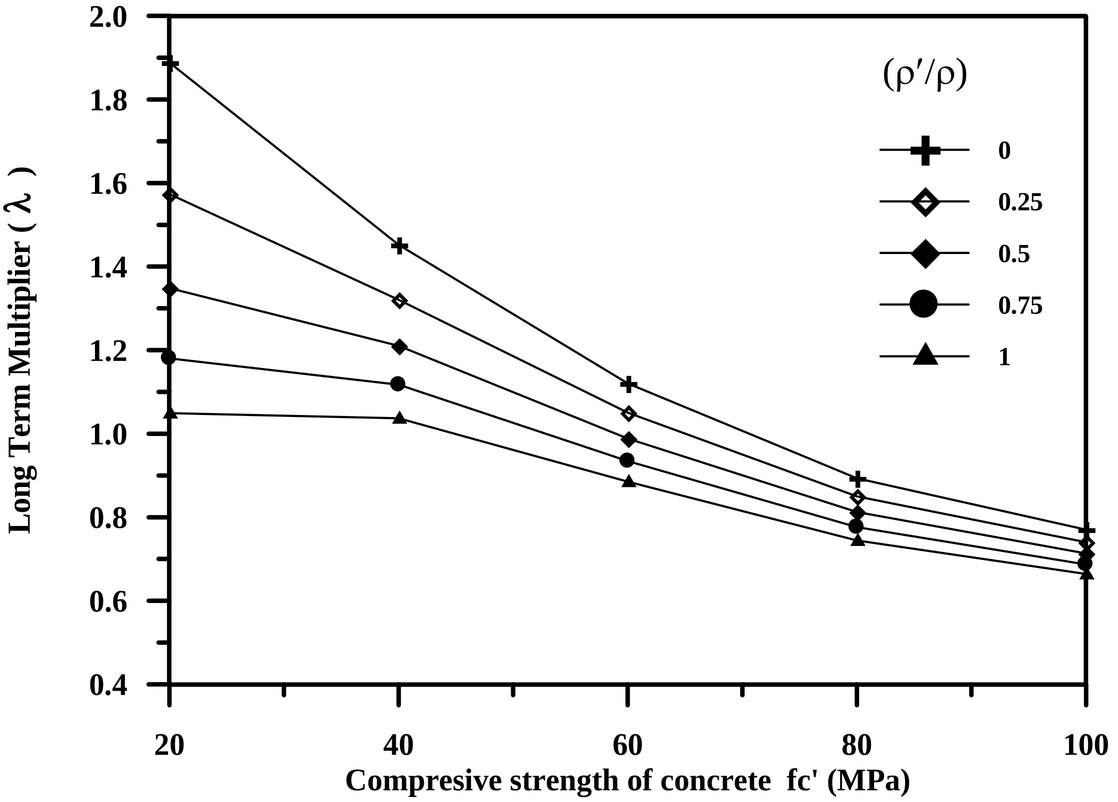
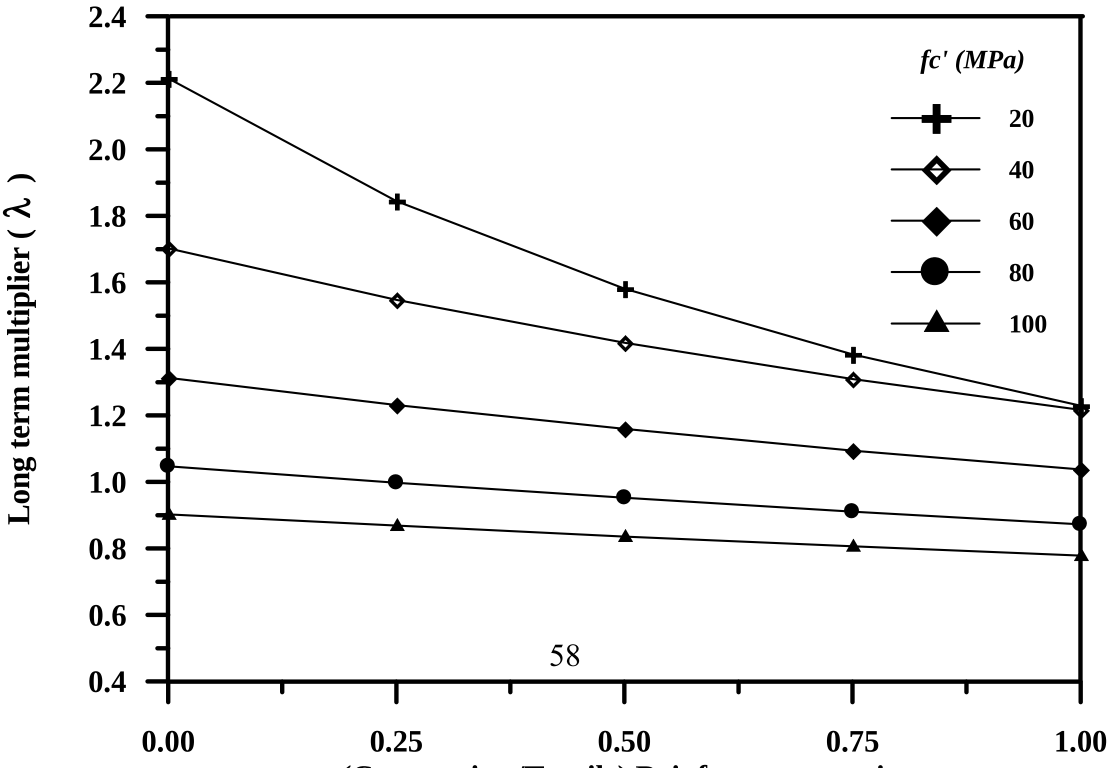


Fig. (11) The relation between long term multiplier with compressive strength of concrete for different ratio of compressive /tensile reinforcement (after 2.5 years of loading)



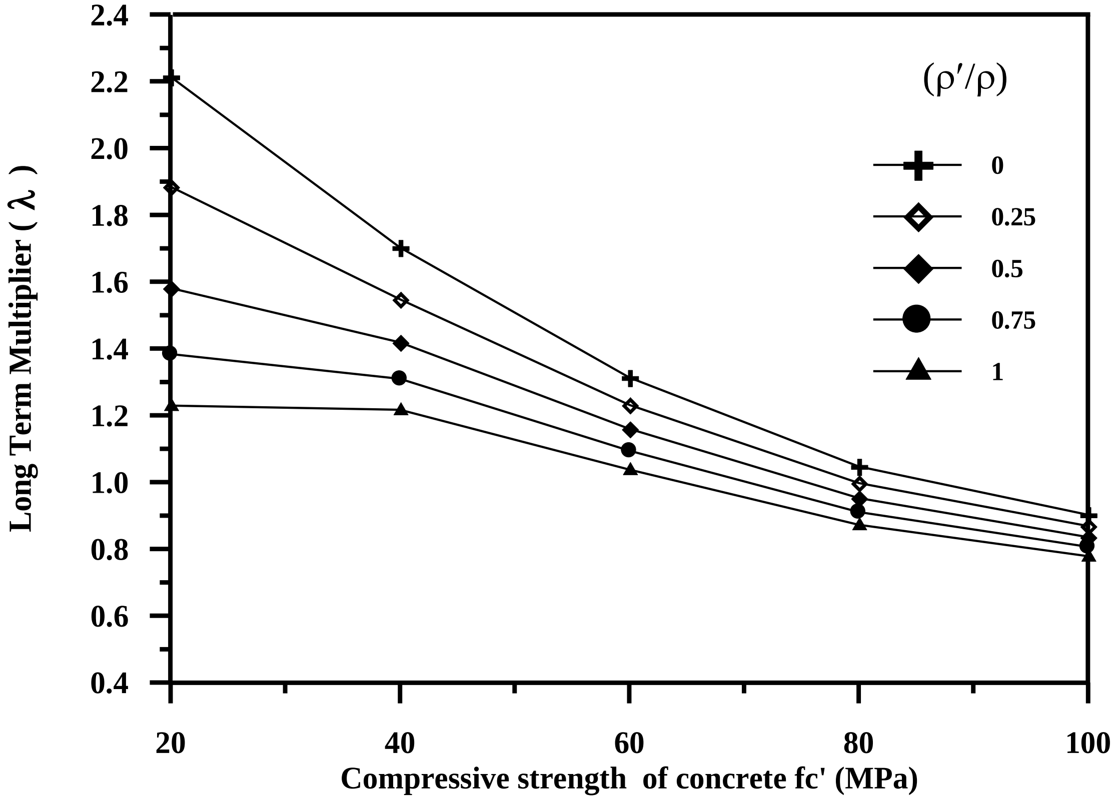


Fig. (13) The relation between long term multiplier with compressive strength of concrete for different ratio of compressive /tensile reinforcement (after 5 years of loading)

From these figures can be noted that:

- The long term deflection at all times of loadings is reduced with increased the compressive reinforcement.
- The effect of compressive reinforcement in reducing the long term deflection depends on the compressive strength of concrete, (it's highly reduced by increasing the compressive strength of concrete, and this can be attributed to increase the compressive strength of the compression zone of the section).

3-Effect of Span /Perimeter of Cross Section.

The simply supported beam shown in figure 1 was reanalyzed for different (span / cross section perimeter) ratio ranged from (7 to13); to study the effect of span / perimeter ratio on the long term deflection of reinforced concrete beams.

Figures (14 and 15) show the effect of span/ perimeter of the cross section ratio on the long term deflection at different times of loading. (The compressive strength of concrete used is 25 MPa and no compressive reinforcement used).

From these figures can be noted that, the increase of span/ perimeter of cross section ratio lead to increase the long term deflection (the long term deflection is increased by about 70% by increase the span/ perimeter from (7 to 13).

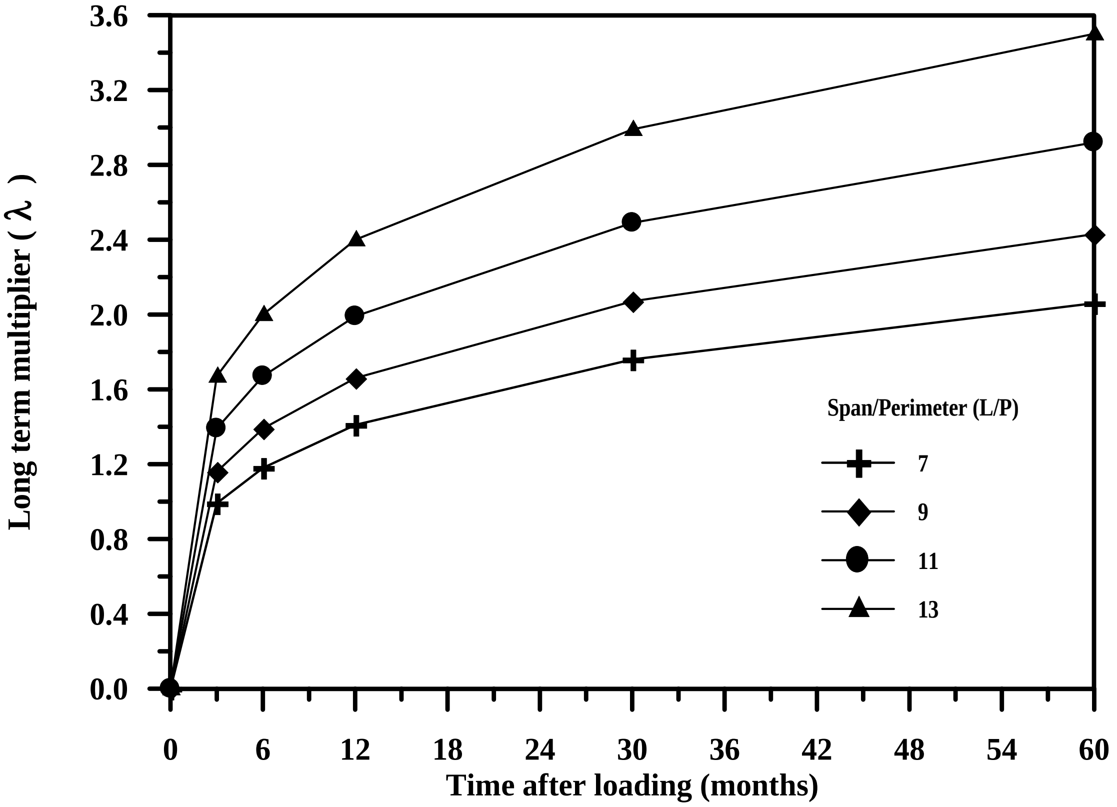
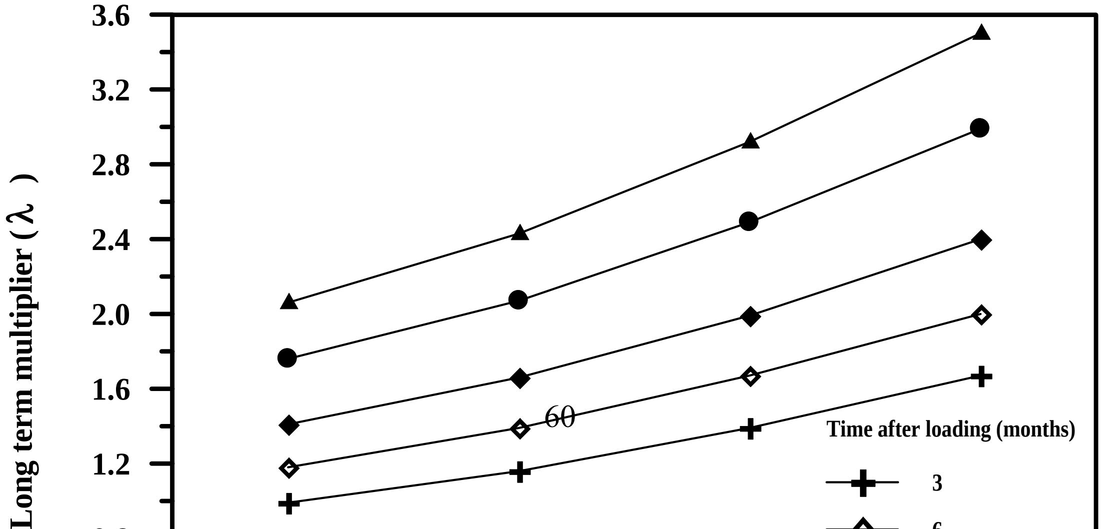


Fig. (14) The relation between long term multiplier with time for a range of (span/perimeter) ratio.



Summary and Conclusions:

- The proposed formula to predict the long term deflection of reinforced concrete beams takes into account several important factor such (compressive strength of concrete, the ratio of the compressive /tensile reinforcement, span, and cross section dimensions).
- A good agreement has been found to exist between measured results of many other researchers and the results of the proposed formula.
- From the data of the test results and the results of the proposed formula, the long term deflection is highly reduced with increase of the compressive strength of concrete.
- The compressive reinforcement reduced the long term deflection of reinforced concrete beams and the percentage of the reduced in the long term deflection decrease with increasing the compressive strength of concrete, (the compressive reinforcement becomes less significant in reducing the long term deflection for high strength concrete).
- The long term deflection of reinforced concrete beams increased with increasing the ratio of (span to cross section perimeter).

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