

Core Polarization Effects on the Inelastic Longitudinal C2 and C4 Form Factors of $^{58,60,62}\text{Ni}$ Nuclei

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Abstract

The form factors for the inelastic electron scattering to $2^+, 4^+$ states in $^{58,60,62}\text{Ni}$ are studied in the framework of shell model. The calculation is performed in $(0f7/2, 1p3/2, 0f5/2, 1p1/2)$ model space as well as extended $6\hbar\omega$ model space. The predictions of longitudinal form factors which include core-polarization effects to first order are compared with the experimental values. It is noticed that the core polarization effects are essential in obtaining a remarkable agreement between the calculated inelastic longitudinal $F(q)$'s and those of experimental data.

Key words: Electron scattering, Core polarization, Form factors

الخلاصة:

تم دراسة عوامل التشكل للاستطارة الطولية غير المرنة C2 و C4 للحالتين $2^+, 4^+$ للأنوية $^{58,60,62}\text{Ni}$ في إطار نموذج القشرة. الحسابات أنجزت في أنموذج الفضاء $(0f7/2, 1p3/2, 0f5/2, 1p1/2)$ فضلا عن توسيعه إلى $6\hbar\omega$. تم إدخال تأثير استقطاب القلب من خلال المرتبة الأولى لنظرية الاضطراب. لقد وجد بان تأثير استقطاب القلب يكون جوهريا للحصول على توافق جيد بين حسابات الاستطارة الطولية غير المرنة $(F(q)/s)$ و القيم العملية لجميع النوى قيد الدراسة.

الكلمات المفتاحية: الاستطارة الالكترونية, استقطاب القلب, عوامل التشكل

1. Introduction

The calculations of shell model, carried out within a model space in which the nucleon are restricted to occupy a few orbits, are unable to reproduce the measured static moments or transition strengths without scaling factors. Comparison between calculated and measured longitudinal electron scattering form factors has long been used as stringent tests of models for transition densities. Various microscopic and macroscopic theories have been used to study excitations in nuclei (Sato *et al.*; 1985). Shell model within a restricted model space is one of the models, which succeeded in describing static properties of nuclei, when effective charges are used. Calculations of form factors using the model space wave function alone are inadequate for reproducing the data of electron scattering (Booten *et al.*, 1994). Therefore, effects out of the model space, which called as core polarization effects, are necessary to be included in the calculations. The intermediate one-particle one-hole states are taken up to $6\hbar\omega$ excitation. These effects are found essential for obtaining a quantitative agreement with the experimental data (Yokoyama *et al.*, 1989; Sato *et al.*, 1994). A microscopic model (Radhi *et al.*, 2001; Radhi; 2003) has been used in order to study the core polarization effect on the longitudinal form factors of fp-shell nuclei. A microscopic model which adopted the first order core polarization is considered to calculate the C2 form factors of the fp-shell nuclei. Inelastic electron scattering from fp shell nuclei had been studied by Sahu et al in 1986. They calculated form factors for $^{50,52,54}\text{Cr}$, ^{54}Fe , ^{56}Fe , $^{46,48}\text{Ti}$, and ^{50}Ti by the use of Hartree-Fock theory, results are in a good

agreement with the experimental data. The form factors for the inelastic electron scattering to 2^+ , 4^+ and 6^+ states in $^{46,48,50}\text{Ti}$, $^{50,52,54}\text{Cr}$ and $^{54,56}\text{Fe}$ were studied by Sahu et al in 1990 and 1987 in the framework of the Hartree-Fock model, also the calculation is performed in the $1f_{7/2}$, $2p_{3/2}$, $1f_{5/2}$, $2p_{1/2}$ model space using a modified Kuo-Brown effective interaction. Magnetic dipole excitation of $N = 28$ isotones ^{48}Ca , ^{50}Ti , ^{52}Cr , ^{54}Fe and ^{51}V , were studied by Muto et al in 1985, in terms of the shell model by assuming $f_{7/2}^{n-m} (p_{3/2} f_{5/2} p_{1/2})^m$ configurations with $m = 0, 1$ and 2 on an inert ^{40}Ca core. The aim of present work is to use a realistic effective nucleon-nucleon (NN) interaction as a residual interaction to calculate the core polarization (CP) effects through a microscopic theory, with a selection of model space effective interaction which generates the model space wave functions (shell model wave functions) and highly excited states. The (MSDI) are used in this case as a residual interaction. The strength of the MSDI denoted by A_T , B and C are set equal to $A_0=A_1=B= 6.2$ MeV and $C=0$. The single particle wave function are those of the harmonic oscillator potential (HO) with size parameter b chosen to reproduce the measured ground state root mean square charge radii of these nuclei. The one-body density matrix (OBDM) elements ($\chi^{\Lambda} \Gamma_f \Gamma_i (\alpha_f, \alpha_i)$) are calculated using the shell model code OXBASH (Brown *et al.*, 2003).

2. Theory

The electron scattering form factor for a given multipolarity λ and momentum transfer q is expressed as (Forest et al; 1966)

$$|F_{\lambda}(q)|^2 = \frac{1}{2j_i + 1} \left(\frac{4\pi}{Z^2} \right) \left| \langle \Gamma_f \parallel \hat{T}_{\lambda}^{\xi} \parallel \Gamma_i \rangle \right|^2 F_{f.s} F_{c.m}^2 \dots\dots(1)$$

Where $F_{f.s} = e^{-0.43q^2/4}$ is the finite nucleon-size correction and $F_{c.m} = q^2 b^2 / 4A$ is the center of mass correction, A is the mass number and b is the harmonic oscillator size parameter.

The effect of the core polarization on the form factors is based on a microscopic theory, which combines shell-model wave functions and configuration with higher energy as particle-hole perturbation expansion. The reduced matrix element of the electron scattering operator \hat{T}_{λ}^{ξ} is expressed as a sum of the fp-model space (p) contribution and the core-polarization (cp) contribution, as follow (Radhi *et al.*, 2001)

$$\langle \Gamma_f \parallel \hat{T}_{\lambda}^{\xi} \parallel \Gamma_i \rangle = \langle \Gamma_f \parallel \hat{T}_{\lambda}^{\xi} \parallel \Gamma_i \rangle_{ms} + \langle \Gamma_f \parallel \delta \hat{T}_{\lambda}^{\xi} \parallel \Gamma_i \rangle_{cp} \dots\dots(2)$$

with ξ selection the longitudinal (L), electric(E) and magnetic(M) transverse form factors, respectively. The Greek symbols were used to denote quantum numbers in

coordinate space and isospace, i.e. $\Gamma_i \equiv J_i T_i$, $\Gamma_f \equiv J_f T_f$ and $\lambda \equiv J T$. The fp-shell model space element can be expressed as linear combination of the single-particle matrix element (Wildenthal,1984)

$$\langle \Gamma_f \parallel \hat{T}_\lambda^\xi \parallel \Gamma_i \rangle_{ms} = \sum_{\alpha_f, \alpha_i} \chi^\lambda \Gamma_f \Gamma_i(\alpha_f, \alpha_i) \langle \alpha_f \parallel \hat{T}_\lambda^\xi \parallel \alpha_i \rangle \quad \dots(3)$$

where $\chi^\lambda \Gamma_f \Gamma_i(\alpha_f, \alpha_i)$ are the structure factors (one body density matrix element), given by,

$$\chi^\lambda \Gamma_f \Gamma_i(\alpha_f, \alpha_i) = \frac{\langle \Gamma_f \parallel [a^+(\alpha_f) \otimes \tilde{a}(\alpha_i)]^\lambda \parallel \Gamma_i \rangle}{\sqrt{2\lambda + 1}} \quad \dots(4)$$

The α_f and α_i label are single-particle states for the fp-shell model space. Similarly, core-polarization matrix element as follow:

$$\langle \Gamma_f \parallel \delta \hat{T}_\lambda^\xi \parallel \Gamma_i \rangle_{cp} = \sum_{\alpha_f, \alpha_i} \chi^\lambda \Gamma_f \Gamma_i(\alpha_f, \alpha_i) \langle \alpha_f \parallel \delta T_\lambda^\xi \parallel \alpha_i \rangle \quad \dots(5)$$

Up to the first order perturbation theory, the single-particle matrix element for the higher-energy configuration is given by Ellis et al in 1971

$$\begin{aligned} \langle \alpha_f \parallel \delta T_\lambda^\xi \parallel \alpha_i \rangle = & \\ & \langle \alpha_f \parallel \hat{T}_\lambda^\xi \frac{Q}{E_i - H_o} V_{res} \parallel \alpha_i \rangle + \langle \alpha \Gamma_f \parallel V_{res} \frac{Q}{E_f - H_o} \hat{T}_\lambda^\xi \parallel \alpha \Gamma_i \rangle \end{aligned} \quad \dots(6)$$

The operator Q is the projection operator on the space outside the model space. E_i and E_f are the energies of initial and final states. For the residual interaction V_{res} the MSDI and M3Y are adopted.

The two term in right hand side of Eq.(6) can be written as follow (Brussard *et al.*, 1977):

$$\begin{aligned} \sum_{\alpha_1, \alpha_2 \Gamma} \frac{(-1)^{\alpha_1 + \alpha_2 + \Gamma}}{e_{\alpha_1} - e_{\alpha_f} - e_{\alpha_1} + e_{\alpha_2}} (2\Gamma + 1) & \begin{Bmatrix} \alpha_f & \alpha_i & \lambda \\ \alpha_2 & \alpha_1 & \Gamma \end{Bmatrix} \sqrt{(1 + \delta_{\alpha_1 \alpha_f})(1 + \delta_{\alpha_2 \alpha_i})} \langle \alpha_2 \parallel T_\lambda \parallel \alpha_1 \rangle \\ & \times \langle \alpha_f \alpha_1 \parallel V_{res} \parallel \alpha_i \alpha_2 \rangle_\Gamma \\ + \text{Terms with } \alpha_1 \text{ and } \alpha_2 \text{ exchanged with an over all minus sign,} & \end{aligned} \quad \dots(7)$$

Where α_1 runs over particle states and α_2 over hole state and e is the single-particle energy .

$$e_{nlj} = (2n + l - \frac{1}{2})\hbar\omega + \begin{cases} -\frac{1}{2}(l+1)\langle f(r) \rangle_{nl} & \text{for } j = l - \frac{1}{2}, \\ \frac{1}{2}l\langle f(r) \rangle_{nl} & \text{for } j = l + \frac{1}{2}, \end{cases} \dots(8)$$

With $\langle f(r) \rangle_{nl} \approx -20A^{-2/3}$ and $\hbar\omega = 45A^{-1/3} - 25A^{-2/3}$

The electric transition strength is given by,

$$B(C\lambda) = \frac{|(2\lambda + 1)!!|^2 Z^2}{4\pi k^\lambda} |F_\lambda(k)|^2$$

Where $k = E_x/\hbar c$.

3. Results and discussion

The core polarization effect are calculated with the MSDI as residual interaction. The parameters of MSDI are denoted by A_T , B and C. where T indicates the isospin (0,1). These parameters are taken to be $A_0=A_1=B= 6.2\text{MeV}$ and $C=0$. In all of the following diagrams the dashed line give the results obtained using the fp-shell wave function and the results including cp effects are shown by solid curve.

3.1 Form factors for the $0^+ \longrightarrow 2^+$ transition

A comparison between the experimental and theoretical form factors for the C2 transition for $^{58,60,62}\text{Ni}$ is given in figures1-3 respectively. For each of these three nuclei, the form factors show three peaks. Fig.(1) shows longitudinal C2 electron scattering form factors as a function of momentum transfer for ^{58}Ni . We observe that the first peak occur at $q=0.7\text{fm}^{-1}$, the second at 1.5fm^{-1} and the third peak at 2.5fm^{-1} . It is noticed from fig.(1) that the inclusion of cp enhance the C2 form factor. This enhancement brings the total form factors very close to the experimental data for the first peak $q < 1\text{fm}^{-1}$ and second peak, while the third peak underestimated. We notice that from fig.(1) the model space fail to describe the form factors in all momentum transfer.

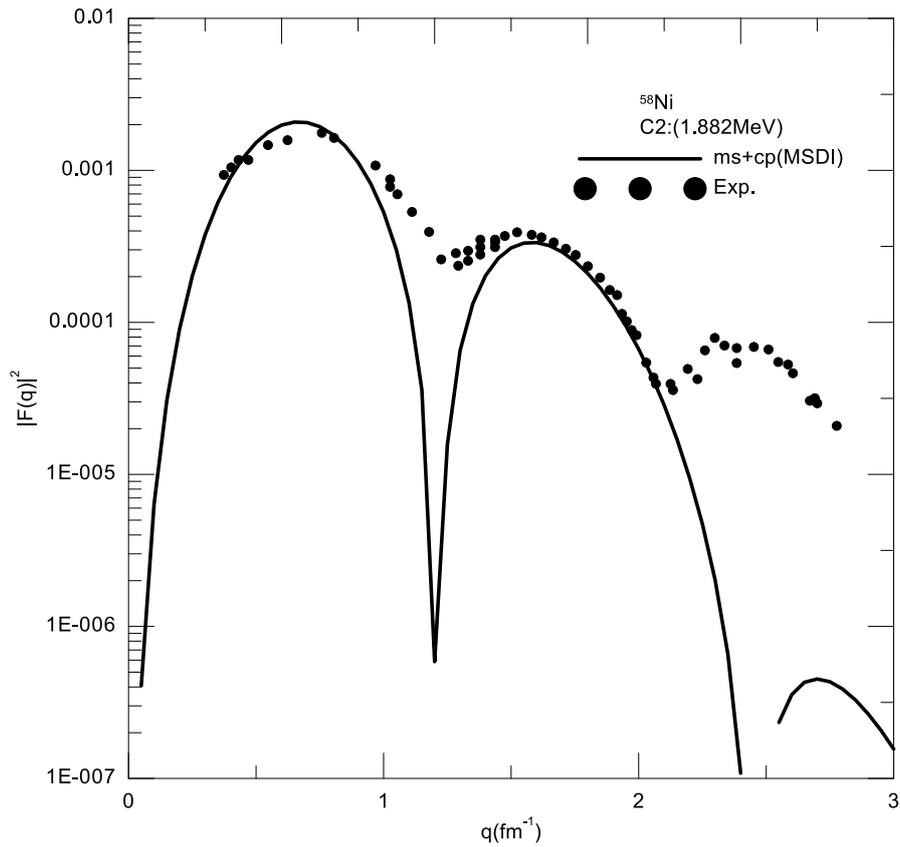


Fig.(1) Inelastic longitudinal form factors for the transition to the 2^+ in the ^{58}Ni , the experimental data are taken from ref. (Ellis *et al.*, 1971)

Fig. 2 displays the calculation of the C2 form factors $J^\pi=2^+, T=2$ at $E_x = 1.078\text{MeV}$ for ^{58}Ni .

The first peak occur at $=0.7\text{fm}^{-1}$ the second at $= 1.7\text{fm}^{-1}$ and the third peak at $=2.5\text{fm}^{-1}$. The model space calculation fail to describe the form factors and the inclusion of the core polarization enhances the calculations and bring the form factors to the experimental values up to momentum transfer $q=2.4\text{fm}^{-1}$, while the third peak is underestimated.

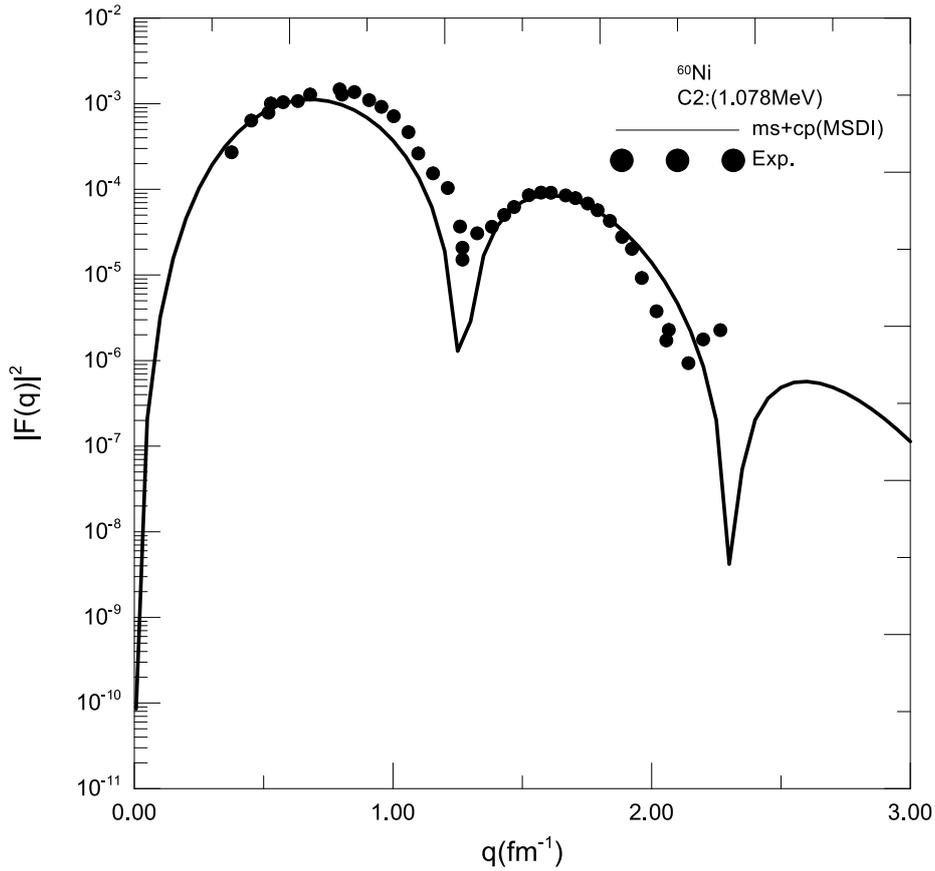


Fig.(2) Inelastic longitudinal form factors for the transition to the 2^+ in the ^{60}Ni , the experimental data are taken from ref. (Ellis *et al.*, 1971)

Fig.(3) displays the calculation of the C2 form factors to the $J^\pi=2^+, T=3$ at $E_x=1.320\text{MeV}$ for ^{60}Ni . We observe three peaks, the first peak occur at $q=0.7\text{fm}^{-1}$ the second at $q=1.7\text{fm}^{-1}$ and the third peak at $q=2.6\text{fm}^{-1}$. The cp effects enhance the C2 form factors in all momentum transfer, where we notice that the results of cp effects give good agreement with exp. data especially for the first peak up to $q=1.8\text{fm}^{-1}$, while the second peak is overestimated and the third peak is underestimated. However the results for the first peak in ^{62}Ni are shifted toward higher momentum transfer and the second peak is good in behavior with experimental data but overestimated in our calculations.

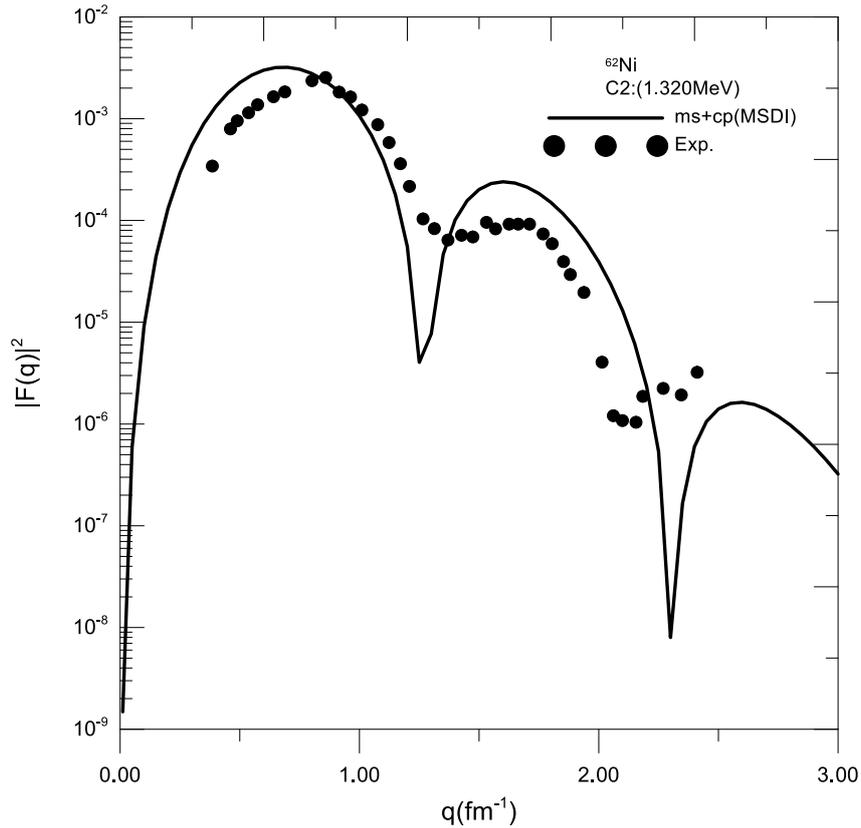


Fig.(3) Inelastic longitudinal form factors for the transition to the 2^+ in the ^{62}Ni the experimental data are taken from ref. (Ellis *et al.*, 1971)

3.2 Form factors for the $0^+ \rightarrow 4^+$ transition

Fig. (4) compare the calculated and experimental of longitudinal C4 form factors for ^{62}Ni at $E_x=0.554\text{MeV}$. The inclusion of cp enhances the calculations. It is seen that the present calculation is quite successful in reproducing the magnitude of the form factors at the first maximum. However one observes discrepancies in the momentum transfer range $1.3 < q < 1.8$.

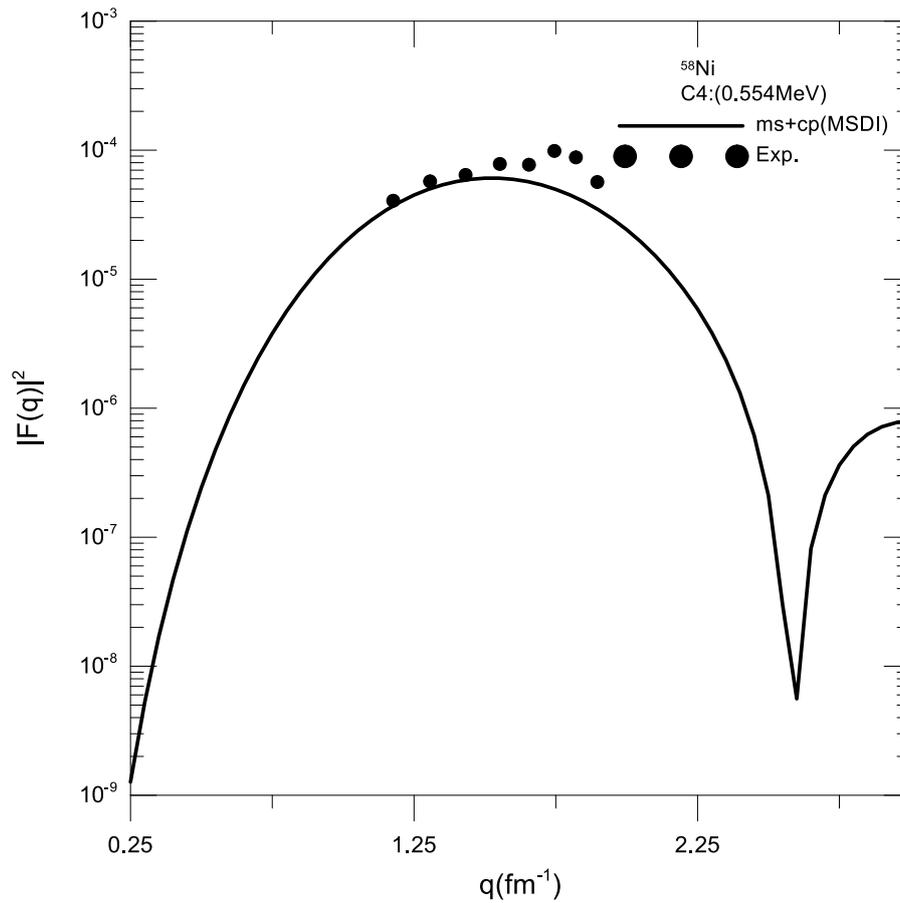


Fig.(4) Inelastic longitudinal form factors for the transition to the 4^+ in the ^{58}Ni , the experimental data are taken from ref. (Forest *et al.*, 1966)

Fig.(5) displays the calculation of the C4 form factors to the $J^\pi=4^+$ T=3 at $E_x=2.675\text{MeV}$. The cp effects enhance the C4 form factors. The results of the fp-shell model space with cp effects give good agreement with the experimental data. However the results of the first and third peaks are in good agreement with experimental data, but the second peak is overestimated in our calculations.

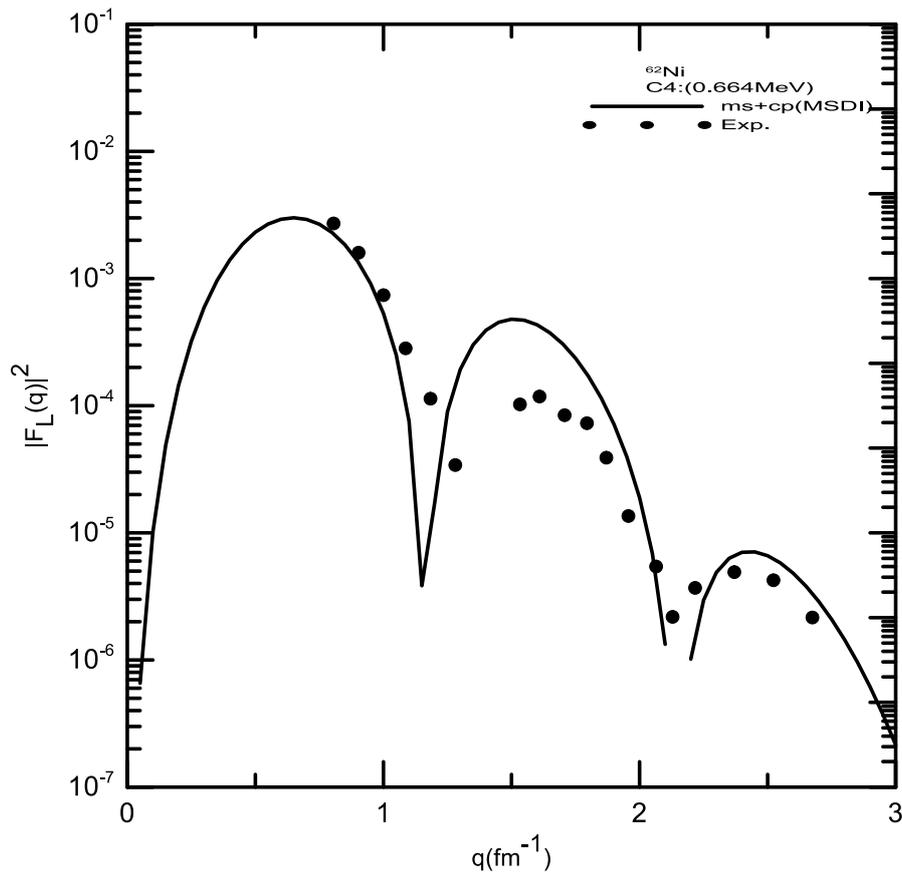


Fig.(5) Inelastic longitudinal form factors for the transition to the 4^+ in the ^{62}Ni , the experimental data are taken from ref. (Forest *et al.*, 1966)

4. Conclusion

The fp-shell models, which can describe static properties and energy level are less successful in describing dynamic properties such as C2 and C4 transition rates and electron scattering form factors. The inclusion of higher-excited configurations by means of cp enhances the form factors and brings the theoretical results closer to the experimental data.

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