f*- Coercive function

Alyaa Yousif Khudayir **Habeeb Kareem Abdullah**
University of kufa, College of Education for Girls, Department of Mathematics

Abstract

In this paper , we introduce the definition of f* - coercive function and introduce several properties of f* - coercive function .

الخلاصة

في هذا البحث، قدمنا تعريف لـ f* - coercive function وقدمنا بعض مبرهنات لـ f* - coercive function

Introduction

Let (X,T) be a topological space. A subset A of a space X is called semi- open if $A \subseteq \overline{A^o}$ and A is called feebly open (f- open) if there exists an open set U of X such that $U \subseteq A \subseteq \overline{U}^s$ where $\overline{U}^s$ stands for the intersection of all semi-closed subsets of X which contain U ,(Navalagi, 1991).

In this paper gives a new definition namely of f* - coercive function .

1-Basic concepts

Definition 1.1,(Levine, 1963).

A set B in a space X is called **semi – open** (s.o) if there exists an open subset O of X such that $O \subseteq B \subseteq \overline{O}$.

The complement of a semi – open set is defined to be semi – closed (s.c.)

Definition 1.2,(Dorsett, 1981).

Let X be a space and $A \subseteq X$. Then the intersection of all semi – closed subsets of X which contains A is called **semi – closure** of A and it is denoted by $\overline{A}^S$.

Definition 1.3,(Dontchev, 1998).

A subset B of a space X is called **pre – open** if $B \subseteq \overline{B}^0$. The complement of a pre –open set is defined to be pre – closed .

Definition 1.4,(Navalagi, 1991).

A subset B of a space X is called feebly open (f-open) set if there exists open subset U of X such that $U \subseteq B \subseteq \overline{U}^s$.
The complement of a feebly open set is defined to be a feebly closed (f-closed)set .

***Proposition* 1.5(Farero, 1987).**

Let X be a space and $B \subseteq X$. Then the following statements are equivalent :

(i) B is f – open set .

(ii) There exists an open set O in X such that $O \subseteq B \subseteq \overline{O}^0$.

(iii) B is semi – open and pre – open .

Definition 1.6(Maheshwari, 1985)
A space X is called f-compact if every f-open cover of X has a finite subcover.

Lemma 1.7(Khudayir, 2008)
Let X be space and F be an f-closed subset of X, then $F \bigcap K$ is f- compact subset of F, for every f-compact set K in X .

Definition 1.8, (Khudayir, 2008)
Let X and Y be spaces, the function $f : X \to Y$ is called st-f-compact if the inverse image of each f-compact set in Y is f- compact set in X.

Definition 1.9 (Khudayir, 2008)
Let X and Y be spaces .A function $f : X \to Y$ is called f - coercive if for every f - compact set $J \subseteq Y$, there exists f- compact set $K \subseteq X$ such that :
$f(X \setminus K) \subseteq Y \setminus J$

Definition 1.10,(Maheshwari and Thakur, 1980; Reilly and Vammanamurthy, 1985; Navalagi, 1998)
Let X and Y be spaces and $f : X \to Y$ be a function, Then f is called f-continuous function if $f^{-1}(A)$ is an f- open set in X for every open set A in Y .

Definition 1.11,(Reilly and Vammanamurthy, 1985; Navalagi, 1998)
A function $f : X \to Y$ is called st-f-closed function if the image of each f- closed subset of X is an f-closed set in Y .

Definition 1.12,(Khudayir, 2008)
Let X and Y be spaces .Then $f : X \to Y$ is called a strong feebly proper (st-f-proper) function if :
(i) f is f-continuous function.
(ii) $f \times Iz : X \times Z \to Y \times Z$ is a st-f-closed function , for every space Z.

Proposition 1.13, (Khudayir, 2008)
Let $f : X \to p = \{ w \}$ be a function on a space X. If f is st-f-proper, then X is an f-compact space, where w is any point which dose not belong to X .

2- The main results

Definition (2.1) :
Let X and Y be spaces .A function $f : X \to Y$ is called f* - coercive if for every f – compact set $J \subseteq Y$, there exists compact set $K \subseteq X$ such that :

$$f(X \setminus K) \subseteq Y \setminus J$$

Example (2.2):
If X is compact space, then the function $f : X \to Y$ is f* - coercive .

Remark 2.3 :

Every f - coercive function is f* - coercive function .

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Proposition (2.4):

Let $f : X \to p = \{ w \}$ be st-f-proper function ,then $f : X \to p = \{ w \}$ is f* - coercive function ; where w is any point which dose not belong to X .

Proof :

By proposition (1.13) and Example(2.2) .

Proposition (2.5):

For any f- closed subset F of a space X , the inclusion function $i_F : F \to X$ is f* - coercive function .

Proof:

Let J be an f-compact subset of X, then by lemma (1.7), $F \cap J$ is f-compact set in F , then $F \cap J$ is compact set in F .

But

$$i_F(F \setminus (F \cap J)) = i_F(F \cap J^c)$$
$$= i_F(F \setminus J) = F \setminus J$$

Since $F \setminus J \subseteq X \setminus J$, *thus* $i_F(F \setminus (F \cap J)) \subseteq X \setminus J.$

Therefore the inclusion function $i_F : F \to X$ is f* - coercive function .

Proposition (2.6):

If $f : X \to Y$ is st-f-compact function, then $f : X \to Y$ is f* - coercive function .

Proof:

Let J be an f-compact set in Y , since $f : X \to Y$ is st-f-compact function, then $f^{-1}(J)$ is f- compact set in X , thus $f^{-1}(J)$ is compact set in X .

Thus

$$f(X \setminus (f^{-1}(J)) \subseteq Y \setminus J$$

Therefore $f : X \to Y$ is f* - coercive function .

Proposition (2.7):

Let X , Y and Z be spaces. If $f : X \to Y$ is f*-coercive and $g : Y \to Z$ is f-coercive function,then gof is a f*-coercive function

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Proof:

Let J be an f-compact set in Z ,then there exists f-compact set K in Y such that:

$$g(Y \setminus K) \subseteq Z \setminus J$$

Since $f : X \to Y$ is f* - coercive function, then there exists a compact set D in X such that $f(X \setminus D) \subseteq Y \setminus K$

Then

$$g(f(X \setminus D)) \subseteq g(Y \setminus K) \subseteq Z \setminus J \ , \ thus \ \ gof(X \setminus D) \subseteq Z \setminus J$$

Therefore $gof : X \to Z$ is f* - coercive function .

Proposition (2.8):

Let $f : X \to Y$ be a f* - coercive function such that F is f- closed subset of X. Then $f_{/F} : F \to Y$ is a f* - coercive function .

Proof:

Since F is f- closed set in Y , then by proposition(2.5), the inclusion function $i_F : F \to X$ is f* - coercive function, since $f : X \to Y$ is f* - coercive function, then by proposition (2.7). $foi_F : F \to Y$ is f* - coercive function.

But $f \, o \, i_F = f_{/F}$, then $f_{/F} : f \to Y$ is f* - coercive function.

Proposition (2.9):

Let X and Y be spaces, such that Y is $T_2 - Space$ and $f : X \to Y$ is continuous , one – one , function .Then the following statements are equivalent :

(i) f is an f*-coercive function .

(ii)f is an f- compact function .

(iii)f is an f- proper function .

Proof:

(i → ii) Let J be an f-compact set in Y. To prove $f^{-1}(J)$ is a compact set in X . Let $\{X_a\}_{d \in D}$ be a net in $f^{-1}(J)$. Since f is f*-coercive function, then there exists a compact set K in X such that $f(X \setminus K) \subseteq Y \setminus J$ then $f(K^c) \subseteq J^c \ \ thus \ \ K^c \subseteq f^{-1}(J^c)$

Then $K^c \subseteq (f^{-1}(J))^c \ \ thus \ \ f^{-1}(J) \subseteq K$. Then $\{X_d\}_{d \in D}$ is a net in K , Since K is a compact set in X , then by [Reilly and Vammanamurthy, 1985,theorem 3.15],the net $\{X_d\}_{d \in D}$ has a cluster point x in X .Thus by [Reilly and Vammanamurthy, 1985,theorem 3.15], $f^{-1}(J)$ is a compact set in X.

Therefore $f : X \to Y$ is an f-compact function .

(ii → iii) By [AL-Badairy, 2005 , proposition 3.1.22]

(iii → i) let J be an f-compact set in Y , since f is f- proper function , then by [AL-Badairy, 2005 ,proposition 3.1.21], f is f- compact function, then $f^{-1}(J)$ is a compact in X . Thus

$f(X \setminus f^{-1}(J)) \subseteq Y \setminus J$

Hence $f : X \to Y$ is f*- coercive function .

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