On Contra (δ, gδ)–Continuous Functions

Zainab Aodia Athbanaih
University of Al-Qadissiya, College of Education

Abstract

In 1980 T . Noiri introduce δ – sets to define a new class of functions called δ–continuous and in 1996 Dontchev and Ganster introduce and studied a new class of sets called δ–generalized closed and in 2000 Dontchev and others introduce $g\delta$ – closed sets. In this paper we use these sets to define and study a new type of functions called contra $(\delta, g\delta)$–continuous as a new type of contra continuous the functions contra δ–continuous is stronger than contra continuous function , contra $g\delta$ – continuous functions and weaker than Rc–continuous and each one of them are independent with $(\delta, g\delta)$ - continuous functions.

Preservation theorems of this functions are investigated also several properties concerning this functions are obtain . The relationships between the δ –closed set with the other sets that have so important to get many results are investigated . finally we study the relationships between this functions with the other functions .

الخلاصة

في عام 1980 قدم تكاشي نويرا المجموعات – δ ليعرف نوع جديد من الدوال هي الدوال المستمرة – δ . وفي عام 1996 Dontchev و Ganster قدموا ودرسوا نوع جديد من المجموعات المغلقة هي (δ – المعممة المغلقة) وفي عام 2000 أيضاً Dontchev وباحثين اخرين قدموا المجموعات المغلقة – $g\delta$. في هذا البحث استخدمنا هذه المجموعات لتعريف نوع جديد من الدوال وهي الدوال الضد مستمرة – $(\delta, g\delta)$ كنوع جديد من الدوال الضد مستمرة (او العكس مستمرة). حيث ان الدالة الضد مستمرة – δ هي اقوى من الدوال الضد مستمرة والضد مستمرة – gδ واضعف من الدوال المستمرة – Rc] وكل واحدة منهما مستقلة عن الدوال المستمرة – (δ و $g\delta$). وللحفاظ على مبرهنات عن تلك الدوال تم الاستقصاء او التحري عنها كذلك حصلنا على بعض الخواص المتعلقة بتلك الدوال ، علاقة المجموعات المغلقة – δ بمجموعات اخرى لها اهمية في الحصول على بعض النتائج . وأخيراً درسنا علاقة هذه الدوال مع دوال اخرى .

1. Introduction

In 1996, Dontchev introduce a new class of functions called contra – continuous functions, in 1999 Jafari and Noiri introduce and studied a new functions called contra super – continuous, and in 2001 they present and study a new functions called contra α -continuous .

In this paper we introduce the notion of contra $(\delta, g\delta)$- continuous, where contra δ–continuous functions is stronger than both contra–continuous and contra $g\delta$ -continuous. We astablish several properties of such functions . especially basic properties and preservation theorems of these functions are investigated . moreover , we investigate the relationships between δ–closed sets and other sets also the relation ships between these functions with the other functions .

2. Preliminaries

Through the present note (X, T_X) and (Y, T_Y) (or simply X and Y) always topological spaces .

A subset A of a space X is said to be regular open (resp. regular closed , δ – open, g -closed, clopen, $g\delta$ - closed) if $A = \text{int}(cl(A))$ (resp. $A = cl(\text{int}(A))$, $A = \bigcup_{i \in I} u_i$ where u_i is

[Noiri,1980] regular open v_i, if $cl(A) \subseteq u$ whenever $A \subseteq u$ and u is open [Dontchev,2000] , open and closed , if $cl(A) \subseteq u$ whenever $A \subseteq u$ and u is δ – open [Dontchev,2000] . Apoint $x \in X$ is said to be δ – cluster point of aset S if $S \cap u \neq \vartheta$, for every regular open set u containing x . The set of all δ – cluster points of S is called δ – closure of S and denoted by $cl\delta(A)$. if $cl\delta(A) = A$, then A is called δ – closed [Noiri,1980], or equivalently A is called δ– closed if $A = \bigcap_{i \in I} v_i$, where v_i is regular closed

The collection of all δ – closed (resp . δ – open , $g\delta$ - closed , g - clopen , g - closed , regular closed) sets will denoted by $\delta c(X)$ (resp . $\delta o(X)$, $g\delta c(X)$, $gc(X)$, $gco(X)$, $Ro(X)$) also the complement of δ – open (resp . $g\delta$ - closed , g - closed , regular closed) sets is called δ – closed (resp . $g\delta$ - open , g - open , regular open) .

Its worth to be noticed that the family of all δ – open subsets of a space X is a topology on X which is denoted by T_δ - space and some time is called semi – regularization of X . As a consequence of definitions we have $T_\delta \subseteq T$.

finally anon – empty topological space (X, T) is called hyper connected or irreducible [Dontchev,2000] if every non – empty open subset of X is dense .

3. Properties of $(\delta, g\delta)$ - Closed Sets

proposition 3.1

Let A be a subset of aspace X :

1- If A is regular open , then A is open .

2- If A is δ – closed , then A is closed .

Proof :-

1- Is clearly

2- Let A be a δ – closed set , $A^c = B$ is δ – open

$B = \operatorname{int}(cl(u_i))$, since $\operatorname{int}(cl(u_i))$ is regular so by part (1) , $\operatorname{int}(cl(u_i))$ is open and countable unions of open sets are open , $\operatorname{int}(cl(u_i)) = B$ is open , so $B = \bigcup_{i \in I}(u_i)$,

$(B^c) = \left(\bigcup_{i \in I} u_i\right)^c = \bigcap_{i \in I} u_i = A$ is closed .

Remark 3.2

The converse of Proposition (3.1) is not true in general . To see this we give the following example .

Example 3.3

Let $X = \{a, b, c, d\}$ and $T_X = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$ be a topology defined on X .

Let $A = \{d\} \subset X$, A is closed set but not δ – closed .

Since $A^c = \{a, b, c\}$ not δ – open .

Also $\{a, b\} \subset X$, is open but not regular open .

Proposition 3.4

Let A be a subset of a space X .

1- If A is regular open , then A is δ – open .

2- If A is δ – closed , then A is $g\delta$ - closed .

3- If A is closed , then A is g - closed .

4- If A is g - closed , then A is $g\delta$ - closed .

Proof :-

1- Let A be a regular open set , $A = \operatorname{int}(cl(A)) = \bigcup_{i=1}^{1}(\operatorname{int}(cl(A)))$ is δ – open

2- Let $x \in X$, $A \subseteq X$ is δ – closed , so $A = cl\delta(A)$

$x \in A$, so $x \in cl\delta(A)$, therefore for every $u \in \tau$, u is regular open and $x \in u$, $A \bigcap u \neq \phi$, this means that $A \subset u$, for every u regular open in X and since A is δ – closed so by lemma (3.1) part (2) .

A is closed so $cl(A) \subset u$. and u is δ – open by part (1) thus $cl(A) \subset u$, then $A \subset u$, for every u δ – openset .

Therefore A is $g\delta$ - closed .

3- Let $A \subset X$, A is closed , let $x \in X$, then $x \in cl(A)$, $\forall u \in \tau$, $x \in u$, $u \bigcap A \neq \phi$, this means that $A \subset u$, and u is open , also since A is closed ,$cl(A) \subset u$, thus $cl(A) \subset u$, when $A \subset u$ and u is open .

Therefore A is g - closed .

4- Let A be a g - closed set , so $A \subseteq u$, where u is open and $cl(A) \subseteq u$, by part (2) from lemma (3.1) there exist M δ – open set , such that $M \subseteq u$ and $A \subseteq M \subseteq u$, so $cl(A) \subseteq M$, $M \subseteq u$ Thus $cl(A) \subseteq M$, when $A \subseteq M$, M is δ – open in X .

Therefort A is $g\delta$ - closed .

Remark 3.5

The converse of lemma (3.4) is not true in general , to see this,we give the following counter example .

Examples 3.6

1- Let $X = \{a,b,c,d\}$ and $T_X = \{\phi, X, \{a,c,d\}, \{b,c,d\}, \{c,d\}\}$ be a topology defined on X .

Let $A = \{a,b\} \subseteq X$ is $g\delta$ - closed but not δ – closed . Also $B = \{a,b,c\} \subseteq X$ is g - closed but not closed .

2- Let $X = \{a,b,c,d\}$ and $T_X = \{\phi, X, \{a\}, \{b\}, \{a,b\}, \{a,b,c\}, \{a,b,d\}, X\}$ be a topology defined on X .

Let $A = \{a,c\} \subseteq X$ is $g\delta$ - closed but not g - closed .

Lemma 3.7 [Dontchev,2000]

Let (X,T) be a topological space , and $(A_i)_{i \in I}$ be alocally finite family , $A_i \subset X, \forall i \in I$:

f $(A_i)_{i \in I}$ is $g\delta$ - closed sets , $\forall i \in I$, then $A = \bigcup_{i \in I} A_i$ is also $g\delta$ - closed .

Proposition 3.8

Let (X,T) be a topological space , and $(A_i)_{i\in I}$ be alocally finite family , $A_i \subset X, \forall i \in I$:

If $(A_i)_{i\in I}$ is δ - closed set , $\forall i \in I$, then $A = \bigcap_{i\in I} A_i$ is also δ - closed .

Proof :-

Let $(A_i)_{i\in I}$ is δ – closed set , $\forall i \in I$, A_i is δ – closed this means that $A_i = cl\delta(A_i)$, $\forall i \in I$, so $\bigcap_{i\in I} A_i = \bigcap_{i\in I} cl\delta(A_i)$, since by lemma 3.1 part (2) A is closed so $cl\delta(A_i) = cl(A_i)$. Thus $\bigcap_{i\in I} cl\delta(A_i) = \bigcap_{i\in I} cl(A_i) = cl\left(\bigcap_{i\in I} A_i\right) = cl\delta\left(\bigcap_{i\in I} A_i\right)$ so $\bigcap_{i\in I} A_i = cl\delta\left(\bigcap_{i\in I} A_i\right)$, therefort $\bigcap_{i\in I} A_i$ is δ – closed .

Proposition 3.9

The following properties are equivalent for a subset A of a space X :

1- A is clopen .
2- A is regular open and regular closed .
3- A is δ – open and δ – closed .
4- A is δ – open and $g\delta$ - closed .

Proof :-

$1 \rightarrow 2$: clearly since A is open and closed .
$2 \rightarrow 3$: by proposition (3.4) part (1) .
$3 \rightarrow 4$: by proposition(3.4) part (2) .
$4 \rightarrow 3$: A is δ – open from part (4) , let A is $g\delta$ - closed set so $cl(A) \subset u$, when $A \subset u$ and u is δ – open by proposition (3.4) (1) , there exist $M \subseteq u$, M is regular open such that $A \subseteq M \subseteq u$, since $cl(A) \subseteq cl\delta(A) \subseteq M$ when $A \subset M$, M is regular open in X .
Thus $cl\delta(A) \subseteq M$, when $A \subset M$, M is regular open , therefore A is δ – closed .
$3 \rightarrow 2$ and $2 \rightarrow 1$: clearly by proposition (3.1)

Lemma 3.10 **[**Dontchev,2000**]**

For a topological space (X,T) the following conditions are equivalent :

1- X is T_δ - space .
2- X is almost weakly hausdorff and semi – regular [NavalaG,2000] .

Lemma 3.11

Let (X,T_δ) be a topological space , $A \subseteq X$:

1- If A is closed , then A is δ – closed .
2- If A is $g\delta$ - closed , then A is δ – closed .

Proof :-

1- Let $A \subset X$ be a closed set , $A^c = B \in T_\delta$ - space , since so $B = \bigcup_{i=1}^{n} u_i$, $u_i \in Ro(X)$ open , $B^c = (\bigcup u_i)^c = \bigcap_{i=1}^{n} (u_i)^c = \bigcap_{i=1}^{n} v_i$ where $v_i \in Rc(X)$, so $A - B^c = \bigcap_{i=1}^{n} v_i$, therefort A is δ – closed .

2- clearly

Proposition 3.12

Let A be an open or dense (resp . δ – closed) subset of a space X , and $B \in \delta o(X)$ (resp . $B \in g\delta c(X)$) then $A \cap B \in \delta o(A)$ (resp . $A \cap B \in g\delta c(A)$) .

Proof :-

Clearly that if A is dense set .

To prove that $A \cap B \in \delta o(A)$, let $B \in \delta o(X)$, so $B = \bigcup_{i=1}^{n} u_i$, $u_i \in Ro(X)$, if A is open in X ,

$$(A \cap B)_A = \left(A \cap \left(\bigcup_{i=1}^{n} (cl(\text{int}(u_i))) \right) \right) \cap A = \left(\bigcup_{i=1}^{n} ((\text{int}(A)) \cap \text{int}(cl(u_i))) \right) \cap A = \bigcup_{i=1}^{n} (\text{int}(cl(A) \cap u_i) \cap A) \subset$$

$$\left[\bigcup_{i=1}^{n} (\text{int}(cl(A \cap u_i))) \right] \cap A$$

So $(A \cap B)_A \subset \left[\bigcup_{i=1}^{n} (\text{int}(cl(A \cap u_i))) \right] \cap A$

Also

$$\left[\bigcup_{i=1}^{n} (\text{int}(cl(A \cap u_i))) \right] \cap A \subset \bigcup_{i=1}^{n} (\text{int}(cl(A \cap cl(u_i)) \cap A)) \subset \bigcup_{i=1}^{n} (\text{int}(cl(A) \cap cl(u_i)) \cap \text{int}(A))$$

$$\subseteq \bigcup_{i=1}^{n} (\text{int}(cl(A) cl(u_i)) \cap A) = \bigcup_{i=1}^{n} ((\text{int}(cl(A)) \cap \text{int}(cl(u_i))) \cap A) = \bigcup_{i=1}^{n} ((\text{int}(cl(A))) \cap A) \cap ((\text{int}(cl(u_i))) \cap A)$$

$$= \bigcup_{i=1}^{n} (\text{int}(cl(u_i)) \cap A)_A = \left(\bigcup_{i=1}^{n} (\text{int}(cl(u_i))) \cap A \right) = (A \cap B)_A$$

so $\left[\bigcup_{i=1}^{n} (\text{int}(cl(A \cap u_i))) \right] \cap A \subset (A \cap B)_A$

Therefore $(A \cap B)_A = \left[\bigcup_{i=1}^{n} (\text{int}(cl(A \cap u_i))) \right] \cap A$

To prove that $A \cap B \in g\delta c(A)$, let $A \in \delta c(X)$ and $B \in g\delta c(X)$, let $p \in clA(A \cap B)$, so

$p \in (cl(A \cap B) \cap A)$

$p \in (cl(A) \cap cl(B) \cap A)$, since $A \in \delta c(X)$, $cl(A) = cl\delta(A) = A$, therefore

$p \in cl\delta(A) \wedge p \in cl(B)$ and $p \in A$.

$p \in cl\delta(A)$, this means that for every $u \in Ro(X)$, $p \in u$ and $u \cap A \neq \phi$ so $A \subset u$ and $u \in Ro(X)$ also $p \in cl(B)$, this means that for every N open in X and $p \in N$,

$N \cap B \neq \phi$, so $B \subset N$ and $p \in A$, by proposition (3.1) (1) $u \subset N$, so $u \cap N \neq \phi$ and $u \cap N \in Ro(X)$, so $(u \cap N) \cap A \in Ro(A)$. Thus $p \in (A \cap B) \cap A \subset (u \cap N) \cap A$ $(u \cap N) \cap A \in Ro(A)$ by proposition (3.4) (1) , $(u \cap N) \cap A \in \delta o(A)$ therefore $clA(A \cap B) \subset (u \cap N) \cap A$, when ever $(A \cap B)_A \subset (u \cap N)_A$. Thus $A \cap B \in g\delta c(A)$.

proposition 3.13

Let $Y \subseteq X$, if $A \in \delta o(Y)$ (resp. $A \in g\delta o(Y)$) and $Y \in \delta o(X)$ (resp . $Y \in gco(X)$), then $A \in \delta o(X)$ (resp . $A \in g\delta o(X)$) .

Proof :-

To prove that $A \in g\delta o(X)$

Let $A \in g\delta o(Y)$, $A^c = v \in g\delta c(X)$, let $G \in \delta o(X)$, where $v \in G$. Then $v \subseteq G \cap Y$.

Since v is $g\delta$ - closed relative to Y .

We have $cl(v) \subseteq G \cap Y$, so $cl(v) \cap Y \subseteq G$.

Since $(Y \cap cl(v)) \cup (X \setminus cl(v)) \subseteq G \cup X \setminus cl(v)$, it follows that $Y \cap X \subseteq G \cup (X \setminus cl(v)) = u \in \delta o(X)$, since $Y \in gco(X)$ then $cl(Y) = \operatorname{int}(Y) = Y \subseteq u$.

But $cl(v) \subseteq cl(Y)$. Therefore $cl(v) \subseteq u = G \cup (X \setminus cl(v))$ which implies that $cl(v) \subseteq G$, $v \in g\delta c(X)$. Thus $v^c = (A^c)^c = A \in g\delta o(X)$.

To prove that $A \in \delta o(X)$ suppose that $A^c = B \in \delta c(Y)$ so $B = cl\delta(B)$ and $Y = cl\delta(Y) \in \delta c(X)$

$$cl\delta(B) \cap (Y)_X = cl\delta(B) \cap (Y \cap X) = cl\delta(B) \cap (cl\delta(Y) \cap X) = (cl\delta(B) \cap cl\delta(Y)) \cap (cl\delta(B) \cap X)$$

$$= cl\delta(B \cap Y) \cap (cl\delta(B) \cap X) = cl\delta(B) \cap (cl\delta(B) \cap X) = (cl\delta(B) \cap X) = B \cap X$$

$B \in \delta c(X)$, thereforte $B^c = (A^c)^c = A \in \delta o(X)$

Lemma 3.14 [Dontchev,2000**]**

If X is almost weakly hausdorff space , let $A \subseteq X$ is $g\delta$ - closed then it is closed set .

Lemma 3.15 [Dontchev,2000**]**

For anon – empty topological space (X,T) the following conditions are equivalent :

1- X is hyperconnected .

2- Every subset of X is $g\delta$ - closed and X is connected .

4. Continuity of contra $(\delta, g\delta)$ - continuous functions

Definition 4.1

A function $f : X \to Y$ is called :

1- contra δ – cont . (resp . contra $g\delta$ - cont .) at a point $x \in X$, if for each open subset v in Y containing $f(x)$, there exists a δ – closed (resp . $g\delta$ - closed) subset u in X containing x such that $f(u) \subset v$.

2- contra δ – cont . (resp . contra $g\delta$ - cont .) if it have this property at each point of X .

Example 4.2

Let $X = Y = \{a,b,c\}$, $T_X = \{\phi, X, \{a\}, \{b\}, \{a,b\}, \{b,c\}\}$ and $T_Y = \{\phi, Y, \{c\}, \{b,c\}, \{a,c\}\}$ are topological spaces defined on X and Y respectively , let $f : X \to Y$ be the Identity function .

Note that f is contra $(\delta, g\delta)$ - continuous functions **.**

Proposition 4.3

For a function $f : X \to Y$, the following are equivalent :

1- f is contra δ – cont . (resp . contra $g\delta$ - cont .) .

2- for every open set v of Y , $f^{-1}(v) \in \delta c(X)$ (resp . $f^{-1}(v) \in g\delta c(X)$).

3- for every closed V of Y , $f^{-1}(v) \in \delta o(X)$ (resp . $f^{-1}(v) \in g\delta o(X)$) .

Proof :-

$1 \to 2$: Let v be an open subset of Y , and let $x \in f^{-1}(v)$, since $f(x) \in v$, there exists $u_x \in \delta c(x)$ (resp . $u_x \in g\delta c(X)$ containing x such that $u_x \in f^{-1}(v)$. We obtain that $f^{-1}(v) = \bigcup_{x \in f^{-1}(v)} u_x$ by lemma (3.7) $f^{-1}(v)$ is δ – closed (resp . $x \in f^{-1}(v)$, $g\delta$ - closed) in X .

$2 \to 3$: Let V be a closed subset of Y . Then Y/v is open , by part (2) , $f^{-1}(Y/v) = X/f^{-1}(v)$ is δ – closed (resp . $g\delta$ - closed) , thus $f^{-1}(v)$ is δ – open (resp . $g\delta$ - open) in X

$3 \to 1$: Let V be a closed subset of Y containing $f(x)$, from part (3) , $f^{-1}(v)$ is δ – open (resp . $g\delta$ - open) , take $u = f^{-1}(v)$. Then $f(u) \subset v$. Therefore f is contra δ – cont . (resp . contra $g\delta$ - cont .) .

Proposition 4.4

Let $f : X \to Y$ be contra δ – cont . (resp . contra $g\delta$ - cont .) functions , if $u \in o(X)$ or dense (resp . $u \in \delta c(X)$) , then $f/u : u \to y$ is contra δ – cont . (contra $g\delta$ - cont .)

Proof :-

To prove that f/u is contra $\delta -$cont .

Let V be aclosed in Y . $(f/u)^{-1}(v) = f^{-1}(v) \cap u$, since f is contra $\delta -$ cont . , $f^{-1}(v) \in \delta o(X)$. By proposition (3.12) , $f^{-1}(v) \cap u \in \delta o(u)$ thus f/u is contra $\delta -$cont .

Remark 4.5

The converse of proposition (4.4) is not true in general . To see this, we give the following counter example .

Example 4.6

Let $X = Y = \{a,b,c\}$, $T_X = \{\phi, X, \{a\}\}$ and $T_Y = \{\phi, Y, \{a\}\}$ are topological space defined on X and Y respectively , let $f : X \to Y$ be the Identity function .

Let $A = \{a\} \subset X$, see that $f/A : A \to Y$ is contra $(\delta, g\delta)$ - cont . But f is not contra $(\delta, g\delta)$ - cont . function .

Proposition 4.7

Let $f:X\to Y$ be a function and let $\{u_\alpha | \alpha\in\Lambda\}$ be a cover of X such that $u_\alpha\in\delta o(X)$ (resp . $u_\alpha\in gco(X)$) , if $f/u_\alpha : u_\alpha\to Y$ is contra δ – cont . (resp . contra $g\delta$ - cont .), for each $\alpha\in I$, then f is contra δ – cont . (resp . contra $g\delta$ - cont .)

Proof :-

Let v be any closed subset of Y , $f^{-1}(v)=\bigcup_{\alpha\in I}\left(f^{-1}(v)\cap u_\alpha\right)=\bigcup_{\alpha\in I}\left(f/u_\alpha\right)^{-1}(v)$, since f/u_α is contra δ – cont . (resp . contra $g\delta$ - cont .) for each $\alpha\in I$. So by proposition (3.8) and proposition (3.12) , $f^{-1}(v)\in\delta c(X)$ (resp . $f^{-1}(v)\in g\delta c(X)$) . Thus we have f is contra δ – cont . (resp . contra $g\delta$ - cont .)

Remark 4.8

The composition of two contra - continuous functions need not be contra $(\delta, g\delta)$ - continuous functions . To see this we give the following counter example .

Example 4.9

1- Let $X=Y=Z=\{a,b,c\}$, $T_X=\{\phi,X,\{a\},\{b\},\{a,b\}\}$, $T_Y=\{\phi,Y,\{b,c\}\}$ and $T_Z=\{\phi,Z\}$ are topological spaces defined on X , Y and Z respectively . Let $f:X\to Y$ and $g:Y\to Z$ be the Identity functions .

Note that f and g are contra δ – cont . , but $g\circ f:X\to Z$ is not contra δ – cont .

Since $(g\circ f)(a)=a\in Z, \exists u=\{a,c\}$ is δ – closed in X .

$(g\circ f)(u)=g(f(u))=g(f\{a,c\})=g(\{a,c\})=\{a,c\}\subset Z$ but $\{a,c\}$ is not open and not δ – closed in Y .

2- Let $X=Y=\{a,b,c\}$, $T_X=\{\phi,X,\{a\},\{b\},\{a,b\}\}$ and $T_Y=\{\phi,Y,\{c\},\{b,c\},\{a,c\}\}$ are topological space defined on X and Y respectively . Let $f:X\to Y$ and $g:Y\to X$ be the Identity functions .

Note that f and g are contra $g\delta$ – cont . functions but $g\circ f:X\to X$ is not contra $g\delta$ – cont . functions .

Remark 4.10

The composition of contra δ – cont . and contra $g\delta$ – continuous need not be contra δ – cont . (resp . contra $g\delta$ - cont .) . To see this we give the following counter example .

Example 4.11

Let $X=Y=Z=\{a,b,c\}$, let $T_X=\{\phi,X,\{a\},\{b\},\{a,b\},\{b,c\}\}$, $T_Y=\{\phi,Y,\{b,c\}\}$ and $T_Z=\{\phi,Z,\{c\},\{b,c\},\{a,c\}\}$ are topological space defined on X , Y and Z respectively . Let $f:X\to Y$ and $g:Y\to Z$ be the Identity functions .

Note that f is contra δ – cont . and g contra $g\delta$ - cont . but $g\circ f:X\to Z$ is not contra $(\delta, g\delta)$ - cont .

Definitions 4.12 [Dontchev,2000] , [Noiri,1980] , [Noiri,1986] , [Ling and Reilly,1996] , [Noiri,2001]

A function $f : X \to Y$ is called :

1- $g\delta$ - cont . (resp . $g\delta$ - irresolute , δ – closed , contra cont .) if $f^{-1}(v)$ is $g\delta$ - closed (resp . $g\delta$ - closed , δ – closed , closed) in X , for each closed (resp . $g\delta$ – closed , δ – closed , open) set v of Y .

2- δ – cont . if for each $x \in X$ and each open set v of Y containing $f(x)$, there exist an open set u of X containing x such that $f(\text{int}(cl(u))) \subset \text{int}(cl(u))$.

3- Super cont . (resp . perfectly cont . , Rc – cont . δ* – cont if $f^{-1}(v)$ is open (resp . clopen , regular closed , δ – open) set in X .

Proposition 4.13

Let $f : X \to Y$ and $g : Y \to Z$ be a functions , the following properties hold :

1- If f is δ – closed (resp . contra δ – cont . if X is T_δ - space) and f is contra $g\delta$ - cont . and g contra $g\delta$ - cont . (resp .g is open cont .) , then $g \circ f$ is contra δ - cont .

2- If Y is T_δ - space , f is $g\delta$ - cont . , and g is contra δ - cont . , then $g \circ f$ is contra $g\delta$ - cont .

3- If f is $g\delta$ - irresolute (resp . contra $g\delta$ - cont . , g - cont .) , and g is contra $g\delta$ - cont . (resp . open cont . , contra cont .) , then then $g \circ f$ is contra $g\delta$ - cont .

4- If Y^{δ} is almost weakly hausdorff space , f is $g\delta$ - irresolute and g is contra cont . , then $g \circ f$ is contra $g\delta$ - cont .

5. Relation ships

Proposition 5.1

If $f : X \to Y$ is contra δ – cont . , then f is contra – cont . (resp . contra $g\delta$ - cont .) .

Proof :-

By definition (4.1) and proposition (3.1) (2) , proposition (3.4) (2) .

Remark 5.2

The converse of theorem (5.1) is not true in general, we will give the sufficient condition before the following example .

Example 5.3

Let $X = Y = \{a,b,c\}$, let $T_X = \{\phi, X, \{a\}, \{a,b\}\}$ and $T_Y = \{\phi, Y, \{c\}, \{b,c\}\}$ be topological spaces defined on X and Y respectively , let $f : X \to Y$ is the Identity function .

Note that f is contra cont . and contra $g\delta$ - cont . but not contra δ - cont .

Proposition 5.4

A function $f : (X,T) \to (Y,T)$ is contra δ - cont . , if and only if $f : (X,T_\delta) \to (Y,T)$ is contra cont . (resp . contra $g\delta$ - cont .) .

Proof :-

$\Rightarrow$ By proposition (5.1)

$\Leftarrow$ By definition (4.11) and lemma (3.11) (1) (resp. by definition (4.1) and lemma (3.11) (2)

Proposition 5.5

If $f : X \to Y$ is a perfectly cont . function , then f is Rc – cont .

Proof :-

By definition (4.12) and proposition (3.9) .

Proposition 5.6

If $f : X \to Y$ is Rc – cont . function , then f is contra δ – cont .

Proof :-

By definition (4.12) and by proposition (3.4) (1) .

Proposition 5.7

If $f : X \to Y$ is contra δ – cont . , then f is contra $g\delta$ – cont .

Proof :-

By definition (4.1) and by proposition (3.4) (2)

Remark 5.8

The converse of theorem (5.5) is not true in general . To see this, we give the following counter example .

Example 5.9

Let $X = Y = \{a, b, c\}$, let $T_X = \{\phi, X, \{b\}, \{c\}, \{b, c\}\}$ and $T_Y = \{\phi, Y, \{a, c\}\}$ be topological spaces defined on X and Y respectively , let $f : X \to Y$ is the Identity function .

Note that f is Rc – cont . and contra $(\delta, g\delta)$ - cont . but not perfectly cont .

Proposition 5.10

Let $f : X \to Y$ be a function , the following statement are equivalent :

1- f is perfectly continuous .

2- f is super cont . and Rc – cont .

3- f is δ^* - cont . and contra δ – cont .

4- f is δ^* - cont . and contra $g\delta$ – cont .

Proof :-

The proof is immediately from proposition (3.9)

Remark 5.11

The converse of theorem (5.10) is not true in general . Also see example (5.9) where f is Rc – cont . and contra $(\delta, g\delta)$ - cont . but not super cont . δ^* - cont . , and perfectly cont .

Remark 5.12

The concepts of contra $(\delta, g\delta)$ - cont . and $(\delta, g\delta)$ - cont . are independent of each other to see this we give the following counter example .

Example 5.13

1- Let $X=\{a,b,c\}$, $T_X=\{\phi,X,\{b\},\{c\},\{b,c\}\}$ be a topology on X , let $f:X\to X$ is the Identity function .

Note that f is δ – cont . but not contra $(\delta,g\delta)$ - cont and $g\delta$ - cont .

2- Let $X=Y=\{a,b,c,d\}$, let $T_X=\{\phi,X,\{a\},\{b\},\{a,b\},\{a,b,c\},\{a,b,d\}\}$ and $T_Y=\{\phi,Y,\{b,c,d\}\}$ be topological spaces defined on X and Y respectively , let $f:X\to Y$ is the Identity function .

Note that f is contra $(\delta,g\delta)$ - cont . but not $g\delta$ - cont . and it is δ - cont .

Proposition 5.14

If $f:X\to Y$ is contra δ – cont . , and Y is connected , then f is δ – cont .

Proof :-

Let $f(x)\in v$, $v\in c(Y)$, since f is contra δ – cont . , there exist $u\in\delta o(X)$, such that $x\in u$ and $f(u)\subset v$ since Y is connected , so the only open and closed sets are ϕ,Y , so $v=cl(v)=\mathrm{int}(v)=Y$.

Thus $f(u)\subset\mathrm{int}(cl(v))$, also there exists $u_1\subset u$ and $x\in u_1$ by proposition (3.4) and $u_1\in Ro(X)$, so $f(u_1)\subset f(u)$, thus $f(u_1)\subset f(u)\subset\mathrm{int}(cl(v))$

Therefort f is δ – cont .

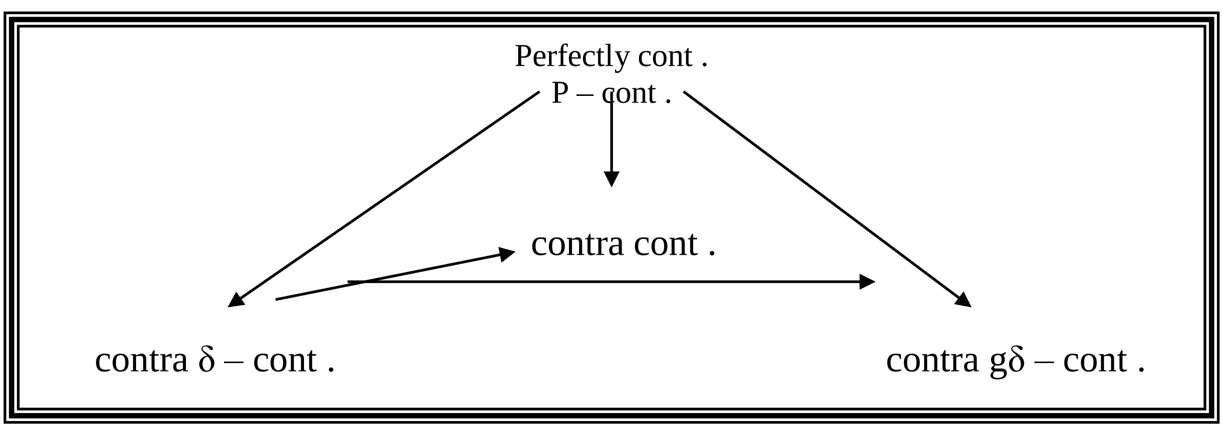
Proposition 5.15

If $f:X\to Y$ is contra $g\delta$ - cont . , and Y is hyper connected , then f is $g\delta$ – cont .

Proof :-

Let $f(x)\in v$, $v\in o(Y)$, since f is contra $g\delta$ – cont . , there exist $u\in g\delta c(X)$, such that $x\in u$ and $f(u)\subset v$, since Y is hyperconnected , $f(u)\subset v=\mathrm{int}(v)\subset cl(v)=Y$ therefort f is $g\delta$ – cont .

By the above stated results , the inter relations of these functions are decided ; the refore , we obtain the following



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