Calculation Energy Levels, The Reduced Electric Transition Probability B(E2) of the Even-Even $^{144-146}$Nd Isotopes Using IBM-1

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Abstract

In this search, $Nd^{146,144}$ isotopes have been studied by the interacting boson model (IBM-1) to determine the energy levels. In addition, by the program IBMT was used for evaluating the reduced electric transition probability B (E2). According to the IBM , The calculated values are compared with the available theoretical and experimental data and showd reasonable agreement.

And was reached ,Nd^{144} isotopes have been shown their membership to the vibrational limit SU(5), while Nd^{146} isotopes belong to the transition region between SU(5)-O(6).

Keywords: Isotope; Energy levels; Interacting boson model; Transition probabilities .

الخلاصة

تم بحث التركيب النووي لمستويات الطاقة والانتقالات الكهرومغناطيسية لنظائر النيديميوم($Nd^{146,144}$) من خلال نموذج البوزونات المتفاعلة الأول(IBM−1), لتحديد مستويات الطاقة لهذه النظائر, بالإضافة إلى استخدام برنامج(IBMT) لحساب قيمة احتمالية الانتقال رباعي القطب الكهربائي المختزل B (E2). ووفقا لبرنامج (IBM) تم الحصول على قيم نظرية لكل من مستويات الطاقة واحتمالية الانتقال رباعي القطب الكهربائي المختزلة وبعد مقارنتها بالقيم العملية لوحظ تطابق تام بين القيمتين وتم التوصل من خلال هذه القيم ان النظير (Nd^{144}) ينتمي للتحديد ((SU(5)) اما النظير (Nd^{146}) فوجد انه ضمن المنطقة الانتقالية (SU(5)−O(6).

الكلمات المفتاحية:النظائر, مستويات الطاقة , نموذج البوزونات المتفاعلة , احتمالية الانتقالية .

Introduction

Arima and Iachello (1979) [Vidya and Gupta ,2012] have developed the interacting boson model (IBM), which is based on the well-known shell model and ongeo metrical collective model of the atomic nucleus. This model is to describe nuclear properties such as spins ,energies of the levels, decay probabilities for the emission of gamma quanta ,probabilities of electromagnetic transition and their reduced matrix elements for different transitions multipole moment and mixing ratios [Casten, and Warner, 1988],[Scholten *et al.*, 1978].The (IBM-1) is used in the present work, this model represents very important step formed in the description of collective nuclear excitations. The underlying U(6) group structure of model basis leads to a simple Hamiltonian which is capable of describing the three specific limits of collective structure vibrational SU(5), rotational SU(3) and gamma unstable O(6) [Yu-Xin and Haigng, 2006],[Arima, and Iachello, 1976].In the simplest version of the interacting boson model (IBM-1), it is assumed that low-lying collective states in even-even nuclei away from closed shells are dominated by excitation of the valence protons and the valence neutrons (particles outside the major closed shell)while the closed shell core is inert. Furthermore, it is assumed that the particle configurations which are most important in shaping the properties of the low-lying states are those in which identical particles are coupled together forming pairsof angular momentum 0 and 2 [Scholten,1980],[Iachello,1979],[Gelbreg and Zemel, 1980].

2. (IBM-1) Model

The IBM-1 model was applied to the positive parity low-lying states in even-even $Nd^{146,144}$ isotopes. The proton, π, and neutron, υ, bosons are treated as one boson and the system is considered as an interaction between s-bosons and d – bosons .Creation (s+,d+) and annihilation (s,d) operators are for s and d bosons.[P.D. Duval and Barre,1982].

2.1. The Hamiltonian Operator of The (IBM-1) :

The Hamiltonian employed for the present calculation is given as [A.D. Jackson and Iachello, 1982],[F.Iachello, 1981]:

$$\hat{H} = \varepsilon\hat{n}_d + a_o\hat{P}.\hat{P} + a_1\hat{L}.\hat{L} + a_2\hat{Q}.\hat{Q} + a_3\hat{T}_3.\hat{T}_3 + a_4\hat{T}_4.\hat{T}_4 \text{.............}(1)$$

Where;

(n_d) is the number of boson; (P.P, L.L, Q.Q, T3.T3 and T4.T4) represent pairing, angular momentum, quadrupole ,octupole and hexadecupole interactions between the bosons respectively; (ε)is the boson energy; and (a_o, a_1, a_2, a_3,a_4) are the strengths of the pairing, angular momentum, quadrupole, octupole and hexadecupole interactions respectively.[Iachello, 1980],[Greiner and Maruhn, 1996].

2.2. Vibrational Limit SU(5) :

Hamiltonian function operator for dynamical symmetry SU(5) in terms of creation and annihilation operators can be given according to the following equation [Arima and Iachello, 1987] .

$$\hat{H}^{(I)} = \varepsilon\hat{n}_d + a_1\hat{L}.\hat{L} + a_3\hat{T}_3.\hat{T}_3 + a_4\hat{T}_4.\hat{T}_4 \text{.....}(2)$$

2.3. SU(5)→O(6) Transition Region :

In this region nuclei has transitional properties between (SU(5)) and (O(6)) and the Hamiltonian is given by [Regan and Beausang, 2003] .

$$\hat{H}^{(I+III)} = \varepsilon\hat{n}_d + a_o\hat{P}.\hat{P} + a_1\hat{L}.\hat{L} + a_3\hat{T}_3.\hat{T}_3 \text{.....}(3)$$

Nuclei fall in this region depends on the ratio (ε / a_0). When this ratio is large the properties will be near to SU(5) limit and when this ratio is small the properties will be near to O(6).

2.4. Transition Rates :

The electric quadrupole transition operator employed in this study is given by [Pfeifer, 1998] :

$$T_\mu^{(E2)} = \alpha_2\left[\hat{d}^+ \times \hat{\tilde{s}} + \hat{s}^+ \times \hat{\tilde{d}}\right]_\mu^{(2)} + \beta_2\left[\hat{d}^+ \times \hat{\tilde{d}}\right]_\mu^{(2)} \text{.....}(3)$$

Where;

(α_2)and(β_2)are the parameters which are used to describe the different terms in operator. The reduced electric quadrupole transition rates between Li→Lf states are given by [Arima, and Iachello, 1987] :

$$B(E2; L_i \to L_f) = \frac{1}{2L_i + 1} \left| \left\langle L_f \left\| \hat{T}^{(E2)} \right\| L_i \right\rangle \right|^2 \quad .....(4)$$

3. Results and Discussion

3.1. Energy Levels :

In this work we have studied the nuclear structure of even-even Nd (A=144) isotope which is classified to Vibrational dynamical and even-even Nd (A=146)isotope which is classified to O(6)→SU(5) transition region .

by comparing the energy ratios $\frac{E0_2^+}{E2_1^+}, \frac{E8_1^+}{E2_1^+}, \frac{E6_1^+}{E2_1^+}, \frac{E4_1^+}{E2_1^+}$ with ideal values [E. Ganio, Wyss and Magierski, 2013],[R.K.Bhat and Rani , 2003] for three dynamical symmetries SU(5), O(6) and SU(3) of IBM-1 (shown in figures (1) to (4)).Table (1) presents the isotopes used in the present work according to its atomic mass number, total number of boson and the corresponding Hamiltonian parameters used in the IBM- Code according to O(6)→SU(5) transition region (in Nd^{146}) and Vibrational dynamical symmetry SU(5) (in Nd^{144}).

Tables (2) and (3) and figures (5) and (6) present values of the energy levels (present work) according to energy bands in comparison with available experimental data.[Chuu *et al.*, 1984],[Sakai ,1984], [Long and Shengjiang,1998] .

This table lists the new energy levels, with their spins and parties. It is to get the best match between the values of the practical and theoretical energy levels Where we have got an exact match for Nd^{144} isotopes for level $2_1^+$and level 4_1^+which values the theoretical and practical were(0.69692 , 0.96965) , (1.30515 , 1.3146) respectively and had other levels compatible well , as well as for Nd^{146} isotopes there is an excellent match for level $2_1^+$and level 6_1^+ which the values of the theoretical and practical were (0.4359 , 0.4537) , (1.71122 , 1.7800) respectively.[Sonzogni, 2001],[Peker, and Tuli, 1997],[Bhat, 2000],[Dermateosian, and Tuli, 1995] .

3.2. The Reduced Electric Transition Probability B (E2):

More information can be obtained by studying the reduced transition probabilities B(E2). The (IBMT-code) have been employed. The parameters E2SD and E2DDwhich are used in the present calculations have been determined. where;
E2SD=α_2 and E2DD= 5 β_2Table (4) presents the values of (E2SD) and (E2DD) used in the present work with the experimental values of B(E2) taken from ref. [Suzan,2005].

4. Conclusions

The interacting boson model version one (IBM-1) gives us a very closing value with the experiment. Since the energy levels depend on the total boson number so that only the ground state band will appear. The even-even $Nd^{146,144}$ isotopes have (60) protons and (70,90) neutrons respectively. The core is taken at major closed shell (82) for protons and neutrons. Therefore, the number of bosons were determined for Nd^{144}andNd^{146}, is equal (12) and (13) bosons respectively .Hamiltonian parameters in table (1) are very small so that these parameters vary to any change may occur in any one of these parameters, so that it is difficult to get the coincidence values between the energy levels in high energy have states .In case of quadruple electrical transitions

B(E2) for even–even nuclei, we found that the values of α_2 and β_2 parameters increase whenever the number of bosons increases in one element isotopes .

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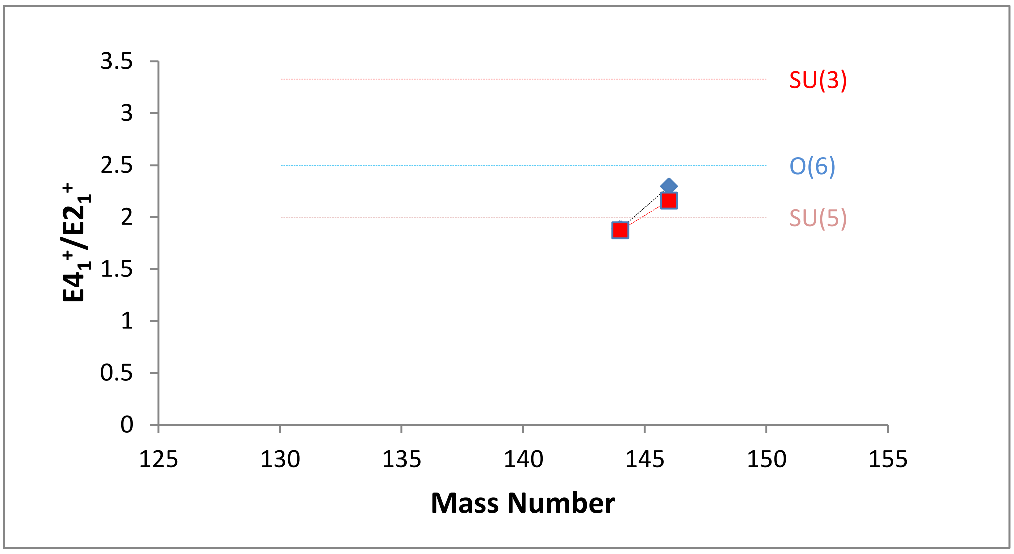

Figure (1): The Comparison of $E4_1^+/E2_1^+$ Theoretically, Experimentally[Bhat and Rani, 2003] and with the Typical Values [Casten, and Warner, 1988] , [Crowley *et al.*,1971] for Each Limit.

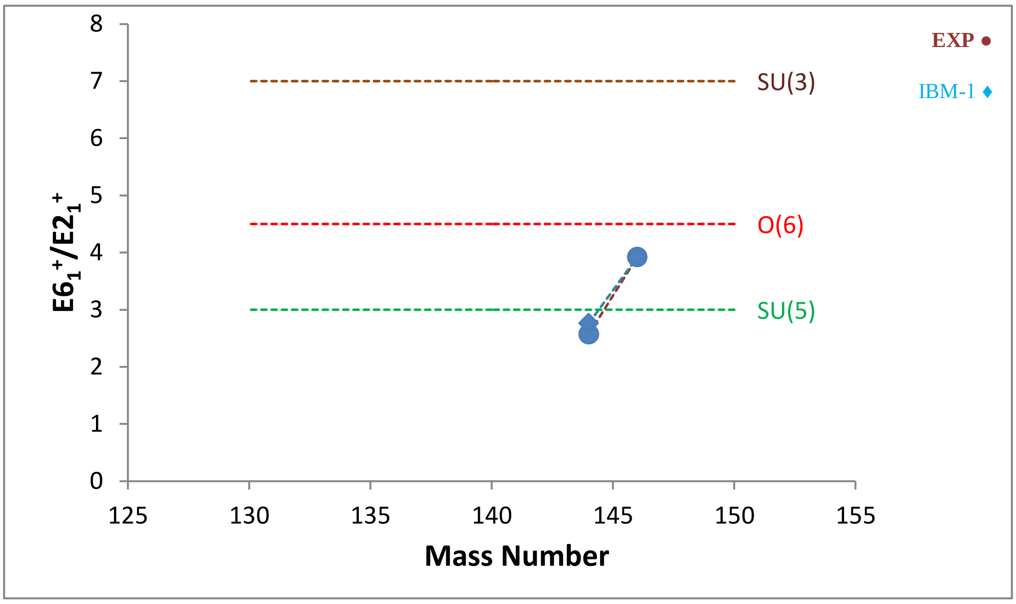

Figure (2): The Comparison of $E6_1^+/E2_1^+$ Theoretically, Experimentally [R.KBhat and Rani , 2003] and with the Typical Values [Casten, and Warner, 1988],[Crowley *et al.*, 1971] for Each Limit.

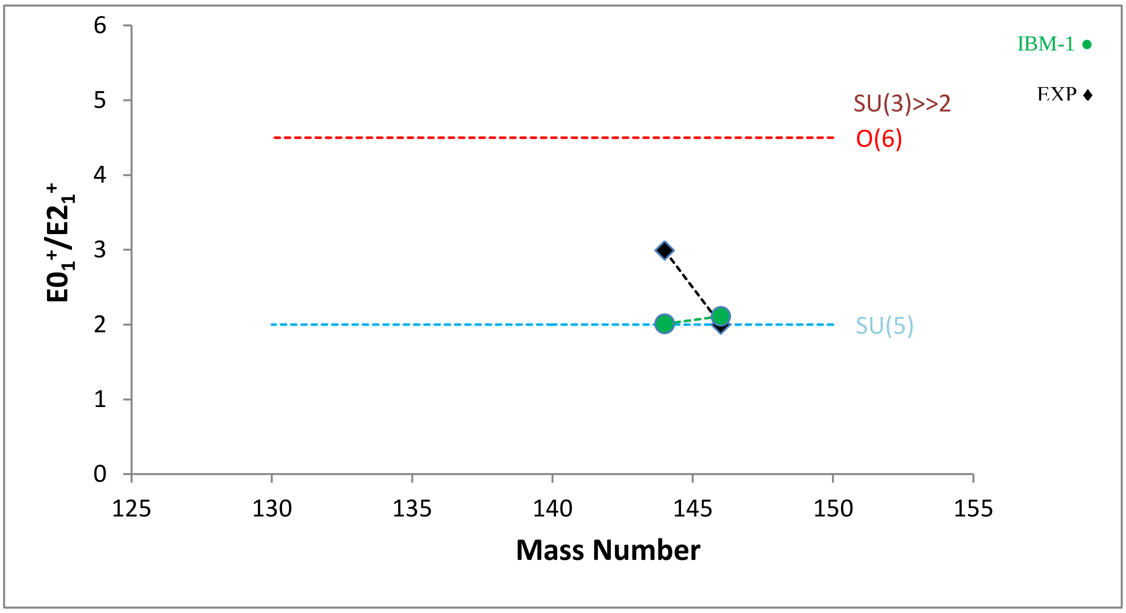

Figure (3): The Comparison of $E0_2^+/E2_1^+$ Theoretically, Experimentally [Bhat and Rani , 2003] and with the Typical Values [Casten, and Warner, 1988],[Crowley *et al.*, 1971] for Each Limit.

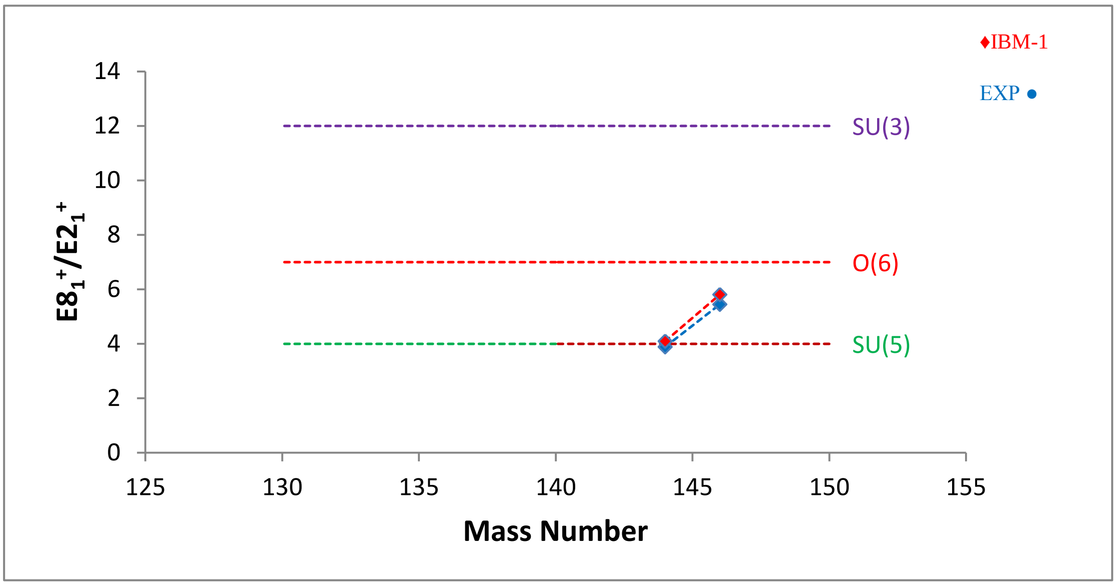

Figure (4): The Comparison of $E8_1^+/E2_1^+$ Theoretically, Experimentally [Bhat and Rani , 2003] and with the Typical Values [Casten, and Warner, 1988],[Crowley *et al.*, 1971] for Each Limit.

Table (1): The Hamiltonian Parameters Used in the IBM-Code for $Nd^{146,144}$ Isotopes.

Nucleus	EPS	P.P.	L.L.	Q.Q.	$T_3.T_3$	$T_4.T_4$	CHI	SO6
6	0.6400	0.0000	0.0010	0.0000	0.0141	0.0173	0.0000	1.0000
7	0.6460	0.4289	0.0224	0.0000	0.0188	0.0000	-0.3298	1.0000

Table (2): Comparison between Experiment [D.S.chuu , Han , Hsieh and King, 1984] and Calculated Energy Levels in MeV for Nd^{144}.

J^{π}	E_{exp} (Mev)	E_{cala} (Mev)
0^+_1	0	0
2^+_1	0.6965	0.69692
4^+_1	1.3146	1.30515
6^+_1	1.7914	1.92469
8^+_1	2.7101	2.85555
10^+_1	3.6727	3.59773
12^+_1	4.3547	4.35122
14^+_1	5.3787	----
0^+_2	2.0846	1.40470
2^+_2	1.5609	1.68821
4^+_2	2.1097	2.16093
6^+_2	2.2183	2.50497
8^+_2	2.8765	3.26032
10^+_2	3.9105	4.42699
0^+_3	2.3281	2.13387
2^+_3	2.0729	2.19725
4^+_3	2.2954	2.58111
6^+_3	2.7754	3.03629
8^+_3	2.9724	3.40278
0^+_4	2.6756	2.88067
2^+_4	2.347	2.90417
4^+_4	2.4517	2.94120
6^+_4	2.8088	3.40973
8^+_4	----	4.28958

Table (3): Comparison between Experiment [D.S.chuu , Han , Hsieh and King, 1984] and Calculated Energy Levels in MeV for Nd^{146} .

J^{π}	E_{exp} (Mev)	E_{cala} (Mev)
0^+_1	0	0
2^+_1	0.4537	0.43595
4^+_1	1.0422	0.94224
6^+_1	1.7800	1.71122
8^+_1	2.4745	2.53568
10^+_1	3.1238	3.30921
12^+_1	3.9022	4.72620
14^+_1	4.6942	5.78180
16^+_1	5.3628	----
0^+_2	0.9154	0.92042
2^+_2	1.3031	1.17864
4^+_2	1.7449	1.61842
6^+_2	2.0835	2.26368
8^+_2	2.5935	2.95801
10^+_2	3.3197	4.09580
12^+_2	3.9937	5.17220
14^+_2	4.6955	----
16^+_2	5.4605	----
0^+_3	1.572	1.43180
2^+_3	1.4705	1.65728
4^+_3	1.9188	1.77088
6^+_3	2.705	2.78601
8^+_3	2.6283	3.04460
10^+_3	----	4.54180
0^+_4	1.6025	1.45648
2^+_4	1.7873	1.97961
4^+_4	1.9889	2.29321
6^+_4	2.9058	3.37260
8^+_4	----	4.69060
10^+_4	----	5.91359

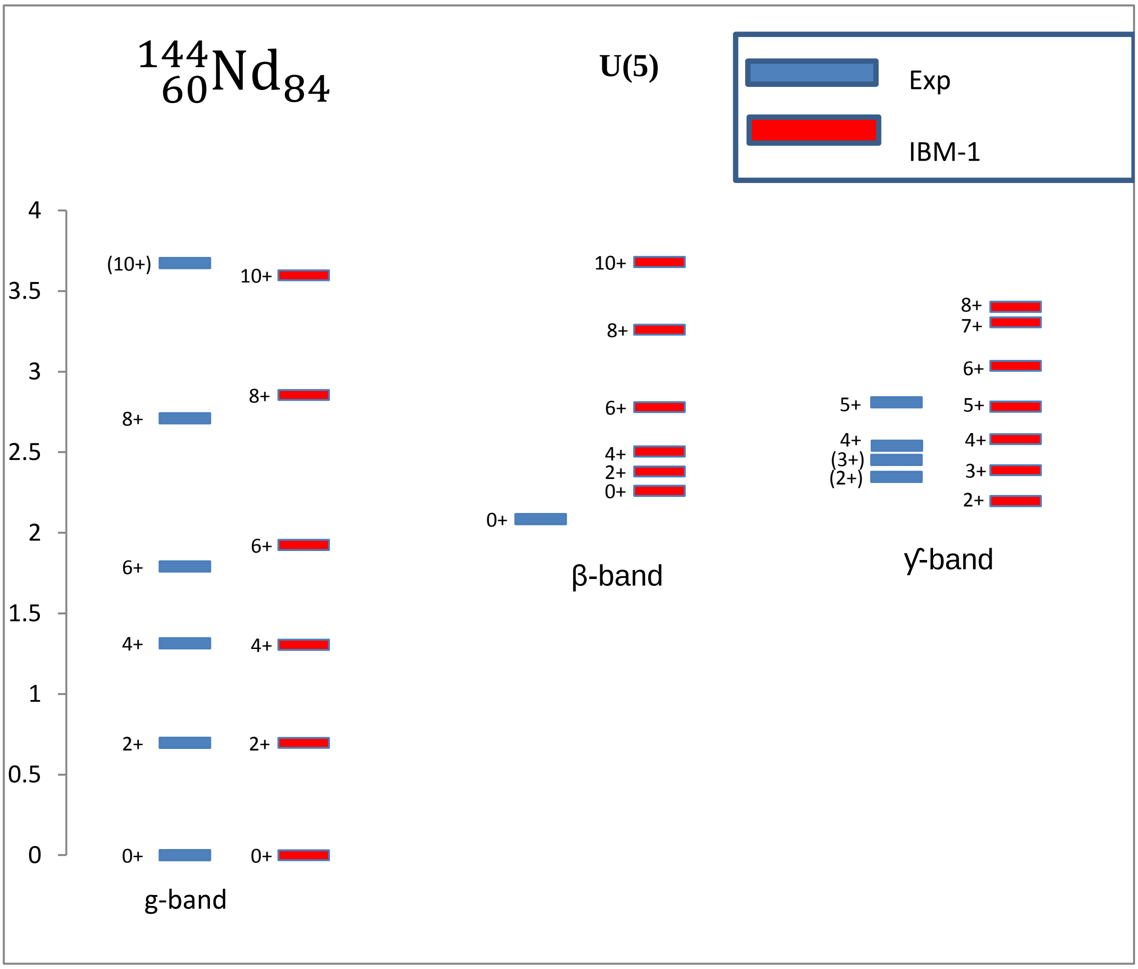

Figure (5): Comparison between Experiment [Sakai , 1984],[Long , 1998] and Calculated Energy Levels for Nd^{144} Isotope.

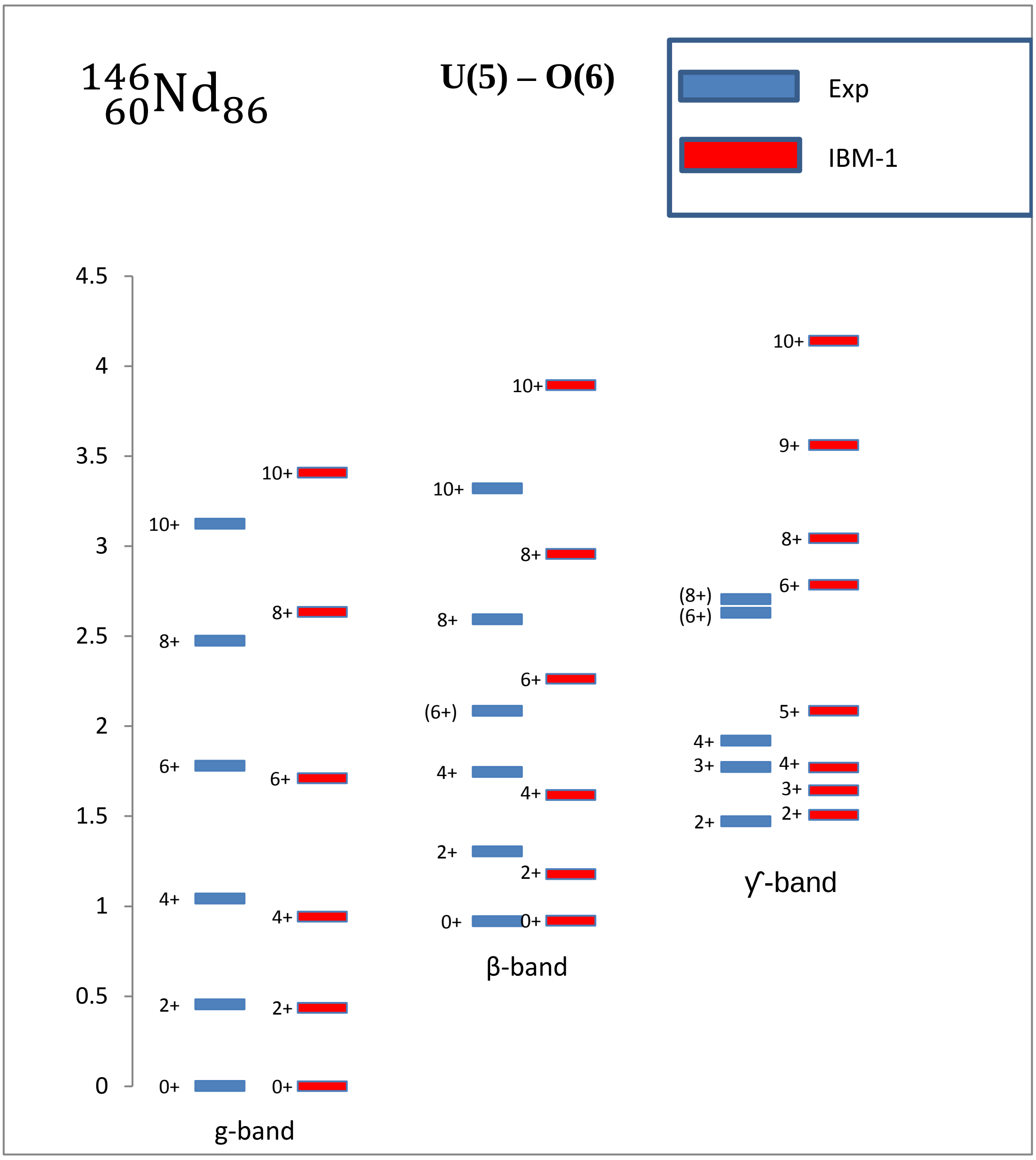

Figure (6): Comparison between Experiment [Sakai , 1984],[Long, 1998]and Calculated Energy Levels for Nd^{146} Isotope.

Table (4): The Experimental Values [Raman, *et al.*, 2001] of B(E2) and the Coefficients (E2SD, E2DD) for $Nd^{146,144}$ Used in the Present Work.

Isotope	$B(E2{:}2_{1^+}\rightarrow 0_{1^+})e^2b^2$	E2SD(eb)	E2DD(eb)
^{144}Nd	0.0495	0.05490	-0.05980
^{146}Nd	0.134	0.08900	-0.30200

Table (5): The Experimental and Calculated B(E2)↓ Using IBMT-Code and the Quadrupole Moment $Q_{21}{}^{+}$ for Nd^{144} Isotope.[Raman, *et al.*, 2001, [Long and Shengjiang ,1998].

i-f	**$B(E2)\downarrow e^2b^2$**	
	Exp.	**P.W**
$2_1^+\rightarrow 0_1^+$	0.0495	0.0497
$2_1^+\rightarrow 0_2^+$	----	0.0609
$2_2^+\rightarrow 0_1^+$	----	0.1916
$2_2^+\rightarrow 0_2^+$	----	0.0175
$2_2^+\rightarrow 2_1^+$	0.09295	0.0748
$2_3^+\rightarrow 0_2^+$	----	0.0337
$2_3^+\rightarrow 0_3^+$	----	0.000
$2_4^+\rightarrow 0_2^+$	----	0.1975
$2_4^+\rightarrow 0_3^+$	----	0.0347
$2_1^+\rightarrow 2_2^+$	----	0.0748
$4_1^+\rightarrow 2_1^+$	0.083	0.0748
$4_1^+\rightarrow 2_3^+$	----	0.0356
$4_2^+\rightarrow 2_1^+$	----	0.1594
$4_2^+\rightarrow 2_2^+$	----	0.0459
$4_2^+\rightarrow 2_3^+$	----	0.0089
$Q2_1^+$	**-0.15**	-0.1546

Table (6): The Experimental and Calculated B(E2)↓ Using IBMT-Code and the Quadrupole Moment Q_{21}^{+} for Nd^{146} Isotope.[Long and Shengjiang, 1998] , [Peker , 1990]

i-f	B(E2)↓e^2b^2	
	Exp.	P.W
$2_1^+ \rightarrow 0_1^+$	0.134	0.1305
$2_1^+ \rightarrow 0_2^+$	----	0.0010
$2_2^+ \rightarrow 0_1^+$	0.0022	0.0032
$2_2^+ \rightarrow 0_2^+$	----	0.0461
$2_2^+ \rightarrow 2_1^+$	0.1989	0.1966
$2_3^+ \rightarrow 0_2^+$	----	0.0886
$2_3^+ \rightarrow 0_3^+$	----	0.000
$2_4^+ \rightarrow 0_2^+$	----	0.0033
$2_4^+ \rightarrow 0_3^+$	----	0.0913
$2_1^+ \rightarrow 2_2^+$	----	0.1966
$4_1^+ \rightarrow 2_1^+$	0.197	0.1966
$4_1^+ \rightarrow 2_3^+$	----	0.0006
$4_2^+ \rightarrow 2_1^+$	----	0.0026
$4_2^+ \rightarrow 2_2^+$	----	0.1206
$4_2^+ \rightarrow 2_3^+$	----	0.0235
$Q2_1^+$	**-0.7809**	-0.78