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Certain Classes of Univalent Functions Associated With Deferential Operator

Abstract- The main objectives of this research work is to present and investigate the certain subfamily of Bazilevic functions by making use the differential operator which generalize of many operators presented by several authors, we have discussed and estimated on the coefficient bounds for this subfamily of functions introduced here and derived some interesting properties. In addition, Consequence of the results known or new is indicated through this work.

Keywords Analytic function, Univalent function, Bazilevic function, Coefficient bounds.

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1.Introduction

Suppose that A represent the family of functions $f(z)$ which has the following formula

$$f(z) = z + \sum_{n=2}^{\infty} c_n z^n, \quad 1 \quad (1)$$

It can be investigative in the open unit disc $\{D = |z| < 1\}$ and standardized such that

$f(0) = f'(z) = 1$. Suppose that A^ρ is the family of the functions f which has the formula

$$f(z)^\rho = (z + \sum_{n=2}^{\infty} c_n z^n)^\rho \quad (2)$$

By using binomial expansion for the equation (2) we have

$$f(z)^\rho = z^\rho + \sum_{n=2}^{\infty} c_n(\rho) z^{\rho+n-1}$$

For function $f \in A^\rho$, we define an operator

$$D_{\mu,\rho}^{m,\delta} : A^\rho \rightarrow A^\rho \text{ such that:}$$

$$D_{\mu,\rho}^{0,\delta} f(z)^\rho = f(z)^\rho$$

$$D_{\mu,\rho}^{1,\delta} f(z)^\rho = D_{\mu,\rho}^{0,\delta} \left(\frac{\mu - \delta\rho + \rho}{1 + \rho} \right) + (D_{\mu,\rho}^{0,\delta} f(z)^\rho)' \frac{\delta\rho}{1 + \rho}$$

$$= z^\rho + \sum_{n=2}^{\infty} \left(\frac{\mu + \rho + \delta(n-1)}{1 + \rho} \right) c_n(\rho) z^{\rho+n-1}$$

and

$$D_{\mu,\rho}^{2,\delta} f(z)^\rho = z^\rho + \sum_{n=2}^{\infty} \left(\frac{\mu + \rho + \delta(n-1)}{1 + \rho} \right)^2 c_n(\rho) z^{\rho+n-1}$$

and in general

$$D_{\mu,\rho}^{m,\delta} f(z)^\rho = z^\rho + \sum_{n=2}^{\infty} \left(\frac{\mu + \rho + \delta(n-1)}{1 + \rho} \right)^m c_n(\rho) z^{\rho+n-1} \quad (3)$$

where $m \in M$, $\mu \geq 0$, $\rho > 0$, $z \in D$

We introduce the subfamily involving family of

Bazilevic functions by using the above operator.

I. Definition

Suppose that $H_m^\rho(\rho, \gamma, \delta, \mu)$ is considered as the subfamily of A containing of functions $f(z)$ which has been holds true to the following condition

$$G \left(\frac{D_{\mu,\rho}^{m,\delta} f(z)^\rho}{\left(\frac{l + \rho + \delta(n-1)}{1 + \rho} \right)^m z^\rho} \right) > \gamma,$$

where

$\delta, \mu \geq 0$, $\rho > 0$ (ρ is real), $m \in Z$, $0 \leq \gamma < 1$.

We now obtain some properties of operators by using the above definition

II. Remark

(i) for $\mu = 0$, $\delta = 0$ in (3), we get

$$D_\rho^m f(z)^\rho = z^\rho + \sum_{n=2}^{\infty} \left(\frac{\rho + n - 1}{\rho} \right)^m c_n(\rho) z^{\rho+n-1}$$

Where

$\rho > 0$, $m \in M$. This operator have been given by Olatunji and Ajai [1].

(ii) for $\rho = 1$ in (3), we obtain

$$D^m f(z) = z + \sum_{n=2}^{\infty} \left(\frac{\rho + \delta(n-1) + \mu}{\mu + 1} \right)^m c_n z^\rho$$

where $\rho > 0$, $m \in M$. Oladipo and Olatunji have been studied the above operator [2].

(iii) for $\rho = 1$, $\mu = 0$ in (3), we get

$$G \left(\frac{D^{m,\delta} f(z)}{(\mu + \delta(n-1))^m z} \right) > \gamma$$

Where D_δ^m is derivative operator of Al-Oboudi,

$H_m(\delta, \gamma)$ is called new subfamily of Bazilevic function, such that $f \in H_m(\delta, \gamma)$

(iv) for $\mu = 0, \delta = 1, \rho = 1$ in (3), we have

$$G\left(\frac{D^m f(z)}{c^m z}\right) > \gamma$$

It is describe as the family of functions considered by references [3] and [4], and the derivative operator D^m is due to Salagean, that is $f \in H_m(\gamma)$

(v) for $\mu = 0, \rho = 1, \delta = 1, m = 0, \gamma = 0$ in (3), we have

$$G\left(\frac{D^0 f(z)}{z}\right) > \gamma \equiv G\left(\frac{f(z)}{z}\right) > 0.$$

Yamaguchi in [5] has been studied the above family of function $f \in H(0)$

We need auxiliary lemma to prove our main results for the functions $f \in H_m^\rho(\rho, \gamma, \delta, \mu)$ in terms of an analytic function in the unit disk D .

III. [6] Lemma

A function $g(z) \in F H_m^\rho(\rho, \gamma, \delta, \mu)$ satisfies the conditions: $G[g(z)] > 0, (z \in X)$,
iff

$$g(z) \neq \frac{\psi - 1}{\psi + 1}, (z \in X, \psi \in C, |\psi| = 1)$$

2. Coefficient inequality for the family $H_m^\rho(\rho, \gamma, \delta, \mu)$

In this section, we give some results that will be used in proving our main result.

I. Lemma

A holomorphic function $f(z)$ as shown with $H_m^\rho(\rho, \gamma, \delta, \mu)$ iff

$$1 + \sum_{n=2}^{\infty} A_n z^{n-1} \neq 0,$$

where

$$A_n = \frac{\psi + 1}{2(1 - \gamma)} \left(\frac{\mu + \rho + \delta(n-1) + \mu}{\mu + \rho} \right)^m c_n(\rho)$$

Proof: Let

$$g(z) = \frac{\frac{D_{\mu, \rho}^{m, \delta} f(z)^\rho}{z^\rho} - \gamma}{(1 - \gamma)},$$

which yields

$$g(z) = \frac{D_{\mu, \rho}^{m, \delta} f(z)^\rho - \gamma z^\rho}{(1 - \gamma) z^\rho}$$

By using Lemma 2.1, we get

$$g(z) = \frac{D_{\mu, \rho}^{m, \delta} f(z)^\rho - \gamma z^\rho}{(1 - \gamma) z^\rho} \neq \frac{\psi - 1}{\psi + 1} \quad (4)$$

From (4) we obtain

$$(\psi + 1) [D_{\mu, \rho}^{m, \delta} f(z)^\rho - \gamma z^\rho] \neq (\psi + 1)(1 - \gamma) z^\rho$$

That is

$$D_{\mu, \rho}^{m, \delta} f(z)^\rho - \gamma z^\rho + \psi D_{\mu, \rho}^{m, \delta} f(z)^\rho - \psi \gamma z^\rho \neq (\psi - \psi \gamma - 1 + \gamma) z^\rho$$

$$z^\rho + \sum_{n=2}^{\infty} \left(\frac{\mu + \rho + \delta(n-1)}{\mu + \rho} \right)^m c_n(\rho) z^{\rho+n-1} - \gamma z^\rho$$

$$+ \psi \left[z^\rho + \sum_{n=2}^{\infty} \left(\frac{\mu + \rho + \delta(n-1)}{\mu + \rho} \right)^m c_n(\rho) z^{\rho+n-1} \right]$$

$$- \psi \gamma z^\rho \neq \psi z^\rho - \psi \gamma z^\rho - z^\rho + \gamma z^\rho$$

$$2z^\rho - 2\gamma z^\rho + (\psi + 1) \sum_{n=2}^{\infty} \left(\frac{\mu + \rho + \delta(n-1)}{\mu + \rho} \right)^m c_n(\rho) z^{\rho+n-1} \neq 0$$

Yields

$$2(1 - \gamma) z^\rho + (\psi + 1) \sum_{n=2}^{\infty} \left(\frac{\mu + \rho + \delta(n-1)}{\mu + \rho} \right)^m c_n(\rho) z^{\rho+n-1} \neq 0 \quad (5)$$

Dividing equation (5) by $2(1 - \gamma) z^\rho$ we get

$$2 + \frac{(\psi + 1)}{2(1 - \gamma)} \sum_{n=2}^{\infty} \left(\frac{\mu + \rho + \delta(n-1)}{\mu + \rho} \right)^m c_n(\rho) z^{\rho+n-1} \neq 0$$

This proves Lemma 2.1.

Setting $\delta = 1, \mu = 0$, in the above Lemma, the following results can be obtained

II. Corollary

A holomorphic function $f(z)$ as shown by family

$$H_m^\rho(\rho, \gamma, \delta, \mu) = H_m^\rho(\rho, \gamma)$$

$$1 + \sum_{n=2}^{\infty} A_n z^{n-1} \neq 0,$$

Where

$$A_n = \frac{(\psi + 1)}{2(1 - \gamma)} \left(\frac{\rho + n - 1}{\rho} \right)^m c_n(\rho)$$

Which has been satisfying for the family of functions considered in references [3] & [4]. Setting $\delta = 1, \rho = 1, \mu = 0$ in Lemma 2.1, we get

III. Corollary

Suppose that $f(z)$ be a holomorphic function in the family $H_m^\rho(1, \gamma, 1, 0) = H_m^\rho(\gamma)$ if

$$1 + \sum_{n=2}^{\infty} A_n z^{n-1} \neq 0,$$

Where

$$A_n = \frac{(\psi + 1)}{2(1 - \gamma)} n^m c_n(1)$$

Also if $\delta = 1, \mu = 0, \gamma = 0$ in Lemma 2.1, we get

IV. Corollary

Let $f(z)$ as used in the family $H_m^\rho(0, 0, 1, 0) =$

$$H_m^\rho(0) \equiv F_m(\rho) \text{ iff}$$

$$1 + \sum_{n=2}^{\infty} A_n z^{n-1} \neq 0,$$

where

$$A_n = \frac{\psi + 1}{2} \left(\frac{\rho + n - 1}{\rho} \right)^m c_n(\rho).$$

Which satisfying for the subfamily of functions considered in [7].

Setting $\delta = 1, \mu = 0, m = 0$ in Lemma 2.1 we obtain.

V. Corollary

Suppose that $f(z)$ as used in the family

$$H_m^\rho(\rho, \gamma, 1, 0) = H_0^\rho(\rho, \gamma) \text{ if}$$

$$1 + \sum_{n=2}^{\infty} A_n z^{n-1} \neq 0,$$

where

$$A_n = \frac{\psi + 1}{2(1 - \gamma)} c_n(\rho)$$

Setting $\delta = 1, \mu = 0, m = 1$ in Lemma 2.1 we obtain.

VI. Corollary

Holomorphic function $f(z)$ was describe by the family $H_m^\rho(\rho, \gamma, 1, 0) = H_m^\rho(\rho, \gamma)$ iff

$$\left| \psi \left| \sum_{t=1}^n \left[\sum_{j=1}^t (-1)^{t-j} (\mu + \rho + \delta(j-1))^m c_n(\rho) b_{t-j} q_{n-t} \right] \right| \right| \leq 2(1 - \gamma)(\mu + \rho)^m$$

Proof: Firstly, we obvious that $(1 - z)^\eta \neq 0$ and

$$(1 + z)^\alpha \neq 0 \quad (z \in D, \eta, \alpha \in G)$$

Hence, if the following inequality

$$1 + \sum_{n=2}^{\infty} A_n z^{n-1} (1 - z)^\eta (1 + z)^\alpha \neq 0, \quad (z \in D, \eta, \alpha \in G)$$

Satisfying, then we get $1 + \sum_{n=2}^{\infty} A_n z^{n-1} \neq 0$

It is clear that (6) is corresponding to

where

$$F_n = \sum_{j=1}^{\infty} (-1)^{n-j} A_j b_{n-j} \left(\sum_{n=0}^{\infty} q_n z^n \right)$$

Hence, by applying the similar technique for the Cauchy product again by formula (8), we find that

Setting $\delta = 1, \mu = 0, m = 0$ in Lemma 2.1 we obtain.

VII. Corollary

Let $f(z)$ as used in the family $H_m^\rho(0, 0, 1, 0) =$

$$H_m^\rho(0) \equiv F_m(\rho) \text{ iff}$$

$$1 + \sum_{n=2}^{\infty} A_n z^{n-1} \neq 0,$$

where

$$A_n = \frac{\psi + 1}{2} \left(\frac{\rho + n - 1}{\rho} \right)^m c_n(\rho)$$

Which satisfying for the subfamily of functions considered in [7].

$$1 + \sum_{n=2}^{\infty} A_n z^{n-1} \neq 0$$

Where

$$A_n = \frac{\psi + 1}{2(1 - \gamma)} \left(\frac{\rho + n - 1}{\rho} \right) c_n(\rho)$$

By using Lemma 2.1, the following theorem will be proved

3. Main Results

I. Theorem

If an analytic function $f(z)$ contents the following condition:

$$\sum_{n=2}^{\infty} \left(\sum_{t=1}^n \left[\sum_{j=1}^t (-1)^{t-j} (\mu + \rho + \delta(j-1))^m \binom{\eta}{t-j} c_j \right] \binom{\alpha}{k-t} \right) \quad (6)$$

Then, f in the family $H_m^\rho(\rho, \gamma, \delta, \mu)$

$$\left(1 + \sum_{n=2}^{\infty} A_n z^{n-1} \right) \left(\sum_{n=0}^{\infty} (-1)^n b_n z^n \right) \left(\sum_{n=0}^{\infty} c_n z^n \right) \neq 0, \quad (7)$$

$$\text{where } b_n = \binom{\eta}{n} \text{ and } c_n = \binom{\alpha}{n}$$

The following result was given since the Cauchy product and the first two factors were reported as

$$\left(1 + \sum_{n=2}^{\infty} F_n z^{n-1} \right) \left(\sum_{n=0}^{\infty} q_n z^n \right), \quad (8)$$

$$1 + \sum_{n=2}^{\infty} \left(\sum_{t=1}^n \left[\sum_{j=1}^t (-1)^{t-j} A_j b_{t-j} \right] q_{n-t} \right) z^{n-1} \neq 0 \quad (z \in D)$$

Thus, if $f(z) \in A$ satisfies the condition

$$\sum_{n=2}^{\infty} \left(\sum_{t=1}^n \left[\sum_{j=1}^t (-1)^{t-j} A_j b_{t-j} c_j \right] q_{n-t} \right) \leq 1$$

Therefore, if

$$\frac{1}{2(1-\gamma)(l+\rho)^m} \sum_{n=2}^{\infty} \left| \sum_{t=1}^n \left[\sum_{j=1}^t (-1)^{t-j} (\mu + \rho + \delta(j-1)) c_n(\rho) b_{t-j} (\psi + 1) q_{n-t} \right] \right|$$

$$\leq \frac{1}{2(1-\gamma)(l+\rho)^m} \sum_{n=2}^{\infty} \left| \sum_{t=1}^n \left[\sum_{j=1}^t (-1)^{t-j} (\mu + \rho + \delta(j-1)) c_n(\rho) b_{t-j} q_{n-t} \right] \right| +$$

Which is equal to (6). This finalizes the proof:

Setting $\delta = 0, \mu = 0$ in Theorem 3.1, the following results can be observed:

II. Corollary

If an analytic function $f(z)$ satisfies with the inequality

$$\sum_{n=2}^{\infty} \left| \sum_{t=1}^n \left[\sum_{j=1}^t (-1)^{t-j} (\rho + j - 1)^m c_n(\rho) b_{t-j} \right] q_{n-t} \right| +$$

$$|\psi| \left| \sum_{t=1}^n \left[\sum_{j=1}^t (-1)^{t-j} (\rho + j - 1)^m c_n(\rho) b_{t-j} \right] q_{n-t} \right| \leq 2\rho^m(1-\gamma)$$

Then $f(z) \in H_m^\rho(\rho, \gamma, 1, 0) \equiv H_m^\rho(\gamma) f(z)$

satisfying with the family of functions considered by references [3] and [4]

Setting $\delta = 1, \mu = 0, \gamma = 0$ in the previous theorem, we obtain

III. Corollary

If an analytic function $f(z)$ satisfies with the inequality

Then $f(z) \in H_1^\rho(\rho, 0, 1, 0) \equiv F_1(\rho)$ which has been considered in references [8] and [9]

On putting $\rho = 1$ in Corollary 3.4, we get

IV. Corollary

If an analytic function $f(z)$ was satisfied with the inequality

$$\sum_{n=2}^{\infty} \left| \sum_{t=1}^n \left[\sum_{j=1}^t (-1)^{t-j} (j) c_n b_{t-j} \right] q_{n-t} \right| +$$

V. Lemma

Let $f(z)$ is holomorphic function the family

$H_m^\rho(\rho, \gamma, \delta, 1, \theta)$ if

$$1 + \sum_{n=2}^{\infty} L_n z^{n-1} \neq 0$$

Where

$$L_n = \frac{e^{i\theta}(\psi + 1)}{2(1-\gamma)\cos\theta} \left(\frac{\mu + \rho + \delta(k-1)}{\mu + \rho} \right)^m c_n(\rho)$$

$$|\psi| \left| \sum_{t=1}^n \left[\sum_{j=1}^t (-1)^{t-j} e^{i\theta} (\mu + \rho + \delta(j-1))^m c_j \right] \left[\frac{\alpha}{n-t} \right] \right| \leq 2(1-\gamma)(l+\rho)^m \cos\theta$$

$$\sum_{n=2}^{\infty} \left| \sum_{t=1}^n \left[\sum_{j=1}^t (-1)^{t-j} \frac{\psi + 1}{2(1-\gamma)} \left(\frac{\mu + \rho + \delta(j-1)}{\mu + \rho} \right)^m c_n(\rho) b_{t-j} \right] q_{n-t} \right| \leq 1$$

$$|\psi| \left| \sum_{t=1}^n \left[\sum_{j=1}^t (-1)^{t-j} (\mu + \rho + \delta(j-1)) c_n(\rho) b_{t-j} q_{n-t} \right] \right|$$

$$\sum_{n=2}^{\infty} \left| \sum_{t=1}^n \left[\sum_{j=1}^t (-1)^{t-j} (\rho + j - 1)^m c_n(\rho) b_{t-j} \right] q_{n-t} \right| +$$

$$|\psi| \left| \sum_{t=1}^n \left[\sum_{j=1}^t (-1)^{t-j} (\rho + j - 1)^m c_n(\rho) b_{t-j} \right] q_{n-t} \right| \leq 2\rho^m$$

Then $f(z) \in H_m^\rho(\rho, 0, 1, 0) \equiv H_m^\rho(\gamma) \equiv F_m(\rho)$ which satisfying the family of functions was considered by reference [7].

Putting $\rho = 1$ in Corollary 3.3, we obtain.

VI. Corollary

If an investigative function $f(z)$ satisfies the inequality

$$\sum_{n=2}^{\infty} \left| \sum_{t=1}^n \left[\sum_{j=1}^t (-1)^{t-j} (\rho + j - 1) c_n(\rho) b_{t-j} \right] q_{n-t} \right| +$$

$$|\psi| \left| \sum_{t=1}^n \left[\sum_{j=1}^t (-1)^{t-j} (\rho + j - 1)^m c_n(\rho) b_{t-j} \right] q_{n-t} \right| \leq 2\rho$$

$$|\psi| \left| \sum_{t=1}^n \left[\sum_{j=1}^t (-1)^{t-j} (j) c_n b_{t-j} \right] q_{n-t} \right| \leq 2$$

Then $f(z) \in H_1^1(1)$ which has considered in reference [9].

Now we will conclude derive The following Lemma to be used in next result.

$$(0 \leq \rho < 1; -\frac{\pi}{2} < \theta < \frac{\pi}{2}, \psi \in C; |\psi| = 1)$$

Proof: It is easily to seen that the proof required as the similar method in the proofing of Lemma 2.1.

VII. Theorem

If an investigative function $f(z)$ was satisfied with the following condition:

$$\sum_{n=2}^{\infty} \left| \sum_{t=1}^n \left[\sum_{j=1}^t (-1)^{t-j} (\mu + \rho + \delta(j-1))^m \left(\frac{\eta}{t-j} \right) c_j \right] \left[\frac{\alpha}{n-t} \right] \right| +$$

$$(0 \leq \rho < 1; -\frac{\pi}{2} < \theta < \frac{\pi}{2}, \psi \in C; |\psi| = 1),$$

then $f(z) \in H_m^\rho(\rho, \gamma, \delta, 1, \theta)$

Proof: The proof follows immediately from the similar way in the proof of Theorem 3.1, we get

$$\sum_{n=2}^{\infty} \left| \sum_{t=1}^n \left[\sum_{j=1}^t (-1)^{t-j} L_j b_{t-j} \right] q_{n-t} \right| \leq 1 \text{ as previously}$$

$$b_n = \binom{\eta}{n} \text{ and } q_n = \binom{\alpha}{n}$$

The coefficient L_n is as given in Lemma 3.6. It is easy seen that the result follows directly

$$\begin{aligned} & \frac{1}{2(1-\gamma)(\mu+\rho)^m} \sum_{n=2}^{\infty} \left| \sum_{t=1}^n \left[\sum_{j=1}^t (-1)^{t-j} e^{i\theta} (\mu+\rho+\delta(j-1))^m c_n(\rho) b_{t-j} (\psi+1) q_{n-t} \right] \right| \\ & \leq \frac{1}{2(1-\gamma)(\mu+\rho)^m \cos \theta} \sum_{n=2}^{\infty} \left| \sum_{t=1}^n \left[\sum_{j=1}^t (-1)^{t-j} e^{i\theta} (\mu+\rho+\delta(j-1))^m c_n(\rho) b_{t-j} \right] q_{n-t} \right| + \\ & |\psi| \left| \sum_{t=1}^n \left[\sum_{j=1}^t (-1)^{t-j} e^{i\theta} (\mu+\rho+\delta(j-1))^m c_n(\rho) b_{t-j} \right] q_{n-t} \right| \leq 1 \end{aligned}$$

, This proves Theorem 3.7.

VIII. Remark

Setting $\theta = 0$ in Theorem 3.7 we get Theorem 3.1

The following result is due to $\delta = 1, \mu = 0$ in Theorem 2.13 :

VIII. Corollary

If an analytic function $f(z)$ satisfies the condition

$$\begin{aligned} & \sum_{n=2}^{\infty} \left| \sum_{t=1}^n \left[\sum_{j=1}^t (-1)^{t-j} e^{i\theta} (\rho+j-1)^m c_n(\rho) b_{t-j} \right] q_{n-t} \right| + \\ & |\psi| \left| \sum_{t=1}^n \left[\sum_{j=1}^t (-1)^{t-j} e^{i\theta} (\mu+\rho+\delta(j-1))^m c_j(\rho) b_{t-j} \right] q_{n-t} \right| \leq 2(1-\gamma)(\rho)^m \cos \theta \end{aligned}$$

. Then $f(z) \in H_m^0(\rho, \gamma, 1, 0, \theta) \equiv H_m^\rho(\rho, \gamma, \theta)$.

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