Calculation the Parameters of the Filters Shaping and the Properties of Random Processes on the Output Linear Filters by Using Mathematical Modeling

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ABSTRACT

Using mathematical modeling of wireless devices and systems, where it was to provide detailed explanations about calculating parameters shaping filters features and characteristics of the operations on the exits, and the study has to solve many computational problems conventional related to the following topics:
1. Calculate the properties of random processes on the output linear filters when exposed to the input of the white Gaussian noise. 2. Calculate the parameters of the filters shaping used to simulate the random processes with giving her the spectral-correlation properties. 3. Calculation parameters of optimal detectors of radio pulses. 4. Optimal estimation of the unknown parameters of the distribution.

Keywords: Random Process, Linear Filters, White Gaussian Noise, Filters Shaping, Optimal Detectors.

حساب معلمات تشكيل المرشحات وخصائص العمليات العشوائية على خرج المرشحات الخطية بأستخدام النمذجة الرياضية

الخلاصة

اجريت الدراسة بأستخدام النمذجة الرياضية للاجهزة والانظمة اللاسلكية حيث تم تقديم تفسيرات مفصلة حول ميزات حساب معلمات تشكيل الفلاتر وخصائص العمليات على مخارجها، وتمت الدراسة من اجل حل العديد من المشاكل الحسابية التقليدية المرتبطة بالمواضيع التالية وهي :
١. حساب خصائص العمليات العشوائية على مخارج المرشحات الخطية عندما تتعرض لإدخال ضوضاء جاوس البيضاء. ٢. وحساب معلمات تشكيل المرشحات المستخدمة لمحاكاة العمليات العشوائية مع إعطاء خصائص الارتباط الطيفي لها. ٣. حساب معلمات اجهزة الكشف المثالية للنبضات الراديوية. ٤. التقدير الامثل لتوزيع المعلمات غير المعروفة.

INTRODUCTION

For the design of wireless systems where the signal is distorted due to physical phenomena, it is necessary to characterize the transmitter, channel and receiver using mathematical models. An understanding of random processes is crucial to many engineering fields-including communication theory, digital signal processing in electrical, computer engineering, vibrational theory and stress analysis in mechanical engineering. The filtering, estimation and detection of random processes in noisy environments are critical tasks necessary in the analysis and design of new communications systems and useful signal processing algorithms. Random processes: filtering, estimation and detection clearly explains the basics of probability and random processes and details modern detection and estimation theory to accomplish these tasks.

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The paper covers four main interrelated topics: Probability and characterizations of random processes, linear filters with random excitations, optimal estimation theory and detection theory to calculate parameters of optimal detectors of radio pulses.
The random processes of almost exclusive interest in modeling receiver noise are the Gaussian processes. Gaussian processes are random processes for which the random variables $N(t_1),N(t_2),..,N(t_k)$ are jointly Gaussian for all $t_1,..,t_k$ and all $k > 0$.
Filtering in general linear models is perhaps the most widely applied branch of filtering, but in the context of linearity the term 'filtering' refers to something that is fundamentally different from the probabilistic models and equations that comprise what mathematicians refer to as filtering theory.

The Model of Discrete White Gaussian Noise

As is known, white noise model is a mathematical abstraction in the form of process, spectrum is uniform for all frequencies and is equal to some constant $N_0/2$, and the correlation function is a delta function with a weight which is determined by the specified constant. Thus, white noise has infinite variance (power). Any real process has the ultimate power, and, consequently, its power spectral density can only be integral decreasing function of frequency. However, the model of the white noise is used, if the width of the noise spectrum is much larger than the width bandwidth some frequency-selective device. We now consider the model of a discrete white Gaussian noise. Discrete white noise, unlike the white noise has finite capacity. Its correlation function is the identity function with a weight equal to the dispersion process. Any two of the reference process are not correlated. This process can be modeled using computer equipment. However, in reality almost always counts a random process, obtained by sampling a continuous process have finite cross-correlation coefficient, and by using only the frequency characteristic of filters and selecting a special kind sampling in accordance with parameters of adjacent filter samples process may not be correlated [1,2,3]. We will demonstrate this using the example of a discrete process samples in a digital receiver. General functional diagram of a receiver is shown in Fig. (1).

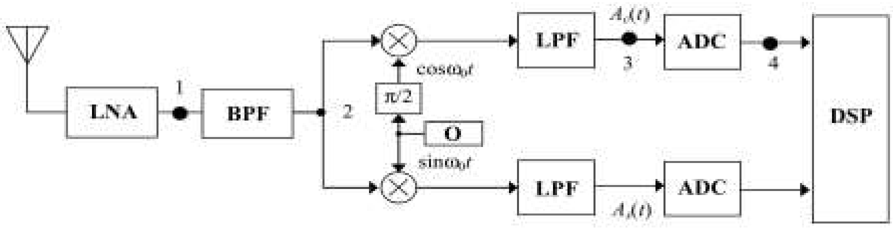

Figure (1): General functional diagram of a receiver

In Fig.(1) denotes: LNA - a low-noise amplifier, the BPF - a band pass filter, LPF - low pass filter, ADC - analog to digital converter, O - Oscillator, DSP - Digital Signal Processing unit.
Suppose that the intrinsic noise of the antenna and the amplifier is much greater than the width bandwidth a band pass filter, amplitude-frequency characteristics of the band pass filter and low pass filter perfectly rectangular. Fig. (2) Shows the power spectral density processes in points 1, 2, 3 and 4, as well as the correlation function of the process in point 4 is shown in Fig. (1).

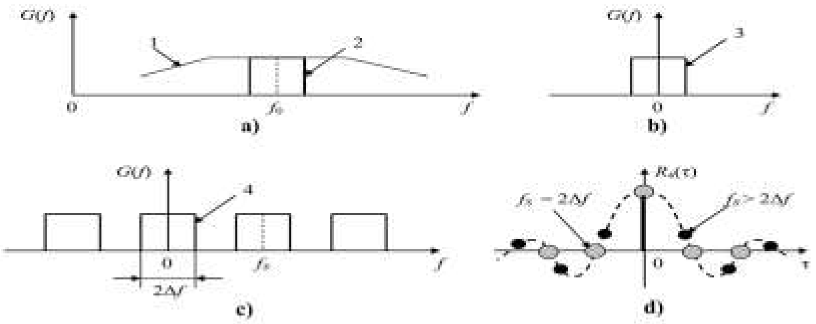

Figure (2): The power spectral density processes

From a consideration of the graphs to the following conclusions, first of all, noise after filtering has a bandwidth defined by the characteristics of the filter, and so the power of the noise is limited. This explains the fundamental difference between the discrete white noise from the white noise – a discrete white noise has a finite variance, as can be obtained by sampling the noise with a finite bandwidth. Consider now the correlation properties of the analog to digital converter process. From Fig. (2, c, d) can be seen that if the sampling frequency is greater than twice the one-sided strip process, its discrete samples will be correlated [4,5,6]. In order to process the neighboring samples were uncorrelated, the sampling frequency must be chosen from the condition $f_s = 2\Delta f$. Only in this case the resulting realization of discrete random process will be uncorrelated, as samples of the correlation function of the discrete process will fall into the low-frequency zero correlation function of the process, having the form $\sin x / x$.

Ideal filters considered in this example, are unrealizable. Therefore, if the amplitude frequency characteristic of the filters used in the circuit of Fig. (1) has some arbitrary shape (e.g., Gaussian), then the process discrete samples are correlated. However, in practice, can be applied certain special filters whose impulse characteristic autocorrelation function (IC) has equally spaced zeroes as a function $\sin x / x$. A common example of such filters are filters with amplitude - frequency characteristic such as a raised cosine [7,8], which is described by the following expression:

$$H(f) = \begin{cases} \frac{T}{2}\left\{1+\cos\left[\frac{\pi T}{\alpha}\left(|f| - \frac{1-\alpha}{2T}\right)\right]\right\} & , \quad \frac{1-\alpha}{2T} < |f| < \frac{1+\alpha}{2T} \\ 1 & , \quad |f| < \frac{1-\alpha}{2T} \\ 0 & , \quad |f| > \frac{1+\alpha}{2T} \end{cases} \qquad \text{....(1)}$$

Where

α - parameter that determines the degree of smoothing of the frequency characteristic, T - the width of the main lobe IC for the first zeros. If α = 0, the amplitude-frequency characteristic of the filter is reduced to a rectangular filter characteristic unrealizable. If α = 1, then the amplitude-frequency characteristic represents one period of a raised cosine. For intermediate values parameter (α) characteristic of the filter has a flat top, and falling slope to the cosine law. IC filter characteristic of the filter with a cosine smoothing has the form

$$h(t) = \left(\frac{\sin \pi t / T}{\pi t / T}\right)\left(\frac{\cos(\alpha \pi t / T)}{1-(2\alpha \pi t / T)^2}\right) \qquad \text{...(2)}$$

When

$\alpha = 0$, this function reduces to a function $\sin x / x$, and for large values of the parameter (α) has a smaller side-lobe level. To using a filter with a cosine smoothing process samples were not mutually correlated, it is necessary to choose the sampling interval equal to $T_s = T$. Cosine smoothing filters are widely used in digital communication, as provide a minimum level of intersymbol interference. Package MATLAB has built-in means for generating realizations discrete white Gaussian noise. Firstly, among the standard functions, included in the programming language MATLAB, there is a function randn (M, N). This function returns when it is called a two-dimensional array of independent random number size (MxN), distributed according to a normal distribution with parameters (0,1). in the case of absence of one or both of the arguments to the function when it is called, it returns a vector of independent random numbers or scalar random number. In the library of SIMULINK in the sources section, there are two blocks for the formation of the implementation white Gaussian noise: (Random Number) and (Band Limited White Noise). Consider the features of the use of and another block. In the parameter list (Random Number block) contains the following items: the noise variance - Variance, mathematical expectation (ME) - Mean, initial conditions - Initial Seed and sampling interval - Sample time. This noise generator generates a discrete implementation (SD), (PSD) which is uniform in the frequency band $[-f_s/2, f_s/2]$ where f_s - sampling frequency - The quantity, asked inverse sampling interval (Sample time). The level of (PSD) $N_0/2$ in this band is given by:

$$\frac{N_0}{2} = \frac{\sigma^2}{f_s} \qquad \dots (3)$$

Where

σ^2 - noise variance (Variance).

Equation (3) is illustrated in (Figure. 3).

In Fig. (3) shows the shaded area under the curve (PSD) is equal to the noise variance is apparent from the expression (4).

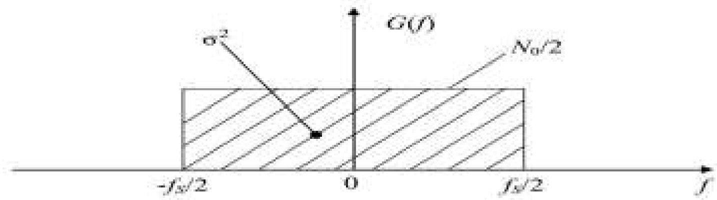

Figure (3): The shaded area is equal to the noise variance

List of block parameters (Band Limited White Noise) is a fundamental difference. In it there instead of the variance parameter called noise power. Although the translation of this phrase means "noise power" (i.e., in other words, the variance), but the true meaning of this parameter is the level of the noise (PSD), i.e. $N_0/2$. Thus, setting the level of (PSD) $N_0/2$ in the band [$-f_S$ / 2, f_S / 2], the noise variance can be determined by the formula

$$\sigma^2 = \frac{N_0}{2} f_s \qquad \dots (4)$$

Using one of the considered noise generators simulating Gaussian (SD) usability determined set of parameters or that block [9,10].

Simulation of Gaussian Random Processes

Assignment: build up the correlation function (SD) at the output of the filter with a given pulse IC. Find dispersion and (ME) process to output filter in the steady state at predetermined (ME) and dispersion inlet. To plot depending on (ME) and dispersion process at the filter output, if the zero initial conditions at time (t_0) at the inlet becomes effective implementation (WGN) with the specified parameters. Find a normalized cross-correlation coefficient values at the output of the filter process, taken after a specified time interval. As an example, consider the spectral-correlation properties of the process at the output of the filter with a rectangular IC. As noted in [11], the correlation functions of the process at the output of the filter up to a constant factor determined by the autocorrelation function IC of the filter. The autocorrelation function IC rectangular shape has a triangular shape. View IC considered filter and its autocorrelation function is shown in Fig. (4).

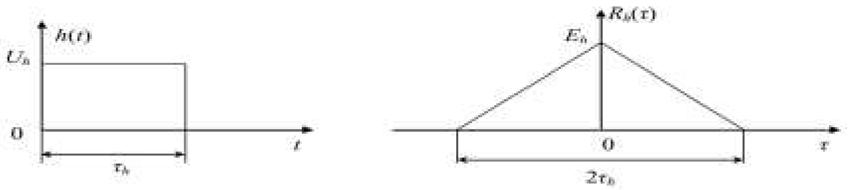

Figure (4): Impulse characteristic and its autocorrelation function

As is well known [7,12], the dispersion process at the output of the linear filter upon exposure at the input white noise with power spectral density $N_0/2$ in steady-state regime is determined by the expression

$$\sigma_2{}^2 = \frac{N_0}{2} E_h \qquad \ldots(5)$$

Where

E_h - their energy IC. Thus, the process of the correlation function at the output of the filter is identical in form with the autocorrelation function IC of their filter and has a maximum, determined by the value (5).Fig. (5) shows graphs of mathematical expectation (ME) and the dispersion process at the filter output from the time, built on the assumption that at time zero to input filter with zero initial conditions applied to the realization of white noise (ME) m_1 and (PSD) $N_0/2$. In the graphs according to mathematical expectation (ME) and the dispersion process at the output of the filter has the transition process, whose duration is determined by the duration of the impulse characteristic of the filter. Graph depending on mathematical expectation from time is a transient response of the filter with a weight equal to the expectation input process. It is therefore obvious that if the process in expectation of zero input, then the output process will have zero expectation. [13].

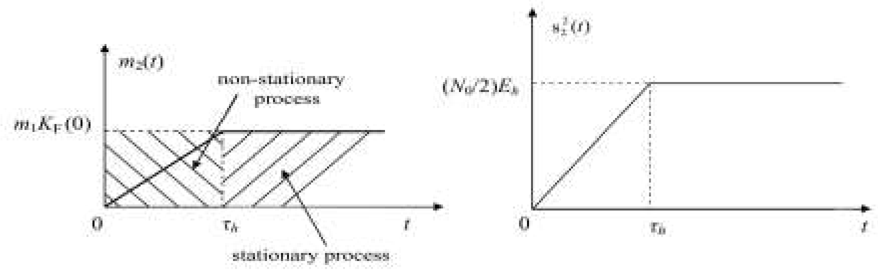

Figure (5): Mathematical expectation & the dispersion process

We define the law change the dispersion process at the filter output depending on the time. For this condition represent the filter with impulse characteristic $h(t)$ in form of parallel connection K filters with impulse characteristic $h(t)$, representing partial portions of the original impulse characteristic, detainees at the time $i\Delta t$, where i - the number of the partial impulse response, Δt - its duration. To the parallel connection of partial filters match the original filter, the output process of the partial filters are added. Functional diagram of a system equivalent to the original filter, is shown in Fig. (6.a). Fig. (6.b) shows partial impulse characteristic of the equivalent circuit of parallel channels. In this case, the time interval $[0, \Delta t]$ there will be a transition process at the output of the first partial filter and at the outputs of the other will zero values. After the end of the transition process to the variance adder output is equal to $\sigma_{2,1}{}^2 = \frac{N_0}{2}\int_0^{\Delta t} h_1^2(t)dt = \frac{N_0}{2}E_{h1}$ where E_{h1} - Energy impulse characteristic $h_1(t)$.

At the same time begin the transition process at the output of the second partial filter after which the dispersion at the outlet of the second filter is equal to $\sigma_{2,2}{}^2 = \frac{N_0}{2}\int_0^{\Delta t} h_2^2(t)dt = \frac{N_0}{2}E_{h2}$.

Impulse characteristic of the partial filters are mutually orthogonal functions, since the relative the product of any two partial impulse characteristic equal to zero. This means that processes at the outputs of the partial filters at the same time shall have the meanings that are responsive to these filters staggered in time and therefore independent white noise input samples.

Given also the fact responses to the cross-sectional values process do not overlap the input values of the process output taken at the same time, are not correlated, but in the case of Gaussian independent process.

Consequently, the dispersion process at the output of the adder will equal the amount of dispersion processes with partial output filters. Then in time moment $k\Delta t$ dispersion process at the output of the adder (and at output initial filter) is equal to $s_{2,k}^2 = \frac{N_0}{2}\sum_{i=1}^{k}\int_{(i-1)\Delta t}^{i\Delta t} h_i^2(t)dt = \frac{N_0}{2}\sum_{i=1}^{k}E_{h,i}$. It is obvious that these arguments are valid only for the case when at the input filter acts white noise. Under the influence of correlated noise law changes the variance of the output process will be more difficult.

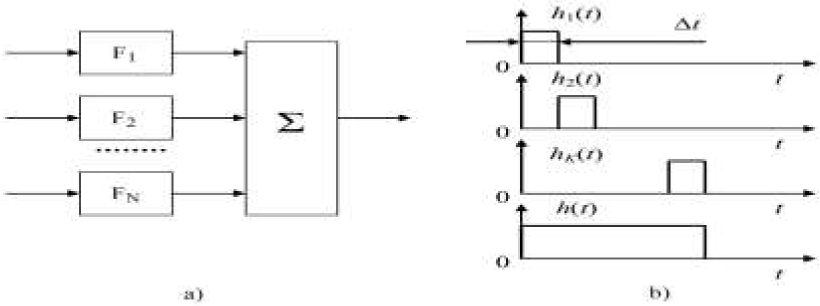

Figure (6): (a) Functional diagram of a system equivalent to the original filter. (b) Partial impulse characteristic of the equivalent circuit of parallel channels.

From the last relation can be seen if the impulse response of the filter is rectangular shape, and all the partial energy of the impulse response is the same, the variance of the process output will increase linearly throughout the duration of the impulse response h (t). In the limit, when $\Delta t \to 0$, the expression for the dispersion law changes process at the filter output will take the form

$$\sigma_2{}^2(t) = \frac{N_0}{2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(u_1)h(u_2)du_1du_2 \qquad \ldots (6)$$

Thus, in the case of a filter with a rectangular impulse response in the interval time from zero to the duration of the impulse response of the expectation and variance varies linearly. In this range the output process is non-stationary. Therefore, in the simulation of a process with given spectral-correlation properties that the initial portion of the implementation output process typically exclude from consideration.

We find also a normalized cross-correlation coefficient between sections output process taken at a time interval Δt_0. Suppose for certainty $\Delta t_0 = 0.35\tau_h$. Normalized correlation function of the process at the output of the filter is obtained by normalizing the correlation function dispersion and a zero time offset is equal to unity. Cross-correlation coefficient of the cross sections of the stationary process t_n and t_k, taken at a time interval Δt_0, equal to the value of the normalized correlation function of the process when the argument, equal to this shift: $r_{nk} = r(\Delta t_0)$. In our example, with the triangular shape of the correlation function of the output of the process $r(\Delta t_0) = r(0.35\tau_h) = 1 - 0.35 = 0.75$. Consider the process at the output of the discrete shaping filter c rectangular impulse response, which is valid at the entrance of the discrete white Gaussian noise. View of the impulse response of the filter and the correlation function of the output of the process is shown in Fig. (7). The maximum value of the correlation function of the random process at the output of the digital filter is equal to $R_{2\max} = \sigma_2^2 = \sigma_1^2 \sum_{i=0}^{N-1} h_i^2$. In Fig. (8) shows plots of the expectation and variance of the output random process from sample number.

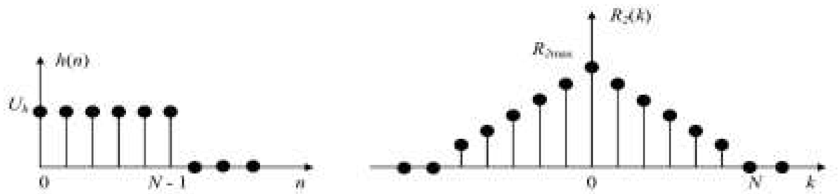

Figure (7): The impulse response of the filter and the correlation function

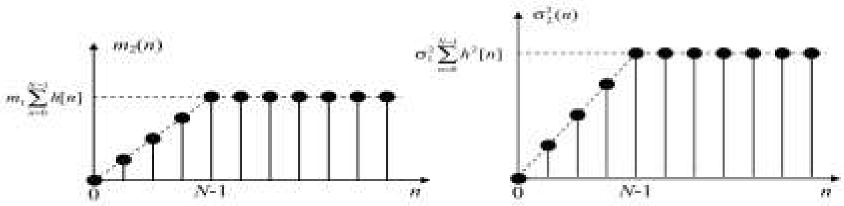

Figure (8): The expectation & variance of the output random process

For an explanation of the graphs in Fig. (8) consider block diagram of the FIR digital filter in Fig. (9). The weighted coefficients of the filter samples of a random process, taken from the line delay elements, are added in the adder. Because of the expectation of the product of a random variable on constant equal to the product of the expectation of the original random variable on the constant, then the expectation of reference process output the n-th multiplier corresponding to the impulse response coefficient equal $m_1 h[n]$. Further, given that the expectation sum of random variables is equal to the sum of the expectation of these values, we obtain the expectation process at the output of the FIR filter in the steady state: $m_2 = m_1 \sum_{n=0}^{N-1} h[n]$. Note that the sum of the filter coefficients determines transfer coefficient at zero frequency, i.e. coefficient transfer constant component. In the transition mode, the duration of which is determined by the duration of the impulse response, we have a partial sum of (k) weighted samples, so the expectation in the k-th time moment is equal $m_2(k) = m_1 \sum_{n=0}^{k} h[n]$. Since in our example all the filter coefficients are equal, transition process changes in the expectation has linear character [14,15]. For building graphics depending on the variance of the output process of the number of reference, consider the following two properties of the dispersion. Firstly, the variance of the product of a random variable by a constant equal to the product dispersion of the initial value by the square of this constant. Second, the variance of the sum of independent random variables equal to the sum of the variances (in the case of correlated random variables is unfair and it is necessary to take into account cross-correlation coefficient). With that said, we obtain the variance of the output process in the steady state $\sigma_2^2 = \sigma_1^2 \sum_{n=0}^{N-1} h^2[n]$. In transition mode does not have the full amount from k terms: $\sigma_2^2(k) = \sigma_1^2 \sum_{n=0}^{k} h^2[n]$. Therefore, in this example, law changes the variance of the output of the process is also linear.

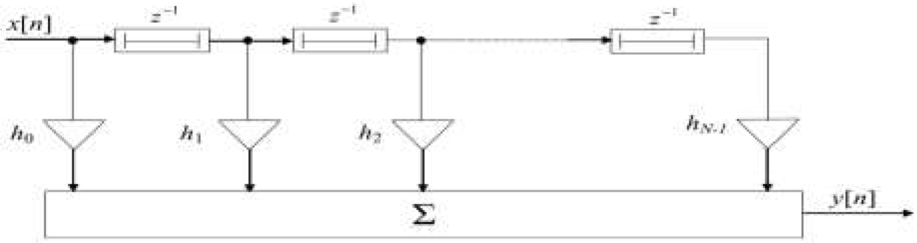

Figure (9): Block diagram of the FIR digital filter

Assignment: Calculating the digital FIR filter for the formation of realization of a Gaussian process with a given a correlation function or given power spectral density. Provide power rationing process at the output filter to Power process inlet. As an example, consider the design of the shaping filter for modeling a random process with power spectral density of the form:

$$W(\omega) = \exp(-a\omega^2) \qquad \ldots(7)$$

Where a - parameter.

As noted in [11,12], the correlation function of the process at the output of the filter up to a constant factor equal to the autocorrelation function filter impulse response. Therefore, to solve this problem we must design the filter impulse response which represents a signal power spectrum is described by expression (7). As is known, the module Fourier transform of a Gaussian pulse is described by a Gaussian curve. In addition, the autocorrelation function and Gaussian pulse representing a Gaussian curve. In this example, we should mention another important property of the Fourier transform, which is the Fourier transform of an even function, is purely real function. Since the function (7) is even, then to calculate the actual impulse response filter need only calculate its actual amplitude spectrum as the square root function (7) and take the inverse Fourier transform. Using the table of Fourier transforms, we obtain that the autocorrelation impulse response function will have the form

$$R(\tau) = \sqrt{\frac{a}{\pi}} \exp(-b\tau^2) \qquad \ldots(8)$$

Where $b = 0.25/a$, and itself as the impulse response is the inverse Fourier transform, equal to the square root of (7), and is given by

$$h(t) = \sqrt{\frac{a}{2\pi}} \exp(-2bt^2) \qquad \ldots(9)$$

Function (9) will be considered as the impulse response of the analog prototype filter. The parameter b in the expression (8) and (9) determines the width of the correlation function of the process being modeled and, therefore, the spectral width of the process. It can be shown that the value of (1 - b) approximately defines the correlation coefficient of two adjacent sampling processes. To obtain digital samples of the impulse response of the shaping filter can use different methods. In [7,12] described a method for calculating the filter coefficients expansion method the spectral power density in a Fourier series. Practically, this method reduces the frequency window to a method which in the practical implementation is desired sampling samples amplitude characteristics and calculation of the inverse discrete Fourier transform (or, in the special case of an even function, discrete cosine transform). It is possible the subsequent weighting impulse response weighted window to decrease the emission frequency characteristic obtained digital filter caused by the influence of the so-called Gibbs effect [12]. Another common way of designing is method window in the time domain. This method is even easier and more convenient for designing FIR filters in case, if known analytical expression for the impulse response of the analog prototype filter. The method consists in weighting the impulse response of the prototype filter weighting window of limited duration and sampling a subsequent within this window. In this example, using the above approach, we obtain samples of the impulse response of the digital FIR shaping filter in the form (10)

$$h(n) = \sqrt{\frac{a}{\pi}}\exp(-2b(n\Delta t)^2) = \sqrt{\frac{a}{\pi}}\exp(-\alpha n^2) \qquad \ldots(10)$$

Where $a = 2b\Delta t^2$. The value of the sampling interval Δt is chosen according to the bandwidth of the simulated process. The number of filter coefficients is chosen based on the requirements of accuracy of approximation required frequency response. Obviously, the window duration must be chosen so that the edge of the window of the impulse response values was negligible. After receiving the vector of coefficients of the digital filter is necessary to calculate its frequency response, as well as the square modulus of the frequency response and assess the accuracy of the approximation of a given PSD output process resulting function. Amplitude factor $\sqrt{\frac{a}{2\pi}}$ in (7) does not principal values. However, the task we have to fulfill the condition of rationing power output process to the power of the input process. This means that the power (variance) of the process at the filter output should be equal to the power input process. As shown above, the dispersion process at the filter output is equal to $\sigma_2^2 = \sigma_1^2 \sum_{n=0}^{N-1} h^2[n]$. It follows that the normalization predetermined conditions necessary to calculate the modified filter coefficients in form

$$h_M(n) = \frac{h(n)}{\sqrt{\sum_{i=0}^{N-1} h^2(i)}} \qquad \ldots (11)$$

It should be noted that the above example is an important from the point of view of the theory of struggle with passive interference in radar, where the correlation function of the form (3) describes the properties of a Gaussian passive interference at discrete values of time shift $\tau = nT_{\Pi}$, where T_{Π}- the pulse repetition period. In this case the value $R(T_{\Pi})/\sigma_2^2$ is a coefficient through a period of correlation passive interference in a pulsed radar system..

Assignment: calculate the recursive digital filter for simulated random process with exponential correlation function and a given value of the correlation coefficient between adjacent samples of the output process. This example shows model another important special case passive interference in radar, namely exponential interference. To calculate the coefficients of the recursive shaping filter for a given correlation function of the process at the output, you can use the method of factorization filter system function [7]. The system function $k(z)$ recursive linear filter with constant parameters can be represented as

$$K(z) = \frac{A(z)}{B(z)} = \frac{\sum_{k=0}^{N-1} a_k Z^{-k}}{1 - \sum_{K=1}^{M-1} b_k Z^{-k}} \qquad \ldots.(12)$$

Where

N - the number of taps of the filter is not recursive, M - the number of taps (order) recursive part. Procedure for the application of this method is as follows.
1. Located z - transform $F(z)$ of the correlation function R(k) of the simulated process:

$$F(z) = \sum_{k=-\infty}^{\infty} R(k) Z^{-k} \tag{13}$$

The resulting function of the complex variable z at points located at the unit circle, belonging to the complex plane, it makes sense to the power spectral density of the simulated process. As mentioned above, in order to find the impulse response filter must obtain the function equal to the square root from spectrum density power. However, the function $F(z)$ is a function of the complex variable z, which implies the following paragraph.
2. Implemented factorization function $F(z)$:

$$F(z) = \frac{A(z)A(z^{-1})}{B(z)B(z^{-1})} = |K(z)|^2 \tag{14}$$

Considering the expression (13), we recall that a complex variable z by the unit circle is expressed as $Z = \exp(j\omega)$. Thus, the function $A(z)$ and $A(z^{-1})$, the function $B(z)$ and $B(z^{-1})$ are a pair of complex conjugate functions, wherein the ratio of $A(z)/B(z)$ determines the function of the filter system. 3. The system function K (z) is converted to the form (12) to find the coefficients of the recursive filter. In this example, the discrete exponential correlation function of the simulated process is given by

$$R[k] = \exp(-\gamma|k|) \tag{15}$$

Where γ - parameter that determines the rate of decay of the correlation function. In accordance with the described method of calculation, find two-sided z - transform of the correlation function (15). For this we use following known relation:

$$F(z) = F^{+}(z) + F(z^{-1}) - R[0] \tag{16}$$

Where $F^{+}(z) = \sum_{k=0}^{\infty} R[k] Z^{-k}$ - One - sided z-transform.

According to The table z-transform we find:

$$F^{+}(z) = \sum_{k=0}^{\infty} \exp(-\gamma k) Z^{-k} = \frac{z}{z - \exp(-\gamma)} \tag{17}$$

If we substitute this expression in (16), after the transformation can be obtained:

$$F(z) = \frac{\left(\sqrt{1-\rho^2}\right)^2}{(1 - z\rho)(1 - z^{-1}\rho)} = |K(z)|^2 \tag{18}$$

Where $\rho = \exp(-\gamma)$. Comparing the expressions (14) and (18) it is easy to see that the numerator and denominator of the complex system function of the designed filter is defined as $A(z) = \sqrt{1-\rho^2}$, $B(z) = 1 - z^{-1}\rho$. Comparing the expressions obtained for the numerator and denominator of the system function with the general view of (11), write down the expressions for the coefficients of direct and recursive part of the filter: $a_0 = \sqrt{1-\rho^2}$, $b_1 = \rho$. As a result of solving the problem, we come to the model recursive filter of first - order, whose work is described by the difference equation of the form:

$$\zeta[n] = \sqrt{1-\rho^2}\,\xi[n] + \rho\zeta[n-1] \ , \ \ \rho = \exp(-\gamma) \tag{19}$$

Where ξ [n] - implementation of the input process, ζ [n] - implementation of the output process. Block diagram of the designed filter is shown in Fig . (10).

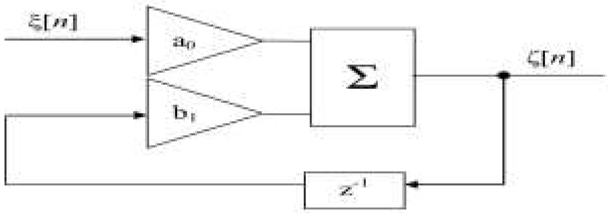

Figure (10): Block diagram of the designed filter

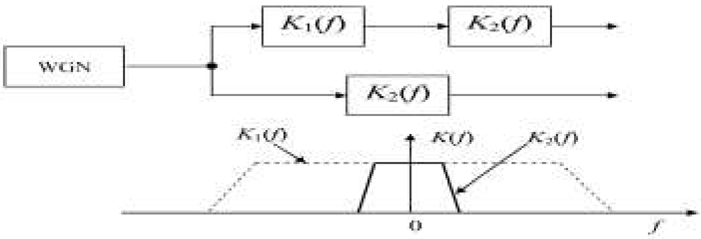

Figure (11): Circuit of a white Gaussian noise generator and frequency characteristics appropriate filters.

As follows from (19), feedback coefficient $b_1 = \rho = \exp(-\gamma)$ is equal to the correlation coefficient of two adjacent sampling processes $\zeta[n]$ at the output of the filter. In accordance with the assignment, simulation process with a given correlation coefficient between adjacent samples of the output process is reduced to the corresponding assignment of the feedback coefficient the circuit in Fig. (10).

Assignment: Fig. (11) shows a circuit consisting of a white Gaussian noise generator, and Dual-channel filtering schemes, and also frequency characteristics appropriate filters.

Assignment: In Fig. (12) shows the functional circuit consisting from generator discrete white Gaussian noise, digital low pass filter with impulse response shown in same figure, delay lines at intervals time equal to the duration of the impulse response filter and half of this duration, adder, mutually - correlation device, device evaluation standard deviations, and also devices multiplication and division. The sampling frequency in the system is equal to $f_s = 100Hz$, level power spectral density of a discrete white Gaussian noise is equal to $N_0/2 = 10^{-2} J/Hz$. The number of samples of the impulse response of the filter $N = 16$. It is necessary to find dispersion process in points 1, 2 and 3 of scheme, and the true value of a parameter estimation, which is calculated in point 4 of scheme.

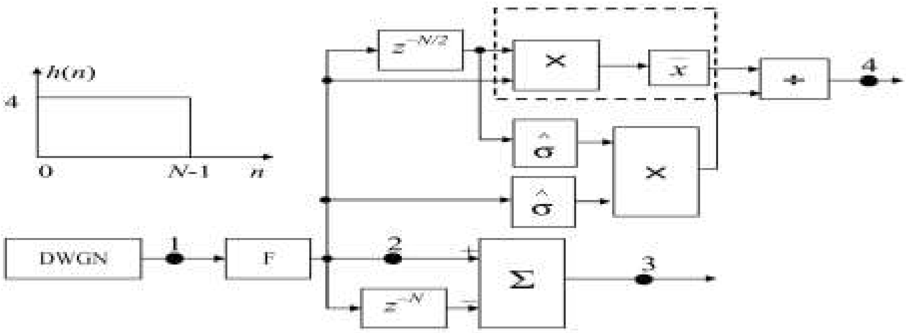

Figure (12): Circuit of generator discrete white Gaussian noise, digital low pass filter with impulse response

The Optimal Detector Simple Radio Pulses

Model optimal detector simple radio pulse will be considered, based on the mathematical apparatus of the complex envelope. Block diagram of the model optimal detector radio pulse signal is shown in Fig. (13). In the model detector consists of a matched filter (MF), the amplitude detector (AD), a threshold device (TD) and a unit for calculating the adaptive threshold detection. At the input of the detector receives the value of real and imaginary parts of the complex envelope of the radio pulse. The output signal of the detector is a binary signal $\lambda = \{0,1\}$. In this case, the zero level corresponds to the solution of the absence of the signal at the input, the unit - decision on availability signal [16,17,18].

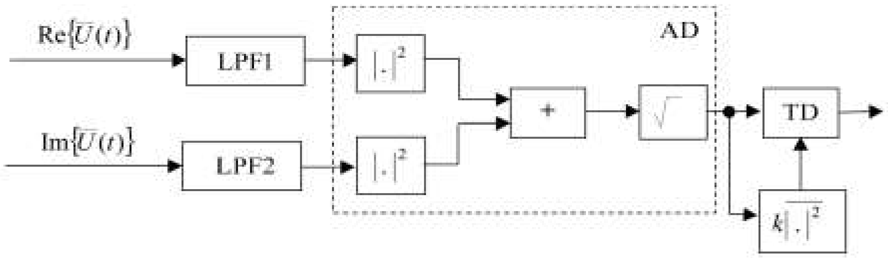

Figure (13): Block diagram of the model optimal detector radio pulse signal

The main parameters consideration of the detector, subject to evaluation includes the following values:

1. The signal-to-noise ratio at the output of a matched filter (MF).
2. The value of the detection threshold U_0, supplied to the lower input a threshold device (TD).
3. The probability of correct detection signal (D).

To calculate these parameters you must set the following set of input data:

1. The signal parameters - the shape, duration, a priori data about amplitude and initial phase. Based on the parameters used in the Signal System, in accordance with the Nyquist theorem is given by the sampling frequency f_s.
2. The parameters of the noise at the input of a matched filter. In general, any random process is completely characterized by multidimensional density distribution probabilities. In the case of stationary discrete white Gaussian noise is enough to know only two parameters of its one-dimensional normal distribution law - the mathematical expectation (ME) and a dispersion, usually believed the mathematical expectation zero.
3. The required level of probability of false alarm.

After a matched filter and amplitude detector included threshold device, whose task is to decision - making of a signal in the input process according to the Neyman-Pearson.

Decision - making rule, optimal according to the Neyman-Pearson provides maximizing the probability of correct detection of a signal at a fixed level of probability of false alarm. Because the noise power is unknown a priori, in the model included block estimates mean square deviations own noise. As an estimate of the mean square deviations this block calculates the sample mean square deviations:

$$\hat{\sigma}_n = \sqrt{\frac{1}{N}\sum_{k=n-N+1}^{n}\left|U[n]\right|^2} \qquad (20)$$

Where N - size averaging window. Threshold value U_0 formed as follows:

$$U_0 = k\hat{\sigma}_n \qquad (21)$$

Where the constant k determined based on the acceptable level of false alarm probability. Assuming that own noise receive path in each quadrature channel is independent Gaussian spectral density, noise output amplitude detector in the absence of the desired signal is distributed Rayleigh. Then the probability of false alarm F is given by:

$$F = \exp - \left(\frac{U_0^2}{\sigma_n^2} \right) \tag{22}$$

Where $\hat{\sigma}_n$ - noise variance at the output a matched filter. Given that $U_0 / \hat{\sigma}_n = k$, from formula (22) you can easily find the value of k. Calculate the probability of correct detection of a signal at a given false alarm probability F (i.e. established threshold U_0). We give a detailed derivation of the relations in the case of detection of a video on a background of white Gaussian noise, and then present the final design relations for the case detection quadratic deviations radio pulse with an unknown initial phase. The decision about the detection of a video signal taken by signal value at output matched filter [19,20]. The maximum value at output matched filter is observed in expiration time of the input signal and the greatest probability of correct detection D this will correspond exactly to the time. At the output of the matched filter at each time point will be observed random variable (section spectral density) distributed according to a Gaussian law. Since the filter is linear, and the mixture signal and noise at the input additive, then the variance of this distribution is always determined only by the input noise variance and filter parameters. With regard to mathematical expectation, it depends on the presence or absence of the signal, and is the parameter for which to be decided.

Mathematical expectation noise is usually equal zero. On the other hand, mathematical expectation process reference at the output matched filter at the time of expiration signal the inlet is defined by a maximum output value, i.e. its energy. On Fig. (14. a), and shows graphs of the probability density function reference in the presence and absence of the desired signal in the input implementation.

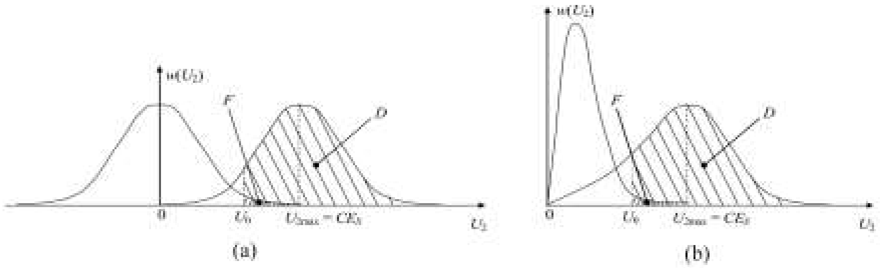

Figure (14): The probability density function

On Fig. (14) Shows the threshold at U_0, the maximum value of the signal at the filter output $U_{2\max} = CE_S$, where C - the proportionality factor determined by the parameters of the filter, E_S - energy of the signal. Also shown is the false alarm probability F and correct detection D in the form of two shaded areas. From Fig. (14) is easy to obtain a general expression for calculating the probability of correct detection. Represent fluctuation at the input matched filter in the form $u_1[n] = \xi[n] + \lambda_s[n]$, where $\lambda = 1$ in the presence of a signal and $\lambda = 0$ in its absence. Then

$$D = p\{U_2 \rangle U_0 | \lambda = 1\} = \int_{U_0}^{\infty} w(U_2 | \lambda = 1) dU_2 \tag{23}$$

Expression (23) can be regarded as a definition of probability correct detection for arbitrary distribution law. In the case of normal distribution upon detection of a video signal condition, $\lambda = 1$ equivalent to the condition $M\{U_2\} = U_{2\max}$. The probability density $w(U_2 | \lambda = 1)$, appearing in (23) – is the conditional probability density that occurs in the presence of a signal. In this case, the detection of a video signal on the background white Gaussian noise, this Gaussian probability density, and in the expression (23) it is advisable to go to the integral of probabilities. Recall that the probability integral $\Phi(z)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{z} e^{\frac{-t^2}{2}}dt$ by definition equal to the probability that the value of the random variable Z, distributed according to a normal distribution with zero mean and unit variance, will not exceed a value of Z. So, the first step in the transformation (23) containing the probability integral, should be replacing the integration limits. Since the probability density function has the property of normalization, i.e. $\int_{-\infty}^{\infty} w(x)dx=1$, it follows from (23) we obtain

$$D=1-\int_{-\infty}^{U_0} w(U_2|M\{U_2\}=U_{2\max})dU_2 \tag{24}$$

To bring any normal distribution to the standard normal must perform alignment and normalization of the original random variable. Centering means subtraction of expectation, and normalization - divide by the standard deviation. In this way, random quantity $Y=\frac{U_0-U_{2\max}}{\sigma_2}$ is as a standard normal distribution. The event, consisting in excess of this random variable values $\frac{U_0-U_{2\max}}{\sigma_2}$, equivalent event consisting of exceeding the threshold U_0 maximum emission signal at the output of the matched filter as a random variable Y derived from a random variable by U_2 by monotone linear transformation. Consequently, the probability of both events is equal to the probability of correct detection and determined from the expression

$$D=1-\Phi\left(\frac{U_0-U_{2\max}}{\sigma_2}\right) \tag{25}$$

By specifying the desired probability of correct detection (eg,0.9), can be from (25) the table of the probability integral find the desired value $U_{2\max}$, and then converted it to the input, i.e., find the desired amplitude of the input signal, which for a given power input noise will determine the threshold signal noise ratio. Conversely, knowing the signal-to-noise ratio at the circuit input, it is possible to find the value $U_{2\max}$ and calculate the probability of correct detection. The easiest way to find the value of $U_{2\max}$ is running the model in the absence of noise at the input. Then it is possible to output waveform signal determine maximum value. In the case of detection quadratic deviations radio pulse with an unknown initial phase required two-channel scheme [8,11], where instead of one matched filter uses two. At the output of such a scheme operates an integrated random process. As before, in the case of useful signal maximum amplitude (modulus) of a complex process $U_{2\max}$ determined by the energy of the signal, however, to calculate this amplitude needs amplitude detector – non-linear device, changing law of distribution of noise. It is known that the modulus Gaussian spectral density in the presence of a useful signal of constant amplitude is distributed according to the law Rayleigh-Rice, and in his absence - in law Rayleigh [21,22]. Graphs corresponding probability densities are shown in Fig. (14.b). Substituting into (23) probability density of the corresponding distributions Rayleigh-Rice can obtain the following expression for the probability of correct detection:

$$D=\int_{\sqrt{2\ln(1/F)}}^{\infty} x\exp\left(\frac{-x^2+q_2^2}{2}\right)I_0(q_2x)dx \tag{26}$$

Where $q_2=\sqrt{3E_S/N_0}$ – signal to noise ratio, $I_0(x)$ – modified Bessel function of the first kind of order zero, E_S - energy of the signal. In the case of signal detection with random amplitude and the initial phase noise distribution at the output amplitude detector in the presence of signal and in his absence, and differs only in the greater dispersion. Probability of correct detection in this case is found from the expression [8]: $D = \exp\{-U_0^2 / [N_0 E_{s_{mean}} (1+\rho^2)]\} = F^{1/(1+\rho^2)}$ (27)

Where $\rho^2 = \frac{E_{S_{mean}}}{N_0}$, $E_{S_{mean}}$ – average (expected) signal energy.

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