Nonlinear Analysis Of Partially Prestressed Partially Steel Fibrous Reinforced Concrete Space Frames Under Cyclic Loading

Mustafa B. Dawood **Muhammad J. Kadhim**
College of Engineering, University of Babylon

Abstract

An investigation of the nonlinear analysis of partially prestress partially steel fibrous reinforced concrete space frames having prismatic and / or non-prismatic (tapered) members and subjected to cyclically varying loading is presented in this study. Plastic zone model is utilized in this study with an incremental and iterative technique to study the inelastic behavior of reinforced concrete frames. An approach namely (**regions approach**) that was previously proposed by is used to take into account the variation of material properties through the depth and width of the section. Equivalent nodal loads (fixed – end forces) are presented for tapered element under uniformly distributed load considering the possibility of existence of plastic zone any where in the member. The effects of shear and torsion forces are taken into account. Kupfer, Hilsdrof and Rusch yield criteria is used as a limitation for the concrete behavior. The hardening / softening rule, flow rule, and tension stiffening rule for concrete are taken into account. Failure can be predicted by the concrete crushing at a certain region. The analytical model adopted in this study for the fibrous concrete represented obviously the behavior of steel fiber prestressed concrete frames under cyclic loading. This could be noted through comparing with the theoretical results of previous studies.

الخلاصة

التحليلِ اللاخطّي للهياكل الخرسانية المسبقة الجهد جزئياً و المسلحِة جزئيا بالألياف الفولاذيةِ و المحتوية على عناصر موشوريةُ و/ أَو غير موشورية (مُسْتَدَقّة) و الخاضعة إلى التحميل المختلفِ دَوريّاً . نموذج المناطقة اللدنة مُسْتَعْمَلُ في هذه الدراسةِ بتقنيةِ تزايديةِ وتكراريةِ لدِراسَة السلوكِ الغير مرنِ للهياكل الخرسانية. تم استخدام طريقة المناطق للأَخْذ في الحسبان إختلافِ الخواص الماديةِ خلال عمقِ وعرضِ المقطع. الأحمال العقدية المكافئة (fixed end forces) للعنصرِ المُسْتَدَقّ تحت الحملِ المُوَزَّعِ بشكل منتظم مع الأخذ بنظر الأعتبارُ إمكانيةَ وجودِ المنطاق اللدنة في أي مكان في العضو. إنّ تأثيراتَ قوى الإلتواءَ والقصَّ يَأْخذانِ في الحسبان. تم استخدام آلية الخضوع التي اقترحها Kupfer، Hilsdrof و Rusch كمحدد لتصرف الخرسانة. كذلك تم الأخذ بنظر الاعتبار قاعدة الصلابة النعومة Hardening / Softening Rule، قاعدة الجريان Flow Rule و تقوية الشد Tension stiffening للخرسانة. آلية الانهيار في هذا البحث تتم بالسحق الموضعي(Crushing Failure) للخرسانة خلال منطقة معينة. النموذج التحليلي الذي تم استخدامه في هذه الدراسة مثل بشكل واضح سلوك الخرسانة مسبقة الجهد و المسلحة بالألياف الفولاذية الواقعة تحت الأحمال الدورية. هذا يمكن ملاحظته من خلال المقارنة مع النتائج النظرية للدراسات السابقة.

Keywords: Concrete, Cyclic Load, Space frames, Prestress, Steel Fiber.

Notation:

$\{a\}$	Flow vector.
$A_{s(ij)}$	Area of steel region (ij).
$\lfloor B_A \rfloor$	Axial strain – displacement matrix.
$b_{c(ij)}$	Width of concrete region (ij).
$b_{sf(ij)}$	Width of steel fibrous concrete region (ij).
$[D_{con}]$	Concrete stress – strain relationship matrix.
$[D_{con}]_{ep}$	Elasto – plastic concrete stress – strain relationship matrix.
$[D_s]$	Steel stress – strain relationship matrix.
$[D_{sfrc}]$	Steel fiber reinforced concrete stress – strain relationship matrix.
$[D_{sfrc}]_{ep}$	Elasto – plastic Steel fiber reinforced concrete stress – strain relationship matrix.
f_c'	Cylinder compressive strength of concrete.
f_t'	Ultimate concrete tensile strength.

G	Shear modulus of elasticity of concrete.
$\overline{G}$	Shear modulus of elasticity of cracked concrete.
H'	Hardening / softening parameter.
K	Shear correction factor.
L	Total length of element.
L_f	Fiber length.
N_{1c}, N_{2c}	Number of concrete regions in depth and width direction respectively.
N_{1ps}, N_{2ps}	Number of prestress steel regions in depth and width direction respectively.
N_{1s}, N_{2s}	Number of steel regions in depth and width direction respectively.
N_{1sf}, N_{2sf}	Number of steel fibrous concrete regions in depth and width direction respectively.
p_i	Incremental internal forces.
$P_{e(ij)}$	Effective prestress force in prestress steel region (i,j).
$t_{c(ij)}$	depth of concrete region (ij).
$t_{sf(ij)}$	depth of steel fibrous concrete region (ij).
V_f	Volume fraction of fiber.
α_{cr}, β_{cr}	Material parameters used in yield criterion.
$\gamma_{cxy}, \gamma_{cxz}, \gamma_{cyz}$	Concrete shear strain in xy, xz and yz – plane.
$\gamma_{sfxy}, \gamma_{sfxz}, \gamma_{sfyz}$	Steel fibrous concrete shear strain in xy, xz and yz – plane.
ε_{cx}	Concrete strain in local direction x.
ε_{sfx}	Steel fiber reinforced concrete strain in local direction x.
ε_{sx}	Steel strain in local direction x.
ε_{cu}	Ordinary concrete or steel fibrous concrete ultimate total strain.
ν	Poisson´s ratio.
σ_0	Equivalent effective stress.
σ_{cx}	Concrete stress in local direction x.
$\tau_{cxy}, \tau_{cxz}, \tau_{cyz}$	Concrete shear stress in xy, xz and yz – plane.
$\tau_{sfxy}, \tau_{sfxz}, \tau_{sfyz}$	Steel fibrous concret shear stress in xy, xz and yz – plane.

Introduction

The behavior of partially prestress partially steel fibrous reinforced concrete (SFRC) members subjected to cyclic loading is extremely complex. It is necessary to make use of numerical solutions in solving non linear governing equations established by the materials nonlinearities. The finite element method has been used by several researchers [Nilson, 1968; Darwin and Pecknold, 1977] to analyze reinforced and prestressed concrete members considering material nonlinearity. While stiffness method with tangent and secant stiffness was used by others [Gunnin, 1970; Sirisreetreerux and Tanbe, 1977; Vecchio, 1987] to analyze the reinforced concrete frames under monotonic loading. In the present work, a computer program in FORTRAN computer language was written to investigates the suitability of the finite element method with tangential stiffness to

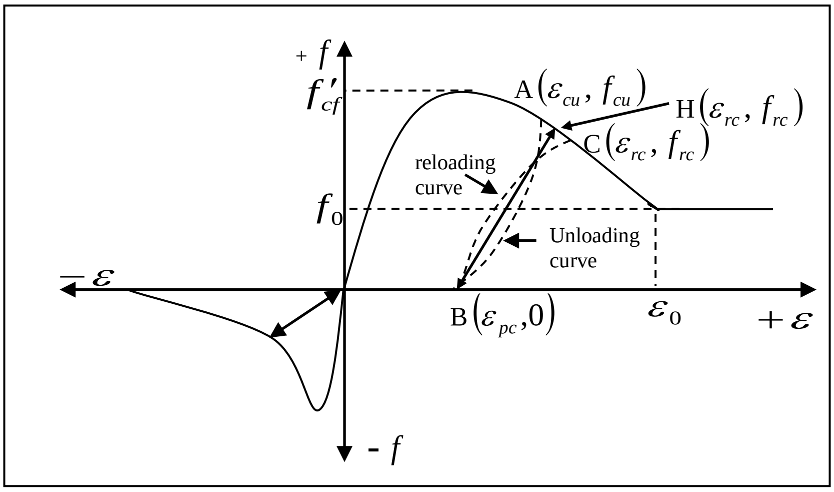

Fig.(1): Stress-strain curve in compression and tension for SFRC under cyclic loading.

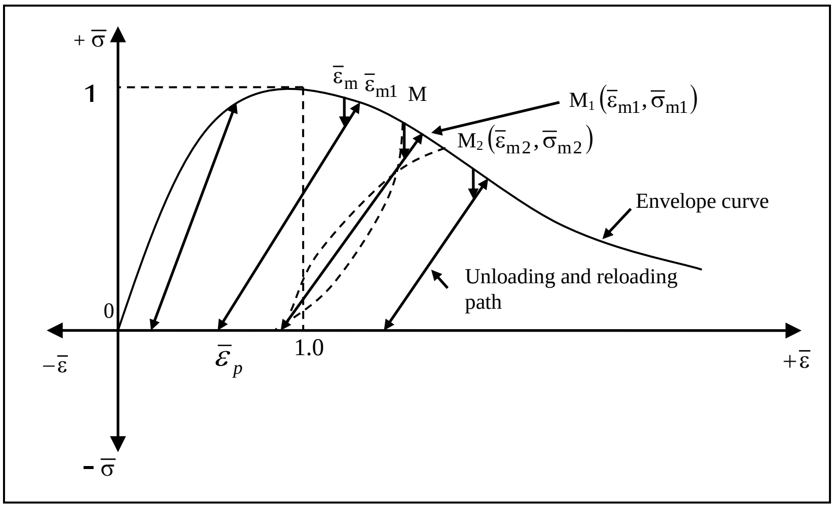

Fig.(2): Adopted normalized stress-strain relation for concrete under cyclic loading.

analyze partially prestress partially steel fibrous reinforced concrete space frames under cyclic loading. Material and geometrical nonlinearity are taken into account by using regions approach and suitable loading model (stress–strain relationship) for concrete and steel. Joint coordinates are updated at the beginning of each load stage.

Stress – strain behavior of fibrous concrete under cyclic loading:

An empirical model suggested by Soroushian and Lee is adopted in this study, Fig.(1) shows the envelop and the cyclic curves. Also this figure shows the cyclic behavior of steel fibrous concrete in tension which is suggested by **Al-Sulayfani and Al-Taee (2005).**

Stress – strain behavior of concrete under cyclic loading:

In the present study, because of the accuracy and simplicity, The model adopted by **[Mahmood] (Al-Sulayfani model, 2005)** is used in the present study to represent the nonlinear behavior of concrete under cyclic loading. Fig. (2) shows the envelope curve and the cyclic behavior curves. A linear path will be adopted for the stress – strain behavior of concrete under tension with the modulus of elasticity equal to the nominal modulus of elasticity in compression. This will be valid up to cracking strength f_t', which has the following expression:

$$f_t' = 0.625\sqrt{f_c'} \qquad (1)$$

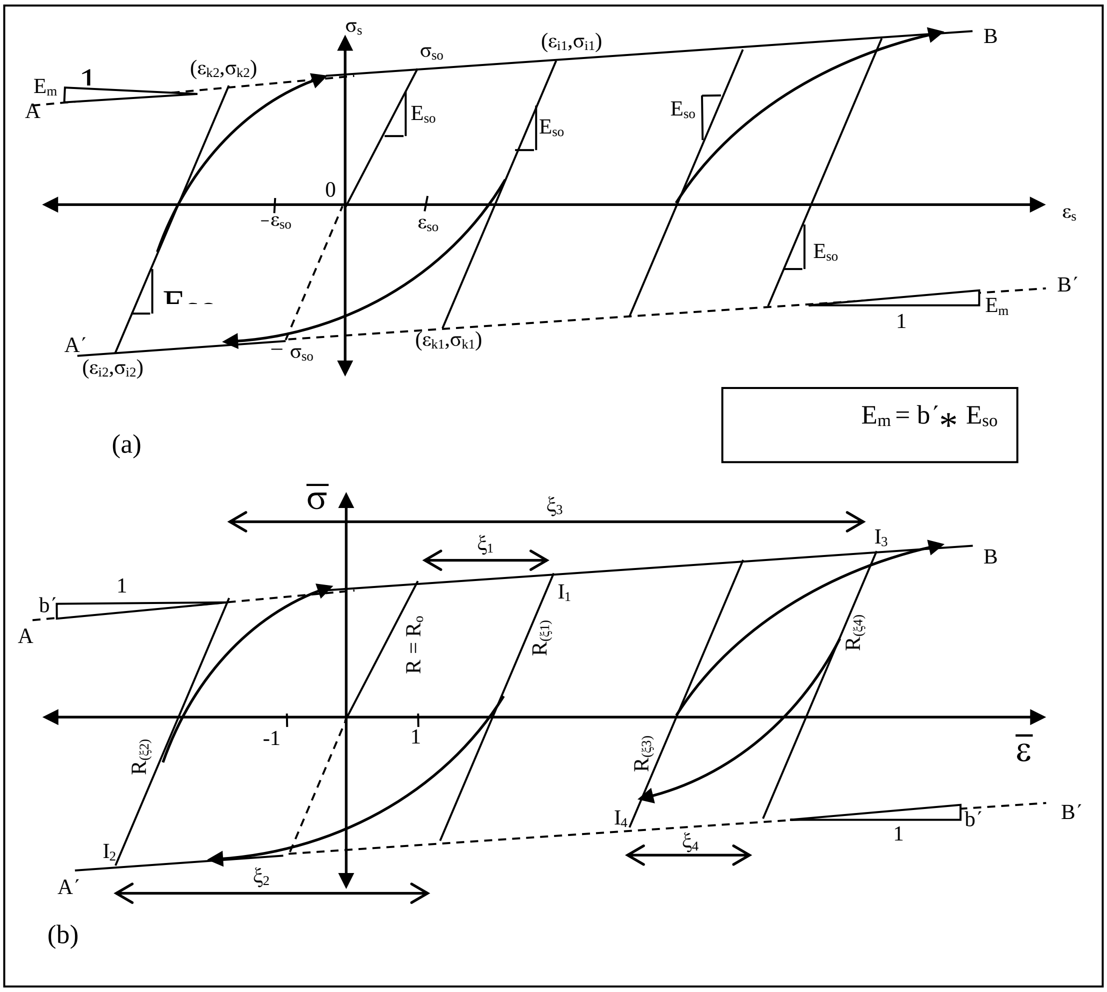

Fig. (3): Cyclic behavior model of steel

(a) Hysteretic behavior model of steel by **Menegotto and Pinto**.

(b) Normalized behavior model of steel by **Menegotto and Pinto**.

Where f'_c is in N / mm^2. Beyond f'_t the concrete is considered to be incapable of transmitting tensile stresses.

Stress-strain behavior of reinforcing and prestressing steel under cyclic loading:

Menegotto and Pinto (1973) model is used to represent the nonlinear behavior of reinforcing and prestressing steel under cyclic loading. This model is shown in Fig. (3). The stress-strain curves of all cycles lie within the two parallel lines A-B and A′B′ which are defined by the monotonic curve and passing through the yield points ($\varepsilon_{so}, \sigma_{so}$) and ($-\varepsilon_{so}, -\sigma_{so}$) respectively. All the curves, which represent the hysteretic behavior of steel, have the same initial slope equal to the slope E_{so} of the monotonic curve.

Effect of effective prestressing force:

The element is divided into five sections and the section is divided into imaginary concrete region, SFRC region, reinforcing steel region and prestressing steel region **[Kadhim, 2007]**. The internal forces of the element (p_i) will be calculated as follows:

$$p_i = \int_0^L \sum_{j=1}^{N_{1c}} \sum_{k=1}^{N_{2c}} b_{c(jk)}\, t_{c(jk)} [B_A]^T [D_{con}] \{\varepsilon_{cx}\} dx + \int_0^L \sum_{j=1}^{N_{1sf}} \sum_{k=1}^{N_{2sf}} b_{sf(jk)}\, t_{sf(jk)} [B_A]^T [D_{sfrc}] \{\varepsilon_{sfx}\} dx + \int_0^L \sum_{j=1}^{N_{1s}} \sum_{k=1}^{N_{2s}} As_{(jk)} [B_A]^T [D_S] \{\varepsilon_{sx}\} dx + \int_0^L \sum_{j=1}^{N_{1ps}} \sum_{k=1}^{N_{2ps}} pe_{(jk)} [B_A]^T dx \quad (2)$$

This internal forces will be compared with the external applied forces and the residual will be applied as external forces. Actually, as a first step this process will be attempt before applying the external load to take into account the effect of effective prestressing force.

The steel fibrous reinforcing concrete and reinforced concrete yield criterion:

The yield criterion determines the stress level at which plastic deformation begins **[Owen and Hinton, 1980]**. In the present study the yield criterion takes the following form:

$$f(\sigma) = \left[\alpha_{cr}\, \sigma_{cx} + \beta_{cr} \left(\sigma_{cx}^2 + 3\tau_{cxy}^2 + 3\tau_{cxz}^2 + 3\tau_{cyz}^2\right)\right]^{1/2} = \sigma_o \quad (3)$$

Where α_{cr} and β_{cr} are material parameters and σ_o is the equivalent effective stress taken from uni – axial test. This yield criterion takes into account the transverse shear effect.

For steel fibrous concrete are **[Ibrahim, 2002**]:

$$\alpha_{cr} = \frac{1-\omega^2}{\omega^2 - 2\omega} \quad , \quad \beta_{cr} = \frac{1-2\omega}{\omega^2 - 2\omega} \quad (4)$$

Where:

$$\omega = e^x \quad (5)$$

$$x = \frac{1}{3.339 - 0.9772 \dfrac{V_f L_f}{D_f}} \quad (6)$$

If the results obtained by **Kupfer** for a failure envelope is employed for the initial yield, the value of the constant α_{cr} and β_{cr} for ordinary concrete are:

$\alpha_{cr} = 0.355\ \sigma_o$ and $\beta_{cr} = 1.355$ (7)

In the present study we will assume that the initial yield surface is attained when the effective stress reaches 30 % of the ultimate stress f'_c.

The hardening/softening rule for concrete

After initial yielding, the stress level at which further plastic deformation occurs is dependent on the current degree of plastic straining. Such a phenomenon is termed strain hardening. In the present study, an isotropic hardening / softening rule is adopted. At first, when a material is stressed beyond its initial yielding surface, the yielding surface will expand until the effective stress reaches the ultimate stress f'_c, after that, the yielding surface will contract due to softening effect until the failure occur. The hardening / softening parameter takes the following form **[Hinton and Owen, 1984**]:

$$H' = E_{ct} / \left(1 - \left(E_{ct} / E_{co}\right)\right) \quad (8)$$

If H′ equal to zero, then the material is stressed to be perfectly plastic. When H′ equal to infinity, then the material is still with in elastic range.

The flow rule for concrete:

The flow rule is considered to construct the stress – strain relationship in the plastic range. The complete elasto – plastic incremental stress – strain relationship can be expressed as **[Hinton and Owen, 1984]:**

For steel fibrous concrete

$$[D_{sfrc}]_{ep} = [D_{sfrc}] - \frac{[D_{sfrc}]\{a\}\{a\}^T[D_{sfrc}]}{H' + \{a\}^T[D_{sfrc}]\{a\}} \tag{9}$$

For ordinary concrete

$$[D_{con}]_{ep} = [D_{con}] - \frac{[D_{con}]\{a\}\{a\}^T[D_{con}]}{H' + \{a\}^T[D_{con}]\{a\}} \tag{10}$$

Where the second term in the bracket represents the stiffness degradation due to plastic deformations.

The crushing conditions for concrete

The crushing type of concrete fracture is a strain-controlled phenomenon. The failure surface is represented by the following equation:

$$\alpha_{cr}\, \varepsilon_{cu}(1-2\nu)\varepsilon_{cx} + \frac{4}{9}\beta_{cr}\left[(1+2\nu+\nu^2)\varepsilon_{cx}^2 + 0.75\left(\gamma_{cxy}^2 + \gamma_{cxz}^2 + \gamma_{cyz}^2\right)\right] = \varepsilon_{cu}^2 \tag{11}$$

When ε_{cu} reaches the ultimate value, which is equal to 1.5 ε_{co}, the concrete is assumed to lose all its characteristics of strength and rigidity.

Fibrous concrete in tension

For cracked fibrous concrete in tension zone, the stress – strain relationship takes the following form:

$$\begin{bmatrix} \sigma_{sfx} \\ \tau_{sfxy} \\ \tau_{sfxz} \\ \tau_{sfyz} \end{bmatrix} = \begin{bmatrix} E_{ts} & 0 & 0 & 0 \\ 0 & \overline{G} & 0 & 0 \\ 0 & 0 & \overline{G} & 0 \\ 0 & 0 & 0 & KG \end{bmatrix} * \begin{bmatrix} \varepsilon_{sfx} \\ \gamma_{sfxy} \\ \gamma_{sfxz} \\ \gamma_{sfyz} \end{bmatrix} \tag{12}$$

Where:[Naji]

$$E_{ts} = \frac{\sigma_{cr}\exp\left(-(\varepsilon_n - \varepsilon_0)/(\eta_o\eta_l\eta_b V_f + 0.001)\right)}{\varepsilon_n} \qquad for \quad \varepsilon_n > \varepsilon_0 \tag{13}$$

η_o varies from 0.33 to 0.5.

η_b varies from 0.5 to 1.

$$\eta_l = \begin{Bmatrix} 0.5 & for\ L_f < L_c \\ 1 - L_c/2L_f & for\ L_f > L_c \end{Bmatrix} \tag{14}$$

Shear modulus of cracked fibrous concrete:

The value of $\overline{G}$ is linearly decreasing with the strain normal to the crack plane and calculated according to the following formula **[Ibrahim, 2002]**:

$$\overline{G} = G\left(1 - \left(\frac{\varepsilon_n}{0.005}\right)^{K_1}\right) \tag{15}$$

Where:

ε_n The fictitious strain normal to the crack plane.

K_1 parameter in range (0.3 – 1).

When the crack is closed, the uncracked shear modulus is again assumed in the corresponding direction.

Tension stiffening rule for fibrous concrete:

Due to the bond effects , cracked concrete carries between cracks a certain amount of tensile force normal to the cracked plane. The concrete adheres to the reinforcing bars and contributes to the over all stiffness of the structure. The assumed shape of the stress – strain hystersis loops for cyclic loading in tension range is shown in Fig. (4).

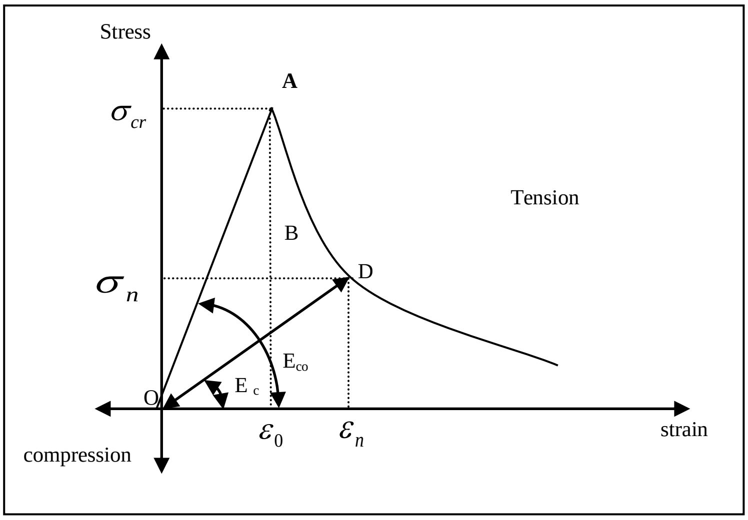

Fig. (4) : Loading and Unloading Behavior of Cracked Fibrous Concrete Illustrating Tension Stiffening Behavior. **[as found in Ibrahim]**.

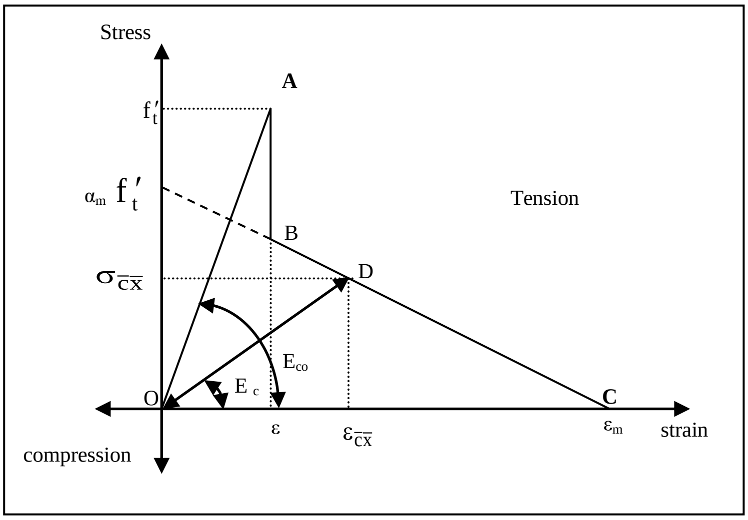

Fig. (5) : Loading and Unloading Behavior of Cracked Concrete Illustrating Tension Stiffening Behavior. **[Hinton and Owen]**.

Ordinary concrete in tension:

For cracked concrete in tension zone, the stress – strain relationship takes the following form:

$$\begin{bmatrix} \sigma_{cx} \\ \tau_{cxy} \\ \tau_{cxz} \\ \tau_{cyz} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \overline{G} & 0 & 0 \\ 0 & 0 & \overline{G} & 0 \\ 0 & 0 & 0 & KG \end{bmatrix} * \begin{bmatrix} \varepsilon_{cx} \\ \gamma_{cxy} \\ \gamma_{cxz} \\ \gamma_{cyz} \end{bmatrix} \tag{16}$$

Shear modulus of cracked concrete:

The value of $\overline{G}$ is linearly decreasing with the strain normal to the crack plane and calculated according to the following formula **[Hinton and Owen,1984]**:

$$\overline{G} = 0.25 * G\left(1 - \frac{\varepsilon_{cx}}{0.04}\right) \tag{17}$$

For $\varepsilon_{cx} < 0.004$

$$\overline{G} = 0 \tag{18}$$

For $\varepsilon_{cx} \geq 0.004$

When the crack is closed, the uncracked shear modulus is again assumed in the corresponding direction.

Tension stiffening rule for concrete:

Due to the bond effects , cracked concrete carries between cracks a certain amount of tensile force normal to the cracked plane. The concrete adheres to the reinforcing bars and contributes to the over all stiffness of the structure. The assumed shape of the stress – strain hystersis loops for cyclic loading in tension range is shown in Fig. (5). **[Hinton and Owen,1984]**

Algorithm for The Proposed Procedure of Analysis:

The adopted approach of the analysis , which is called the incremental approach , treats the problem of nonlinear behavior as a sequence of linear problems. During every loading step of the sequence , the structure supports a new increment of external loads. Each step is based on material and geometry properties appropriate to that step , i.e. , the stiffness of the structure is updated at the beginning of each step. The procedure of analysis can be illustrated through the figure (6).

Applications:

Example No.1:

A plane one storey one bay steel fibrous reinforced concrete frame was tested experimentally by **Sabnis** and **White**, in 1969 and analysis theoretically by **Al-Sulayfani** and **al-Taee**. Figure (7) shows the data used in the present study to analysis this frame. The loading conditions for frame (F1) are shown in figure (8). According to the new proposed procedure of analysis, frame (F1) is subdivided into 18 elements (i.e. six elements for each member). Each beam or column section in the frame is subdivided into ten concrete regions in width direction by ten concrete regions in depth direction and eight steel regions. The horizontal displacements obtained by **Sabnis** and **White (1969)** experimentally and by **Al-Sulayfani** and **Al-Taee (2005)** theoretically and the results obtained from the proposed analytical procedure are presented in figure (9). From figure (9) the analysis using (18 elements) gives good agreement with the experimental results.

Example No.2:

In 2005, a prestressing fibrous reinforced concrete plane frame was analyzed by **Al–Sulayfani** and **Al-Taee**. This frame is reanalyzed using the proposed approach. The details of this frame are shown in figure (10). The test loading conditions for frame (F2) are shown in figure (11). This frame is subdivided into 18 elements(i.e. six elements for each member). Each section in this frame is subdivided into (100)concrete regions, ie. (10) regions in depth direction by (10) regions in width direction; and eight reinforcing steel regions, and one prestress steel region.

Theoretical load – deflection curves from **Al–Sulayfani** and **Al-Taee** (2005) and the theoretical load –deflection curve from the present study are shown in figure (12). The analysis using (18 elements) give good agreement with the results obtained by **Al–Sulayfani** and **Al-Taee** (2005).

Example No.3:

In this example the effect of partial depth of steel fibrous concrete on the behavior of partial prestress concrete space frame under cyclic loading will be studied through drawing the load–deflection curves for frame with various partial depth of steel fibrous concrete varies from zero to the total depth of the member. This frame is analyzed up to crushing failure. On the other hand this parametric study shows the ability of the present analysis procedure and program to solve the problem of partial

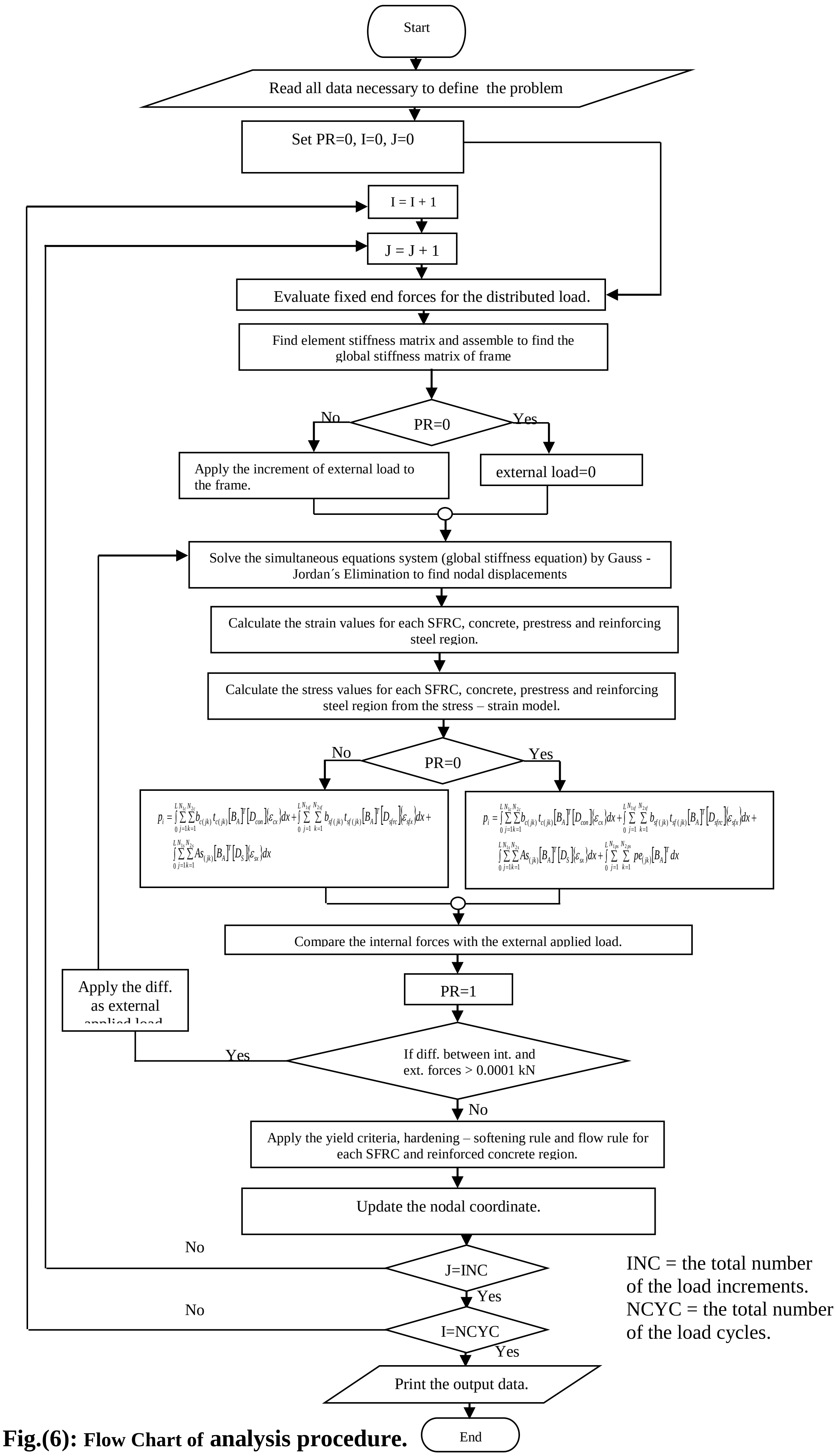

Fig.(6): Flow Chart of analysis procedure.

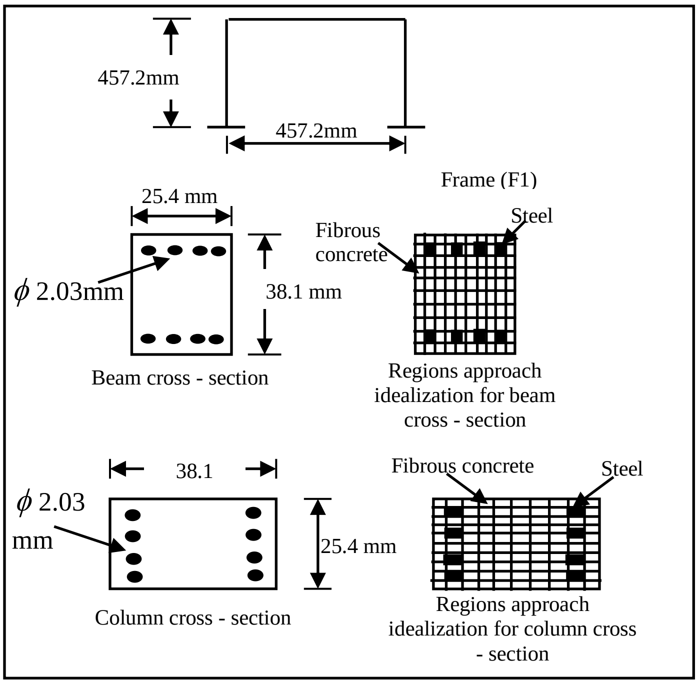

Fig. (7) : Details of Frame (F1)

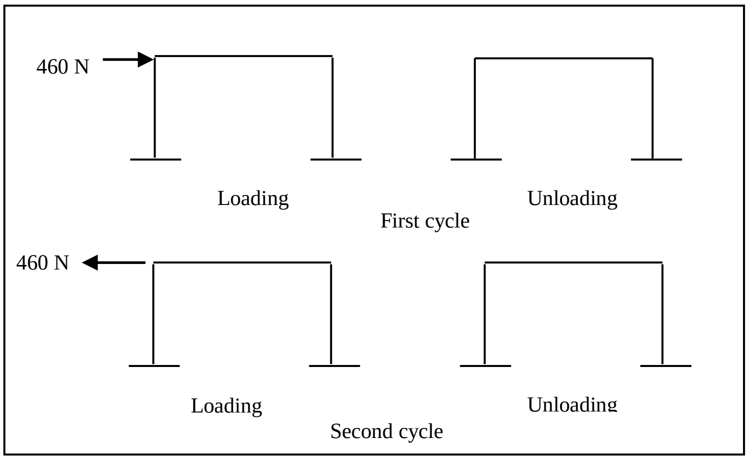

Fig. (8) : Test Loading Conditions for Frame (F1)

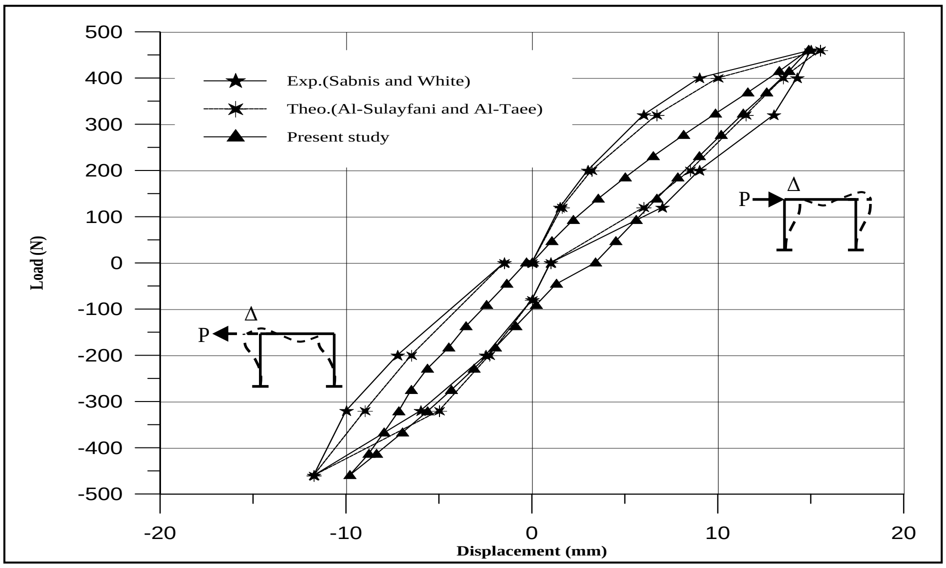

Fig. (9):Load – Displacement Curve for Frame (F1) Obtained by the previous researchers and The Present Analysis Using (18 Elements).

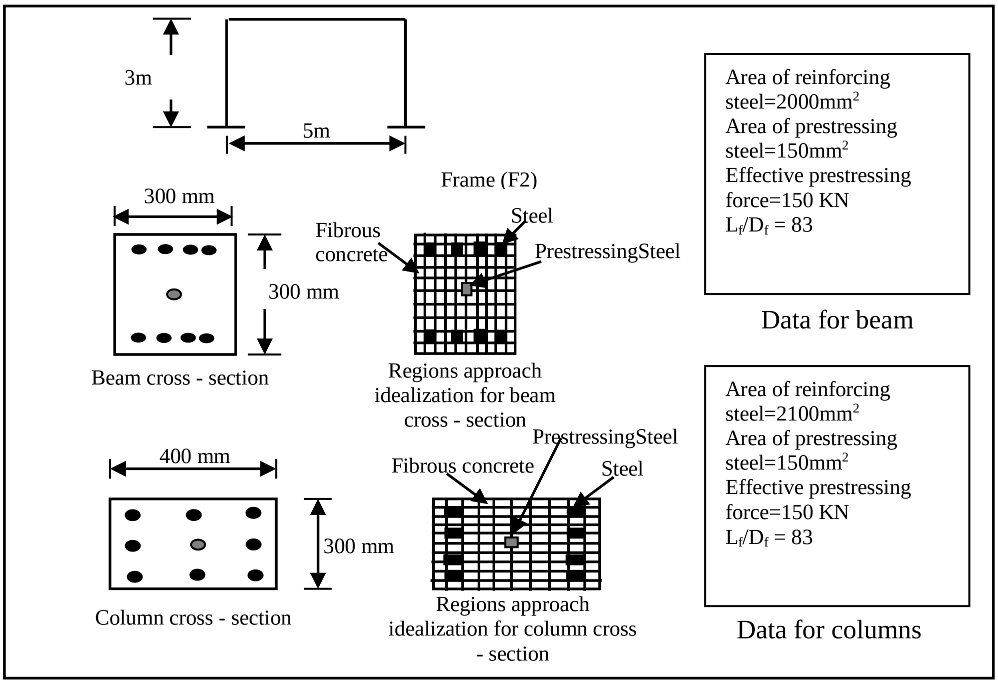

Fig. (10) : Details of Frame (F2)

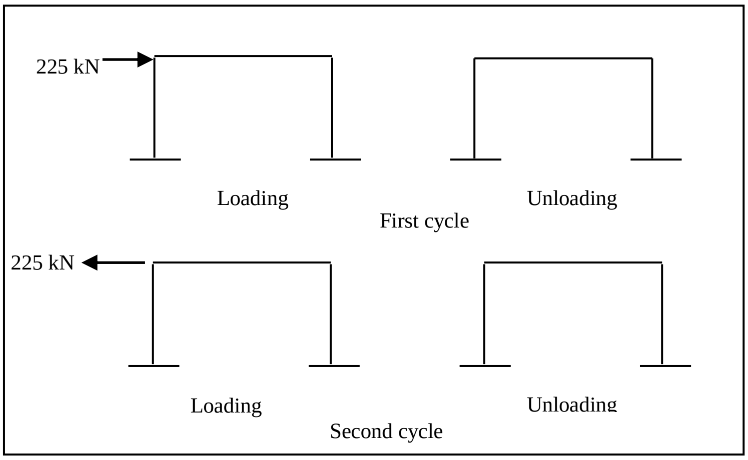

Fig. (11) : Test Loading Conditions for Frame (F2)

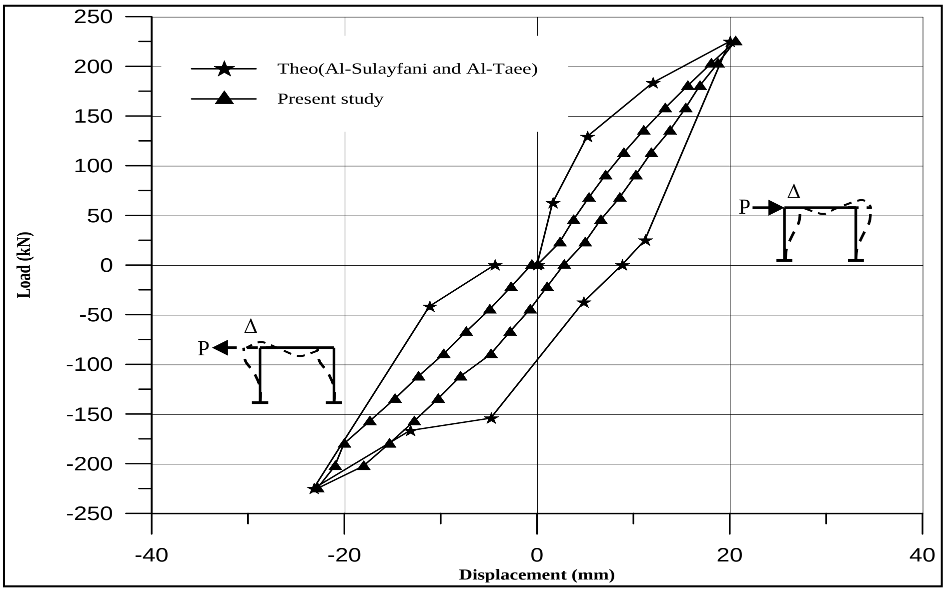

Fig. (12):Load – Displacement Curve for Frame (F2) Obtained by the previous researchers and The Present Analysis Using (18 Elements).

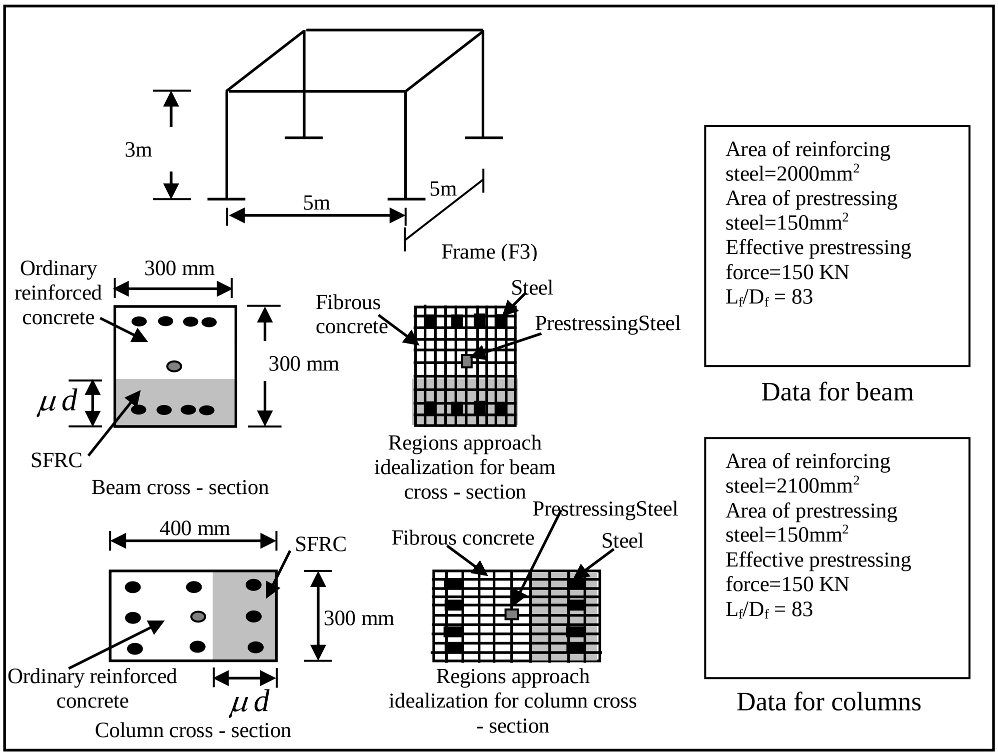

Fig. (13) : Details of Frame (F3)

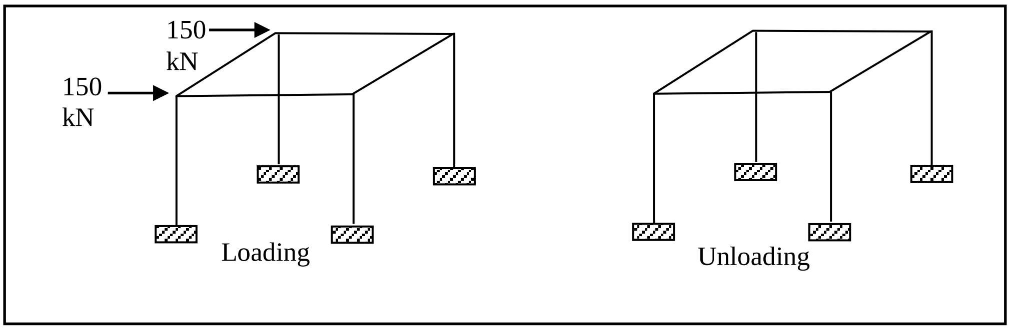

Fig. (14): Loading Conditions for Frame (F3).

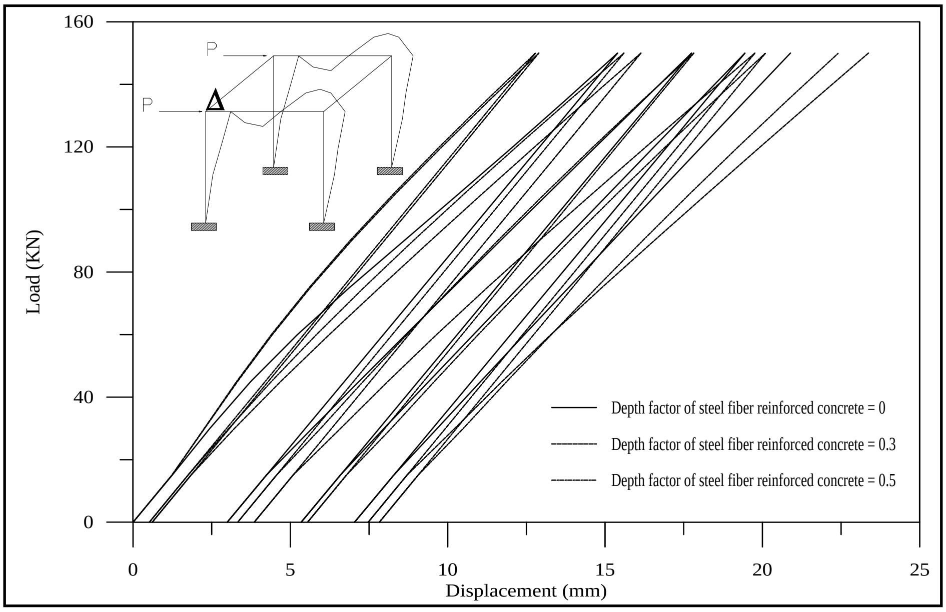

Fig. (15):Load – Deflection Curves for Frame (F3) With Various Depth of Steel Fibrous concrete.

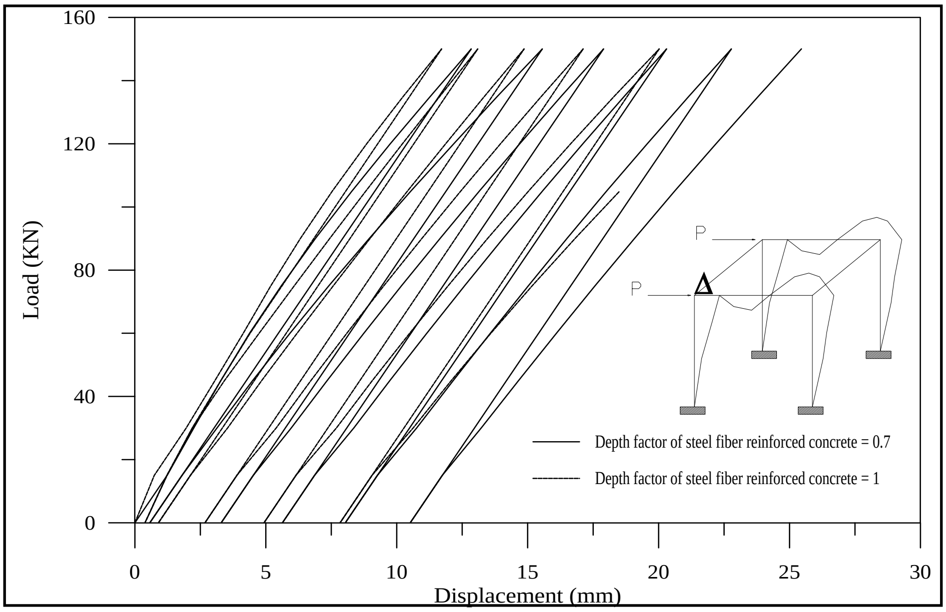

Fig. (16):Load – Deflection Curves for Frame (F3) With Various Depth of Steel Fibrous concrete.

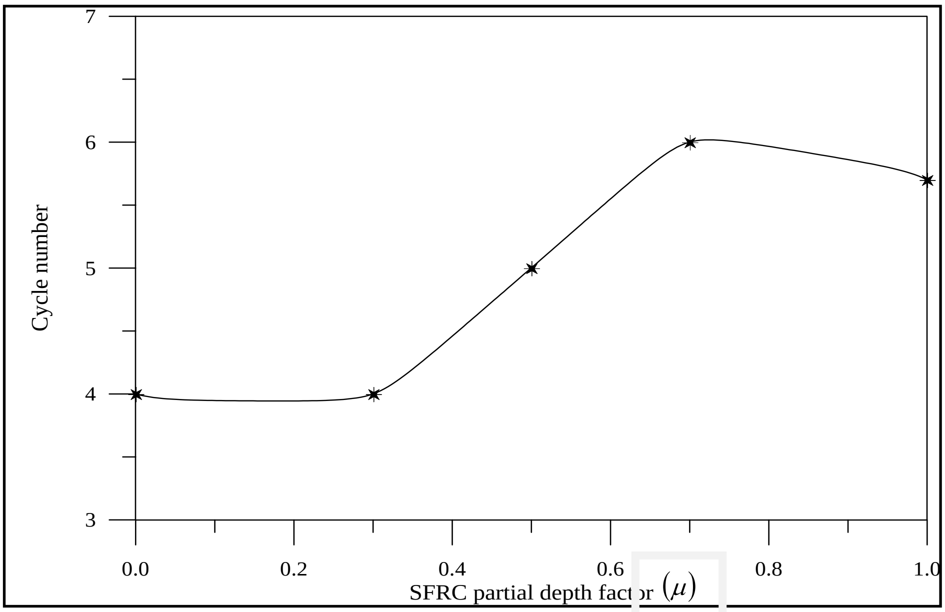

Fig. (17):Cycle Number at Crushing State – SFRC Partial Depth Factor Curves for Frame (F3).

steel fibrous partial prestressing reinforced concrete space frames under cyclic loading. The details of this frame are shown in figure (13). The test loading conditions for the frame are shown in figure (14). This frame is subdivided into 48 elements(i.e. six elements for each member). Each section in this frame is subdivided into (100) concrete regions, ie. (10) regions in depth direction by (10) regions in width direction; and eight reinforcing steel regions, and one prestress steel region. Figures (15 and 16) shows the load – deflection curves obtained from analyzing frame (F3)with various depth of steel fibrous concrete. Figure (17) show the relation between number of load cycle at which the crushing failure occurs and steel fibrous concrete depth factor (μ). This figure shown that the optimum depth factor is about (0.7) approximately but the following equation which was proposed by **Padmarajaiah** and **Ramaswamy** (2002) given that the optimum depth factor is 0.3.

$$\mu = \frac{(t-1)}{(t-\gamma)} \tag{19}$$

Where:

$\gamma = \sqrt{3(2t-1)}/2$, $t = 1.725(RI)+1$, $RI = V_f L_f / D_f$

This difference between the optimum SF depth factor obtained from figure (17) and that obtained from equation (19) come from that this equation was derived for members under monotonic loads. From this parametric study, for the member under cyclic loading, the SF should be add to the top and bottom of the member not to one side.

Conclusions

1. The analytical model adopted in this study for the fibrous concrete represented obviously the behavior of steel fiber prestressed concrete frames under cyclic loading. This could be noted through comparing with the theoretical results of previous studies.
2. The region approach is so efficient in nonlinear analysis of concrete members, since the stress distribution along the sections seems to be rather close to the real state.
3. The optimum steel fibrous concrete depth factor for member under cyclic loading (approximately 0.7 when the addition in one side of the member)is different from that obtained by the equation that was proposed by **Padmarajaiah** and **Ramaswamy** (2002) (0.3). So the addition of steel fibrous concrete in top and bottom of the member or in the form as a ring around the circumference of the member can be studied.

References

Abdul Kareem Darweesh Mahmood, Sep. 1989, “ Nonlinear Analysis of Reinforced Concrete Frames Under Cyclic Loading ”, M. Sc. Thesis, College of Engineering, University of Mosul.

Al-Sulayfani, Bayar J. and Al-Taee, Hatim T. , June 2005, "Non linear Behavior of Steel Fibrous Prestressed Framed Under Cyclic Loads", Al-Rafidain Engineering Journal, Vol.14, No.1, pp.12-24.

Bahnam, M. (Lctures in Concrete Technology for Post Graduated Studies”, Civil Eng. Dept., Eng. College, Babylon University.

Darwin, and Pecknold, 1977, A., "Analysis of Cyclic Loading of Plane RC Structures", Computers and Structures, Vol. 7,pp. 137 - 147.

Gunnin, B. L. , 1970, "Nonlinear Analysis of Plane Frames", Ph.D. Dissertation, The University of Texas at Austin.

Ibrahim, Ayoub Abbas, 2002, "Nonlinear Analysis of Steel Fiber Reinforced Concrete Members Under Cyclic Loading", Msc. Thesis, University of Babylon.

Kadhim, M.J., 2007, "Nonlinear Analysis of Reinforced Concrete Space Frames Under Cyclic Loading Including Time Dependent Effects", Ph.D. Thesis, Civil Eng. Dept., Engineering College, University of Babylon.

Menegotto, M. And Pinto, P. E., 1973, "Method of Analysis for Cyclically Loaded Reinforced Concrete Plane Frames Including Changes in Geometry and Non-Elastic Behavior of Elements under Combined Normal Force and Bending", Proceedings, IABSE Symposium on Resistance and Ultimate Deformability of Structures Acted on by Well – Defined Repeated Loads, Final Report, Lisbon, pp. 15-20.

Naji, J.H., 1997, "Nonlinear Finite Element Analysis of Steel Fiber Reinforced Concrete Beams", Proc. To the 4th Scientific Engineering Conf., University of Baghdad, 18-20 Nov.

Nilson , A.H., Sept. 1968, "Non Linear analysis of reinforced concrete by finite element method",ACI Journal, pp. 757 – 766.

Owen D.R.J. and Hinton, E. 1980, " Finite Elements in Plasticity: Theory and Practice ", Pineridge Press Limited, Swansea, U.K..

Owen D.R.J. and Hinton, E. 1984, " Finite Element Software for Plates and Shells ", Pineridge Press, Swansea, U.K..

Padmarajaiah, S. K., and Ramaswamy, A., 2002, "Comparative Study on Flexural Response of Full and Partial Depth Fiber – Reinforced High – Strength Concrete", Journal of Materials in Civil Engineering, Vol.14, No.2, pp.130-136.

Sabnis, G.M. and White, R.N., Sep. 1969, "Behavior of Reinforced Concrete Frames Under Cyclic Loads Using Small Scale Models", ACI Materials Journal, Vol.66, No.9, pp. 703-715.

Sirisreetreerux, and Tanabe, 1977, "Nonlinear Analysis of Reinforced Concrete Frames", Concrete Engineering and Pavement Division, Vol.11, pp. 320 – 323.

Soroushian, P., and Lee, C.D., 1989, "Constitutive Modeling of Steel Fiber Reinforced Concrete under Direct Tension and Compression", Recent Development in Fiber Reinforced Cements and Concretes, Elevier Scinece Publishers Ltd., Essex, pp. 363-377.

Vecchio, F.J. , Nov.–Dec. 1987, "Nonlinear Analysis of Reinforced Concrete Frames Subjected to Thermal and Mechanical Loads", ACI Structural Journal, pp. 492 501.