# Using Inverse Design Method to Investigate the Objective Focal Properties for Asymmetrical Magnetic Field

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## Abstract

In this research depending on inverse design method to investigate the objective focal properties for asymmetrical magnetic field by using the axial distribution of the magnetic field for (Grivet-Lenz model ) . The influence of the magnetic field model parameters on the objective focal properties and the pole pieces profile of asymmetrical magnetic electron lenses has been taken into account .

The results have shown that the most effective parameters on the design configuration are the two half widths of the axial magnetic field distribution .Also , the pole pieces of various asymmetrical double pole piece magnetic lenses can be reconstructed according to the different half widths of the field on the object and image sides .

## الخلاصة

اعتمد هذا البحث طريقة التصميم العكسي في دراسة الخواص البؤرية الشيئية لمجال مغناطيسي لامتناظر بالاستعانة بالتوزيع المحوري للمجال المغناطيسي لأنموذج – كرفت لنز . لقد اخذ بنظر الاعتبار تأثير معلمات النموذج المستخدم على الخواص البؤرية الشيئية والمظهر الجانبي لأقطاب العدسات المغناطيسية اللامتناظرة .

أظهرت النتائج إن المعاملات الأكثر تأثيرا على شكل التصميم تتمثل بعرضي النصف لتوزيع المجال المغناطيسي المحوري كذلك إن أقطاب العدسات المغناطيسية ثنائية القطب اللامتناظرة المتنوعة يمكن إعادة تركيبها حسب عرض النصف المختلف للمجال على جانبي الجسم والصورة .

## 1.Introduction

The synthesis technique is considered to be on the best method for improving the work or performance of the magnetic lenses and its saves time in comparison with the analytical procedure which takes much more time. In this research an analytical function has been used (Grivet-Lenz model) for representing the axial magnetic scalar potential distribution through which its asymmetrical axial magnetic field distribution has been reached and its objective focal properties such as ( spherical and chromatic aberration coefficients and focal length) at two sides of asymmetrical axial magnetic field distribution and then comparison between them under zero and infinite magnification conditions .

Depending on the similarity between the curve shapes of the axial magnetic field distribution and the hyperbolic tangent function, this function has been used for representing the electron beam trajectory $r_z$ as shown by the following form [Szilagyi, 1988]:

$$r_z = \alpha \tanh\left(\frac{z}{\beta}\right) \qquad z_S \le z \le z_E \qquad (1)$$

where $z_S$ and $z_E$ are the initial and final axial coordinates of the optical axis and ($\alpha = w.B_{max} / 2\mu_o$), ($\beta = w / 2$) , where $\mu_o$ is the space permeability and equal $4\pi x10^{-7}$ H.m$^{-1}$ , w is the half width and $B_{max}$ is the maximum magnetic flux density.

Since the magnetic lens is symmetrical, the magnetic field also is symmetrical, thus $|z_S| = z_E$, therefore the lens length $L=2|z_S|$ or $L=2Z_E$ or anther word $B_z(z)=B_z(-z)$. Hence the lens length L is put as optimization parameter in above mentioned of the two parameters w and $B_{max}$. The calculation of the magnetic flux density distribution along the optical axis has been calculate from the following paraxial ray equation [Juma and Mulvey, 1974;Zhigarev, 1975; Juma and Mulvy, 1978]:

$$r'' + \frac{\eta}{8V_r} B_z^2 r = 0 \qquad (2)$$

where $r''$ is the second derivative of the electron beam trajectory, r is the electron beam trajectory, $\eta$ is the charge -to- mass quotient of the electron, $V_r$ is the relativistically-corrected accelerating voltage and Bz is the axial magnetic flux density distribution along the optical axis . Then we can re-write the equation (2):

$$B_z^2 = -\left(\frac{8V_r}{\eta}\right)\left(\frac{r''}{r}\right) \qquad (3)$$

Now by derivative the equation (1) twice we get:

$$r' = \left(\frac{\alpha}{\beta}\right) \sec h^2 \left(\frac{z}{\beta}\right) \qquad (4)$$

$$r'' = -\left(\frac{2\alpha}{\beta^2}\right) \sec h^2 \left(\frac{z}{\beta}\right) \tanh \left(\frac{z}{\beta}\right) \qquad (5)$$

From substitution the equation (5) in the equation (3) we get :

$$B_z^2 = -\left(\frac{8V_r}{\eta}\right)\left(\frac{-\left(\frac{2\alpha}{\beta^2}\right) \sec h^2 \left(\frac{z}{\beta}\right) \tanh \left(\frac{z}{\beta}\right)}{\alpha \tanh \left(\frac{z}{\beta}\right)}\right) \qquad (6)$$

This equation led to:

$$B_z = \left(\frac{16V_r}{\eta\beta^2}\right)^{1/2} \left[\sec h\left(\frac{z}{\beta}\right)\right] \qquad (7)$$

Suppose the relations as shown by the following forms:

$$B_{max} = \left(\frac{16V_r}{\eta\beta^2}\right)^{1/2} \qquad (8)$$

$$z = \frac{w}{2} \qquad (9)$$

$$B_z\left(\frac{w}{2}\right) = \frac{1}{2}B_{max} \qquad (10)$$

Put the equations (8), (9) and (10) in the equation (7) we get :

$$\frac{1}{2}B_{max} = B_{max}\,\sec h\left[\frac{w}{2\beta}\right] \qquad (11)$$

$$\frac{1}{2} = \sec h\left(\frac{w}{2\beta}\right) = \frac{1}{\cosh\left(\frac{w}{2\beta}\right)} \qquad (12)$$

$$\cosh\left(\frac{w}{2\beta}\right) = 2 \qquad (13)$$

$$\left(\frac{w}{2\beta}\right) = \arccos h\,(2) = 1.317 \qquad (14)$$

$$\beta = \left(\frac{w}{2.634}\right) \qquad (15)$$

$$B_z = \frac{B_{max}}{\cosh\left(\frac{2.634\ z}{w}\right)} \qquad (16)$$

The relationship between the half width and the half half width is (w=2a), then equation (16) becomes:

$$B_z = \frac{B_{max}}{\cosh\left(\frac{1.317\ z}{a}\right)} \qquad (17)$$

where $B_z$ is the axial magnetic flux density distribution along the optical axis z, $B_{max}$ the maximum magnetic flux density and a the half half width of the magnetic field. It can be seen that equation (17) representing the (Grivet-Lenz model) and has three main control parameters that $B_{max,}$ a and z. In the present research the parameter has been studied only a and other two parameters were kept constant ( $B_{max}$=1T and L=40mm ) .

## 2.Field Distribution

The axial magnetic flux density distribution $B_z$ along the optical axis z may be represented by the following well-known field model(Grivet-Lenz model) equation (17). The half width is the axial distance from the position of maximum field $B_{max}$ to the point where the field has fallen to half this value. In the present work various asymmetrical field distributions have been taken into account. The maximum flux density ($B_{max}$) is considered to be located at z=0; it is a point of inflection of the magnetic field and the zero coordinate of the optical axis divides the asymmetrical field into two unequal parts.

It should be mentioned that in a symmetrical magnetic lenses $B_{max}$ is situated at the centre of the air gap separating the two pole pieces. However, in asymmetrical lenses $B_{max}$ is situated in the vicinity of the smaller bore pole piece. In terms of coordinates $B_{max}$ may be located at a point on the optical axis other than zero.

Equation (17) is applied to compute the asymmetrical axial flux density distribution $B_Z$ for the two unequal regions about the zero coordinate. In the present work the field asymmetry is obtained by varying the various parameters of the axial field such as its half width and its axial extension. The denominator of equation (17) is $[\cosh(1.317(-z/a_1))]$ for the half width $a_1$ on negative z and it is $[\cosh(1.317(z/a_2))]$ for the half width $a_2$ on positive z .

The total half width a of the asymmetrical axial field is the sum of the two half widths $a_1$ and $a_2$, i.e. $a=a_1+a_2$. When $a_1=a_2$ the axial flux density distribution will be symmetrical whereas if $a_1 \neq a_2$ the distribution will be asymmetrical . In an asymmetrical objective lens, one part of the axial field acts as a condenser and the other part acts as an objective. The function of each part of the axial field depends on the direction of entry of the electron beam .

Figure (1) shows a diagram of an asymmetrical axial magnetic field where $B_{max}$ is located at z=0. The object side field extends along negative z and has a half width $a_1$ whereas the image side field is along positive z and its half width is $a_2$. It is assumed to a good of accuracy that the axial extension of the field along the z-axis is represented by the distance L. The field at distances greater than L is neglected since its effect in practice is insignificant. The area under the curve of the axial asymmetrical flux density distribution is computed by using Ampere's law [Grivet, 1972; Hawkes and Kasper , 1989]:

$$\int_{Z_B}^{Z_E} B_Z \, dz = \mu_o NI = \mu_o (V_1 - V_2) \qquad (18)$$

where NI is the excitation of the magnetic lens in ampere-turns, and $V_1$ and $V_2$ are the magnetic scalar potentials on the two ends of L .

Computations on the two parts of the asymmetrical field have been carried out under various conditions. By keeping $B_{max}$, $a_2$ and L constants, the object side field has been computed at various values of $a_1$. Similarly, the image side field has been computed at various values of $a_2$ by keeping $B_{max}$, $a_1$, and L constants and also the effect of L on the focal properties has been investigated .

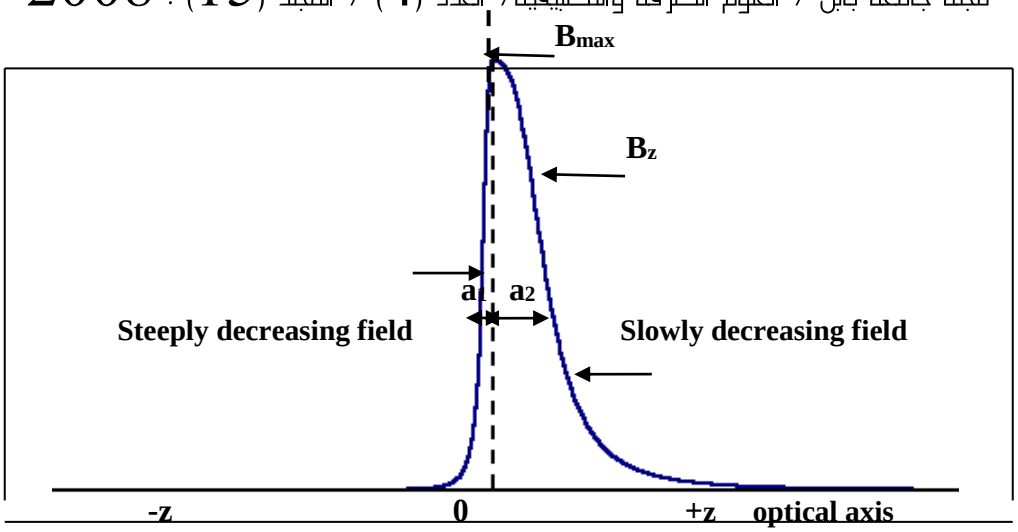



**Figure (1): Asymmetrical axial field distribution of a double pole piece magnetic lens .**

## 3.Pole piece Reconstruction

To determine the lens potential, the region of its axial extension is divided into (n-1) subintervals where n represents the points of the field along the z-axis. By numerical analysis the axial magnetic flux density distribution $B_z$ is represented by a cubic spline function in each subinterval as shown in the following equation [Al-Obaidi, 1995]:

$$B_z = -\mu_o \frac{dV_z}{dz} \qquad (19)$$

and with the aid of the cubic spline technique, the axial magnetic scalar potential may be given by*:*

$$V_{Z(k+1)} = V_{Zk} - G_k \qquad (20)$$

*where:*

$$G_k = \mu_o^{-1}\left[ B_k h_k + B_k^{/}\left(\frac{h_k}{2}\right)^2 + B_k^{//}\left(\frac{h_k}{2}\right)^3 + \left(\frac{B_{k+1}^{//}}{3}\right)\left(\frac{h_k}{2}\right)^3 \right]$$

Here $h_k=z_{k+1}- z_k$; it represents the width of each subinterval. When the lens is symmetrical $V_Z(n)= -V_{Z(1)}=0.5$ NI .

Consider the magnetic field model given in equation (17) is determined under asymmetrical conditions along the optical axis. Since the lens is asymmetrical, then the absolute value of the potentials at the lens terminals are not equal in magnitude (see figure 2). This indicates that the area under the condenser and objective fields are not equal. At the plane where z=0 the potential is zero as shown in figure (2). The lens asymmetry is achieved by maintaining this plane and keeping the condenser field on its left-hand side and the objective field on the right-hand side. Equation (20) is applied on each of the two above-mentioned fields since these two fields are not identical.

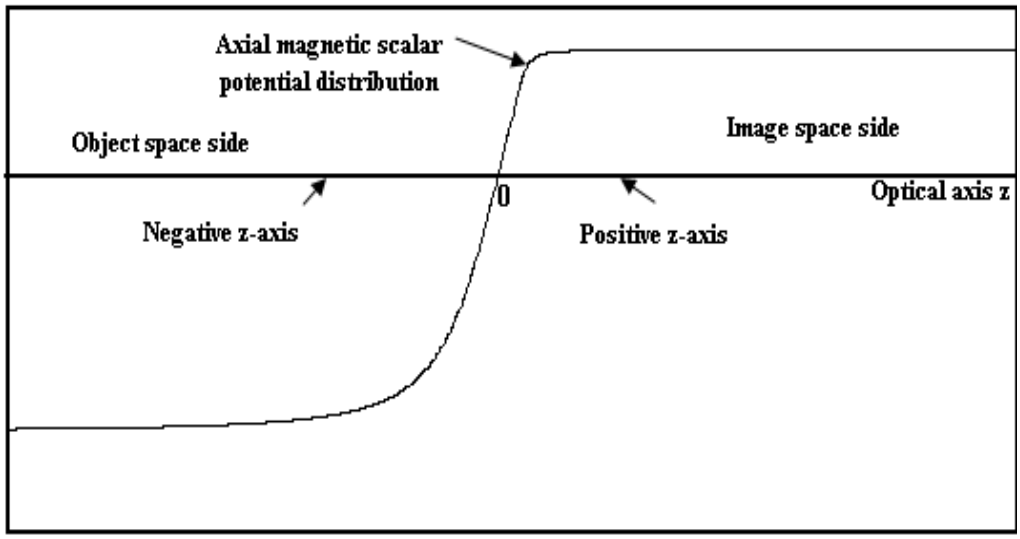



**Figure (2) :Potential distribution of asymmetrical double pole piece magnetic lens.**

## 4. Results and Discussion

The maximum flux density $B_{max}$,the two half widths of the asymmetrical field on the object and image sides $a_1$ and $a_2$ respectively, and the axial length of the field have been taken into consideration as optimization parameters for the asymmetrical lens. The objective focal properties of each lens of specific asymmetry parameters have been investigated under zero and infinite magnification conditions. In the former the electron beam enters the lens field parallel to the optical axis and emerges after crossing the axis once whereas in the latter the beam is on the axis before it enters the lens and emerges parallel to the optical axis. The electron beam experiences two different magnetic fields *in the region of* $B_{max}$ *i.e. about* the zero coordinate on the optical axis. The lenses are investigated in the absence of magnetic saturation where the flux density B(r, NI) generated by a coil of excitation NI ampere-turns is proportional to NI [Hawkes, 1982; Al-Saadi, 2007] :

$$\frac{\partial \mathrm{B}(r, NI)}{\partial (NI)} = \frac{\mathrm{B}(r, NI)}{(NI)} \qquad (21)$$

The linearity of equation (21) holds if the flux density within the pole pieces and shielding material is sufficiently low, and if the magnetic permeability is sufficiently high. The magnetic field strength H within the material may be neglected and the magnetic scalar potential is defined by the following equation:

$$\mathrm{H(r)} = -\mathrm{grad\ V(r)} \qquad (22)$$

When the magnetic lens is symmetrical about the mid-plane the absolute value of the magnetic scalar potential on the object side is equal to that on the image side i.e., the electron beam suffers the same refractive power on both sides of the magnetic field. However, in the case of asymmetrical double pole piece lens these two values are not equal to each other since the magnetic scalar potential on the object or image side would be greater or less than the other depending on the number of ampere-turns under the condenser and objective fields. The sum of the absolute values of the magnetic scalar potentials on the object and image side is equal to the total excitation of the lens according to Ampere's law ( see equation 18 ) .

Figure (3) shows the axial magnetic field distribution Bz for different values of the half width $a_1$($a_1$=1, 2, 3, 4, 5mm) on the object side by maintaining $B_{max}$=1T, $a_2$=1mm and L=40mm. It should be noted that the distribution of the field on the image side is not affected since $a_2$ is kept constant. The total number of ampere-turns as well as the number of ampere-turns under the condenser field increase with increasing $a_1$. This means that the absolute value of the magnetic scalar potential on the object side increases with increasing $a_1$ as shown in figure (4) .

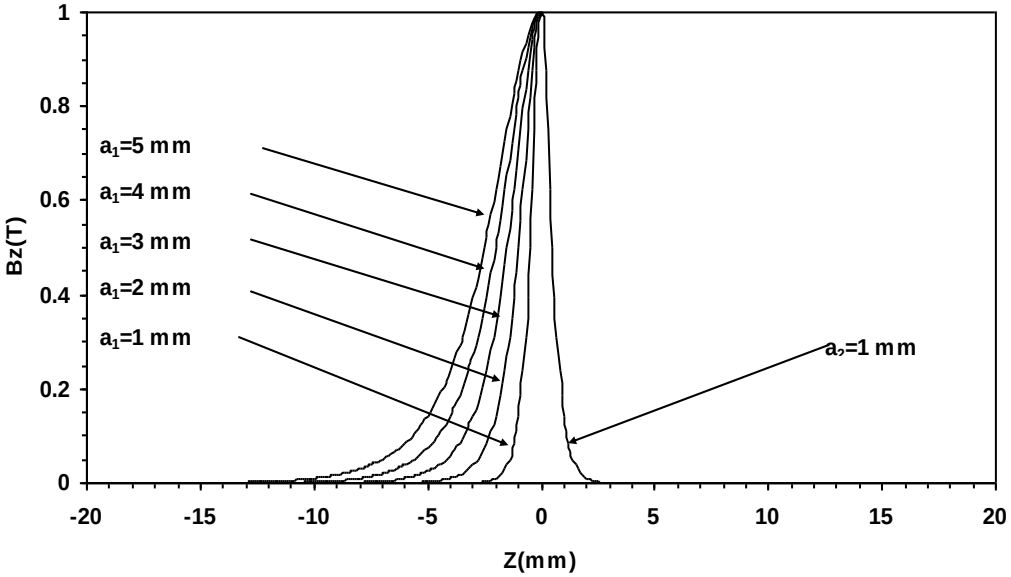



**Figure (3): The axial magnetic field distribution for different values of $a_1$($a_1$=1, 2, 3, 4, 5mm) when $B_{max}$=1T, $a_2$=1mm, and L=40mm.**

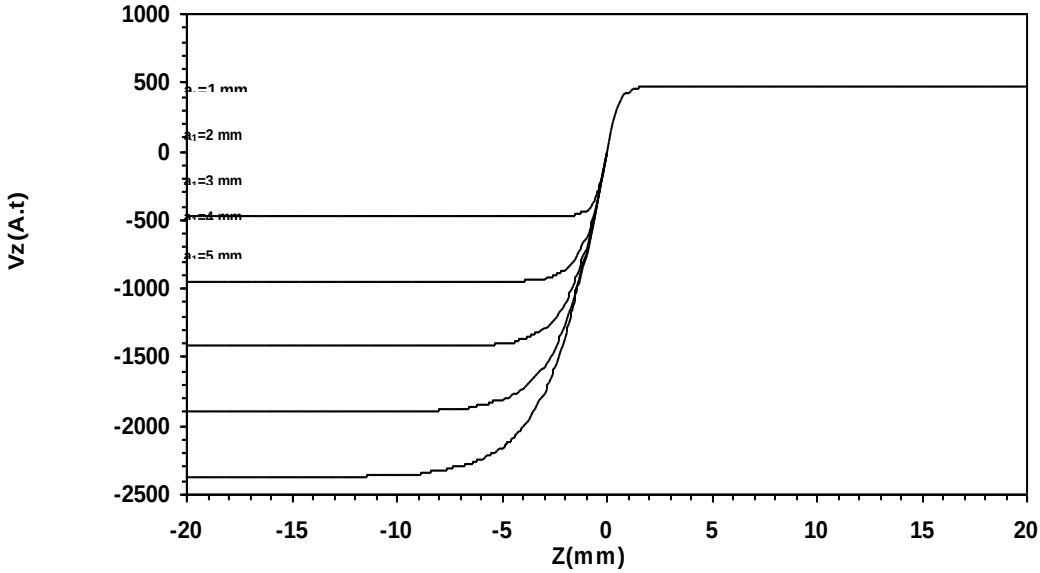



**Figure (4): The axial magnetic scalar potential distribution for different values of $a_1$($a_1$=1, 2, 3, 4, 5mm) when $B_{max}$=1T, $a_2$=1mm, and L=40mm.**

By using the reconstruction procedure of the magnetic lens pole pieces, figure (5) shows the upper half of the pole piece for various values of $a_1$ when the other optimization parameters are kept constants. It should be mentioned that when $a_1$=$a_2$=1mm the resulting

lens is symmetrical; however, for the other values of $a_1$ the magnetic lens becomes asymmetrical with different bore diameters about the mid-plane and the distance between the two faces of the pole pieces increases with $a_1$ [Al-Batat et al , 2005].

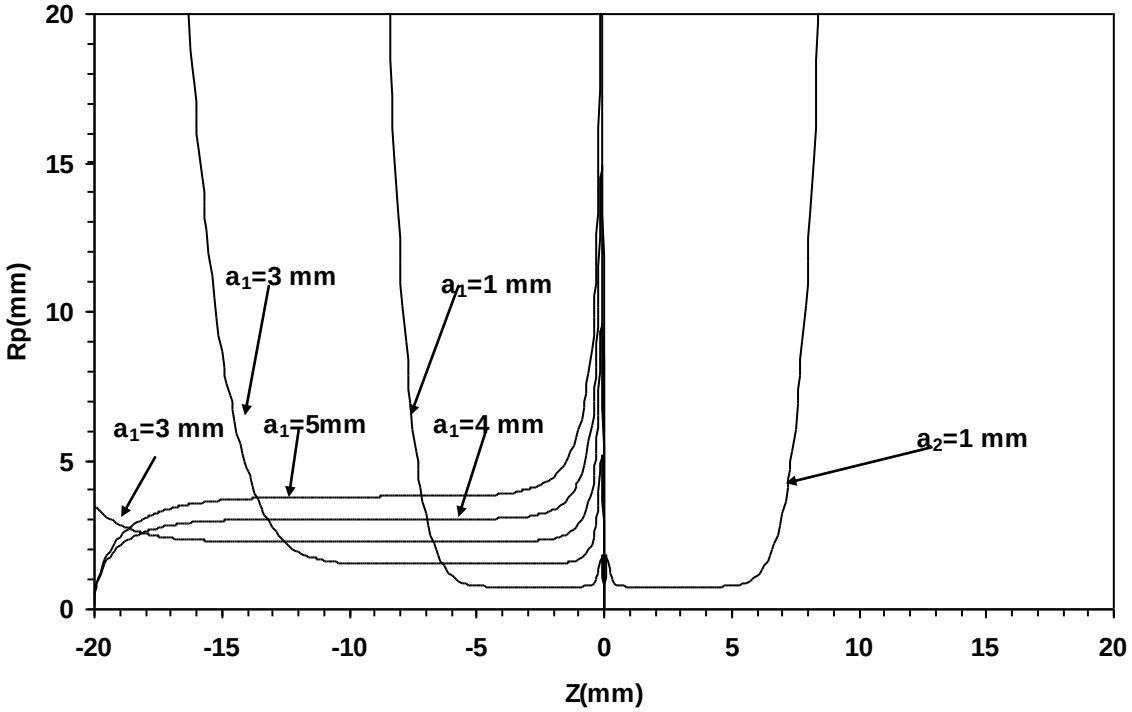
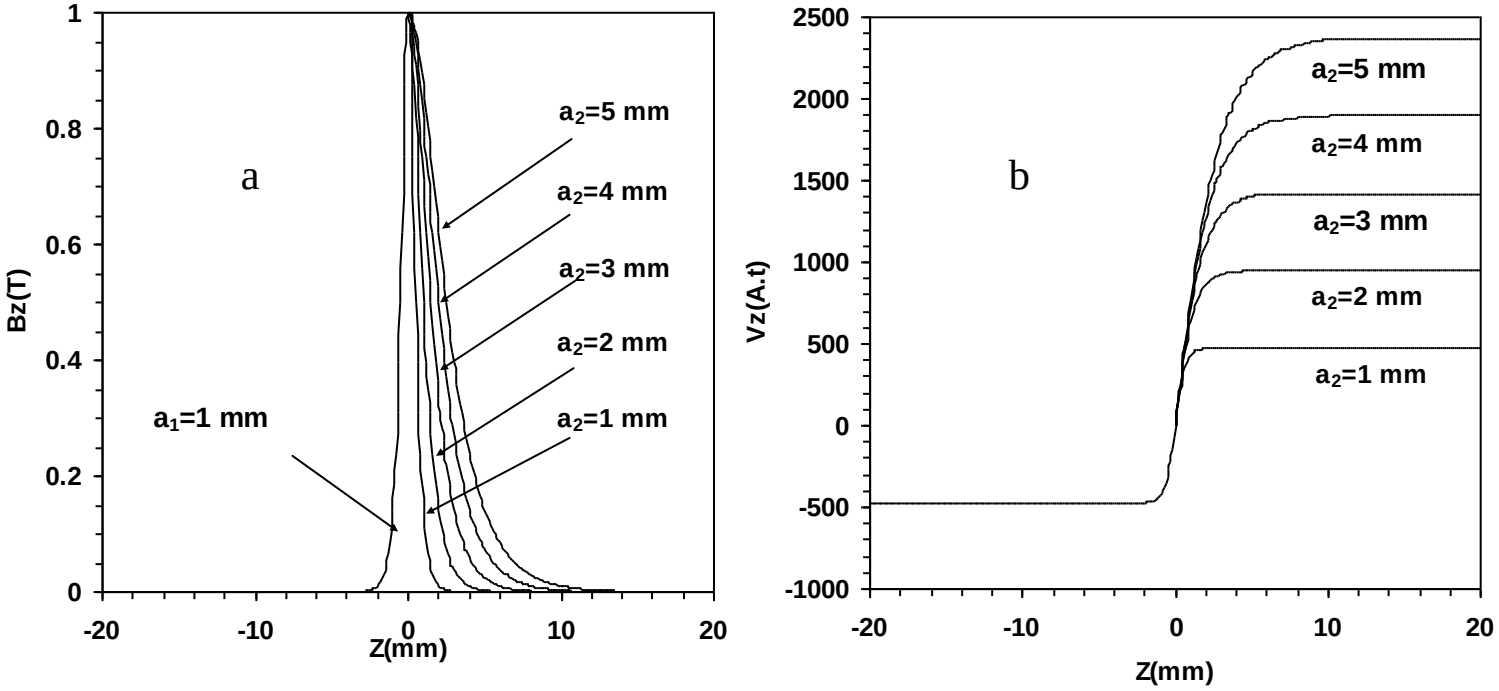



**Figure (5): Profile of pole pieces for different values of $a_1$($a_1$=1, 2, 3, 4, 5mm) when $B_{max}$=1T, $a_2$=1mm, and L=40mm.**

The half width $a_2$ on the image side space is now given the same values as those on the object side space $a_1$ in the mathematical model for calculating the axial magnetic field i.e. $a_2$=1, 2, 3, 4, 5mm as shown in figure ( 6a ) . The other parameters remain the same as those used in calculating the axial magnetic field for different values of $a_1$ i.e. $a_1$=1mm, $B_{max}$=1T, and L=40mm. It is found that the distributions of the axial magnetic field and its axial magnetic scalar potential distribution for different values of $a_2$ are the same as those for $a_1$, but the left side becomes right and the right side becomes left as shown in figure ( 6b ). This result means was found the corresponding of pole piece profile for different values of $a_2$ as shown in figure (7).Variation of the lens excitation NI in ampere-turns with each of the two half widths $a_1$ or $a_2$ as shown in figure (8). It indicates that in order to increase the width of $a_1$ or $a_2$ one should increase the lens excitation NI .




**Figure(6): (a)The axial magnetic field distribution and (b)the axial magnetic scalar potential distribution for different values of $a_2$($a_2$=1, 2, 3, 4, when $B_{max}$=1T, $a_1$=1mm, and L=40mm.**

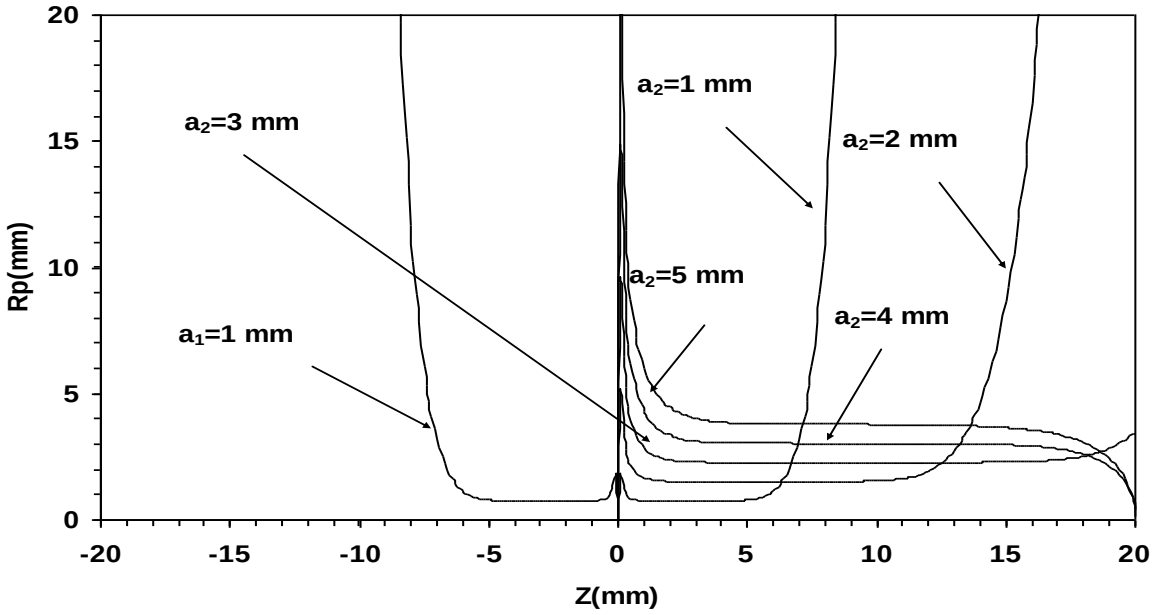



**Figure (7): Profile of pole pieces for different values of $a_2$($a_2$=1, 2, 3, 4, 5mm) when $B_{max}$=1T, $a_1$=1mm, and L=40mm.**

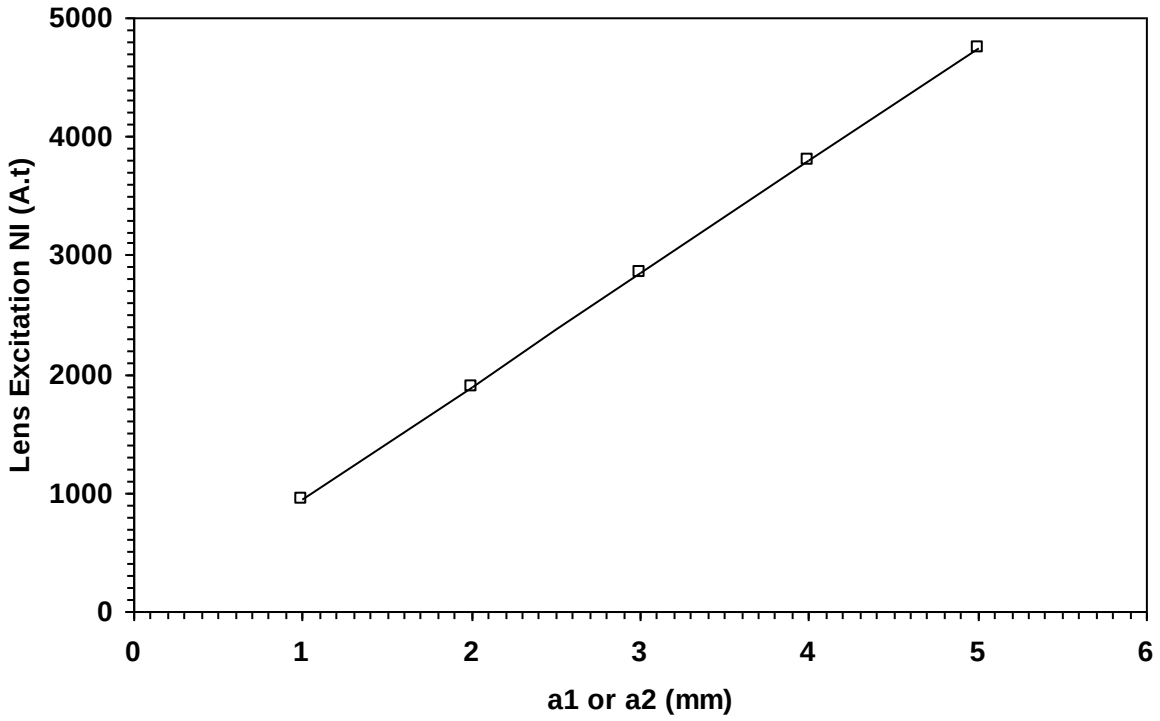



**Figure (8): Variation of the lens excitation NI with the half widths $a_1$ and $a_2$.**

Variation of the objective focal length $f_o$, spherical aberration coefficient $C_s$, and chromatic aberration coefficient $C_c$ with the object space side half width $a_1$ and image space side half width $a_2$ at the excitation parameter $NI/V_r^{1/2} = 30$ for zero and infinite magnification conditions as shown in figure (9). It should be noted that the objective properties under zero magnification condition for different values of $a_1$ are equal to those under infinite magnification condition for the same values of $a_2$ on the image side. Also these properties under infinite magnification condition for different values of $a_1$ are equal to those under zero magnification condition for the same values of $a_2$ on the image side.

For asymmetrical axial magnetic field of different values of $a_1$ and $a_2$, it is noted that the maximum value of the flux density $B_{max}$ has no effect on the objective focal properties and the pole piece profile since the total half width is kept constant. Furthermore, there is no effect of the asymmetrical lens length on the focal properties and the pole piece profile when the lens length is changed by equal values on its two sides [Al-Batat *et al* , 2005].

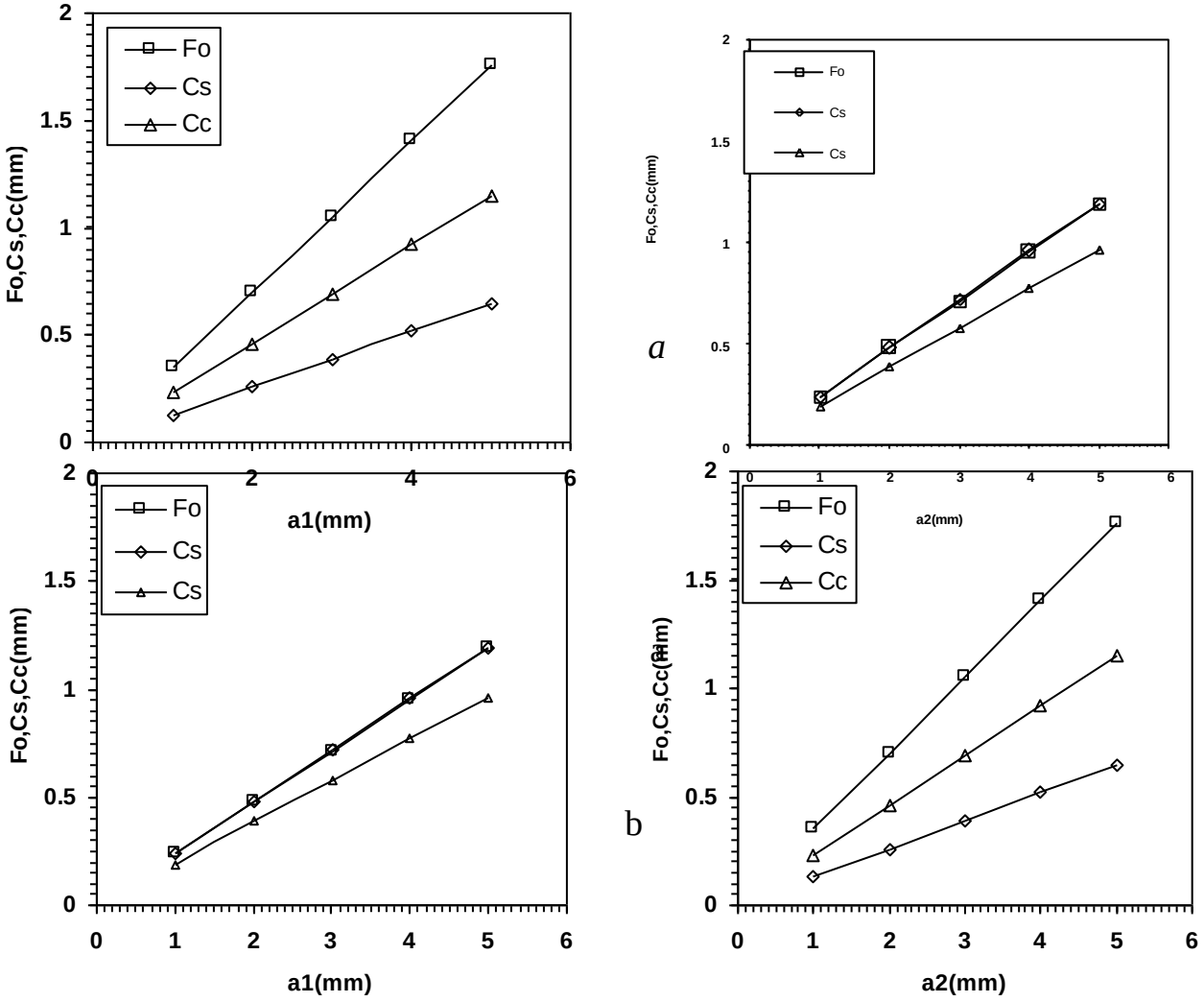



**Figure (9): The objective focal length $f_o$, the spherical aberration coefficient $C_s$, and the chromatic aberration coefficient $C_c$ against the object space side half width $a_1$ and the image side half width $a_2$ when $B_{max}$=1T, and L=40mm:**
**(a) zero magnification condition ; (b) infinite magnification condition.**

In tables (1) and (2) it was found that the values of the relative spherical and chromatic aberration coefficients $C_s/f_o$ and $C_c/f_o$ respectively are quite favorable in high resolution electron microscopes since they are less than unity at various values of $a_1$. This result indicates that the performance of such asymmetrical objective lenses is desirable in the field of electron microscopy [Al-Batat *et al* , 2005].

**Table (1): The objective focal properties for different values of $a_1$ under zero magnification condition at $NI / V_r^{1/2} = 30$ .**

| $a_1$mm | $f_o$ mm | $C_s$ mm | $C_c$ mm | $C_s/f_o$ | $C_c/f_o$ |
|---|---|---|---|---|---|
| 1 | 0.35 | 0.13 | 0.23 | 0.37 | 0.66 |
| 2 | 0.70 | 0.26 | 0.46 | 0.37 | 0.66 |
| 3 | 1.05 | 0.39 | 0.69 | 0.37 | 0.66 |
| 4 | 1.41 | 0.52 | 0.92 | 0.37 | 0.65 |
| 5 | 1.76 | 0.65 | 1.15 | 0.37 | 0.65 |

**Table (2): The objective focal properties for different values of $a_2$ under zero magnification condition at $NI / V_r^{1/2} = 30$ .**

| $a_2$mm | $f_o$ mm | $C_s$ mm | $C_c$ mm | $C_s/f_o$ | $C_c/f_o$ |
|---|---|---|---|---|---|
| 1 | 0.24 | 0.24 | 0.19 | 1.00 | 0.24 |
| 2 | 0.48 | 0.48 | 0.39 | 1.00 | 0.48 |
| 3 | 0.71 | 0.72 | 0.58 | 1.01 | 0.71 |
| 4 | 0.95 | 0.96 | 0.77 | 1.01 | 0.95 |
| 5 | 1.19 | 1.19 | 0.96 | 1.00 | 1.19 |

## 5. Conclusions

In the present work it was found the corresponding between the objective focal properties in the objective side under zero magnification condition with these properties in the image side under infinite magnification condition and vice versa .

The accuracy of the spline solution technique mainly depends on the spline interval width . However , as this width decrease the accuracy will increase and vice versa . This work can be extended further by using other models for asymmetrical axial magnetic fields.

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