A Superposition Theory Application In 3d Random Unsteady Hydrogeologic Heterogeneous Mathematical Model

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Abstract

An unsteady three dimension mathematical model is modified to be fitted for groundwater motion media to recognize the validity of superposition theory in the non-homogeneous unsteady media in the case of a given water table condition for unconfined aquifer. The results are evaluated using Darcy's Law and Theis Solution. After the model has been run for sufficient period, the resulted superimposed drawdown of the numerical and theoretical solutions show a good matching up to 2968 days and show acceptable variations beyond this period.

الخلاصة

النموذج الرياضي الغير ثابت مع الزمن ثلاثي الأبعاد طور ليكون مناسب لوسط حركة الماء الجوفي للتعرف على صلاحية نظرية التراكب في وسط غير متجانس متغير مع الزمن في الحالة المفترضة من مناسيب المياه الجوفية للمكمن المائي المفتوح. أن النتائج قد قيمت باستخدام قانون دارسي ومعادلة ثايس. بعد تشغيل النموذج لمدة كافية فان الهبوط المتراكب الناتج للحل النظري والعددي أعطى تطابقا جيدا لحد 2968 يوم وتطابقا مقبولا بعد ذلك.

Introduction

A well-known concept of superposition theory is recognized and applied in different respects of life through the worldwide. Study of earthquake, pollution disasters, light, sink and source in gases and fluids, waves,…etc are some typical examples of superposition application in nature.

The superposition theory is applied to study the shear stress distribution over an axial tension plate. The model results illustrate that after the tension load are applied normal to xy plane, the stresses distribution of a finite element analysis when compared with a reference solution show good identity except that near the plate edge (Jacob, 1997). *Zhang* presents an earthquake-wave-motion model through a natural basin which has a rugged topography. The model deals an existing geologic formation of an earthquake source to analyze the seismic effects of an earthquake through a heterogeneous media (R. Zhang, 2000). The diffusion of the optical tomography in non-homogenous media has been investigated. Gadolinium-enhanced magnetic resonance images of the breast have been employed to simulate the heterogeneity of the media. Image quality and quantification accuracy worsens are used to widen the theoretical background basics in non-homogeneity (Andreas, *et al*., 2001). There are three methods which can be recognized; the moment, the level set, and the computational methods for interface problems in high frequency waves under the highlight of the Eulerian Computation of high frequency waves in heterogeneous media. These approaches are all based on high frequency asymptotic limits (Shi Jin, 2007).

Many programs have been written for aquifer simulation by a mathematical model, using a finite difference approach. In this study, the program of (Prickett, and. Lonngquist, 1971*)* has been modified and used. Darcy's law and Theis Equation are used in the theoretical analysis and manipulations

Aim of the Research

The aim of this research is to investigate the validation of superposition theory in hydro-geologic heterogeneous media using a traditional comparison between theoretical and numerical solution based on finite difference approach.

Description of the Case Study

A random hydrogeologic regime of an certain natural hydrogeologic properties is used as a pattern of the study as presented in Fig.(1). The covered area of this regime is about 184 km^2. Actually the figure presents natural and essential boundaries that are perfect for the modeling process. The area is bounded by a river and stream at the Northern-East and Northern-West respectively and a drain represents the southern border of the region.

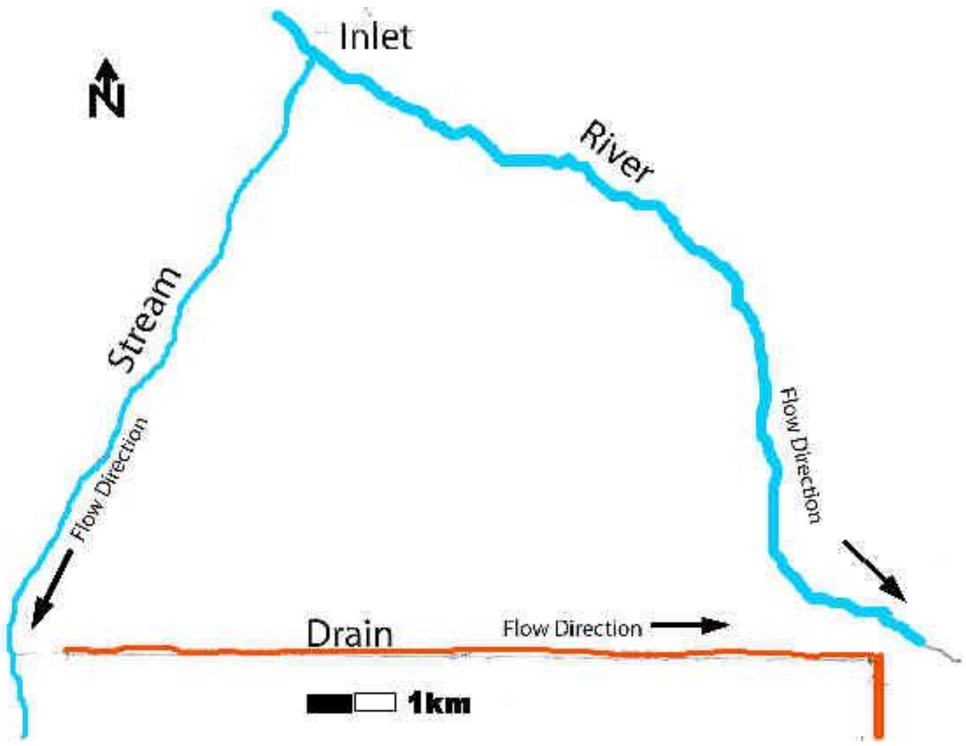


Fig. (1) Location Map of the Random Study Media

Mathematical Model Preparation

The development of a mathematical model requires several preparations before any simulating and analyzing processes. The program is written by using a finite difference approach for aquifer simulation. It is issued to be flexible and modifiable for input and output data. The model in general is designed to compromise by applying superposition theory for pumping wells in heterogeneous hydrogeological subsurface media.

Meshes Design

Once, the modular starts with the discretization of the model domain into finite difference meshes over the area map under consideration. A suitable number of meshes should be chosen depending on the area extent and degree of accuracy. Uniform mesh spacing of 0.5 km in both XY direction is used as shown in Fig.(2).

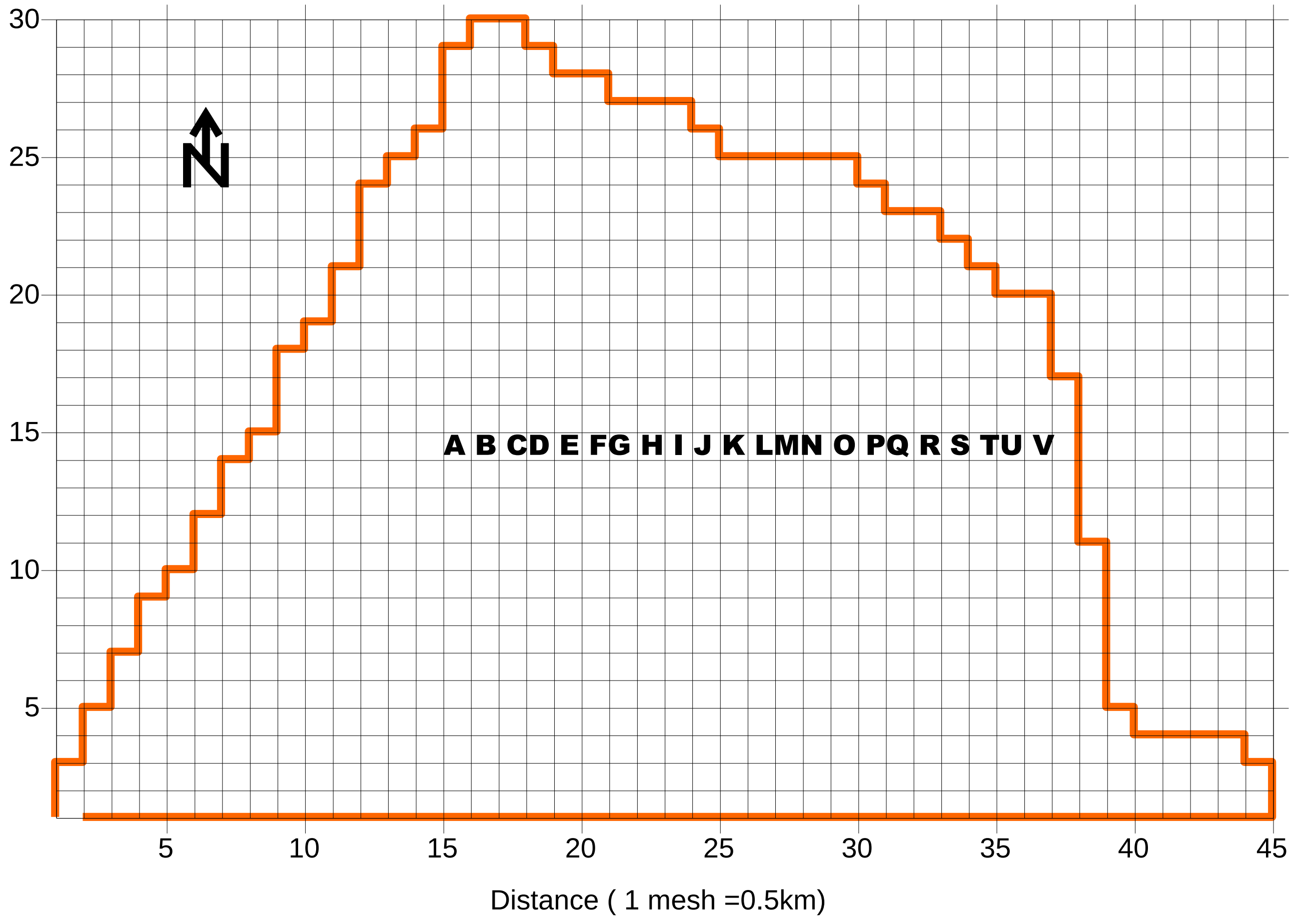


Fig.(2) Mesh Design

Base Map Implementation

The most important process is the adoption of the base map which is defined as the number of cells in model domain cover the boundary of the modeled area, and it is assigned in the modeling process by their individual xy coordinates as shown in{Table (1), Appendix A}

Basics of the Work

In the current work, the theory of Darcy's Law and solution of Theis in unsteady flow in subsurface media is used as a basis. Theis solution is summarized in t as follows:-

$$U = \frac{r^2 S}{4\pi t} \quad \ldots\ldots\ldots\ldots 1$$

$$s = \frac{Q}{4\pi T} w(u) \ldots\ldots\ldots\ldots 2$$

Where: U is the Theis factor , r is the distance from the pumping well(L), S is storage coefficient, t is the time since pumping starts(T), s is the drawdown(L), Q is the production capacity(L^3/T), T is the average transmissivity at the vicinity of the production well(L^2/T), and w(u) is the well function(L). So the drawdown in any point around the production well in the vicinity of the cone of depression can be obtained immediately if the previous parameters are known.

Assumptions in Aquifers

it is assumed that there are:

1- Many constant production wells which are scattered over the considered area.
2- Equidistance pumping wells are considered in both X and Y directions.
3- The resultant drawdown at the point of interest is produced by the accumulated drawdown of discharging wells operated at the same.
4- If the point of interest is located at the center of a production well, the corresponding drawdown should be considered in the analytical solution of the drawdown determination.

Modeling Process of the New Technical Work

The model area should be firstly selected with certain characteristics such as the nature of the surrounding and the main hydraulic boundaries inside and outside of the area; this may or may not be a true simulated area. A random media of Fig.(1) is selected to be as an interesting modeled area for the availability of the necessary data. Briefly, the modeling process is undertaken sequentially as hereinafter.

Data Files Preparation

a) Hydraulic Boundaries Particularizing for Modeling Process

The natural hydraulic heads and water levels at all of the natural rivers, streams, existing wells and the drain are fixed and adjusted**.** The hydraulic heads of the natural river stream and drain Table (2), can be interpolated over the meshes surrounded the modeled area. The meshes bounded of the entire area which are also represent the location of natural hydraulic boundaries are illustrated in [Table (1), Appendix A]. .

Table (2) Natural hydraulic Heads of the Boundaries, in (m asl) [see Fig.(1)]

Boundary	Inlet	Outlet
River	30	26
Stream	30	24
Drain	22	20

b) Recharge Boundary Condition

Najah (2008) outlined the basis of how the recharge boundary nodes are specified during the modeling process.

Calibration & Steady State Condition Adjustment

Calibration of any groundwater model should be carried out before any strategic scenario is performed (AL Assaf, 1976). Briefly he indicated the differences between the natural and simulated W. L are less than 10%. The model is run for a long period with an initial W.L. of 30 masl but the steady state condition is obtained after 88042.89days of operation as included in {Table (3) Appendix A}. Fig.(3) shows the good matching between the natural and simulated WL of the media.

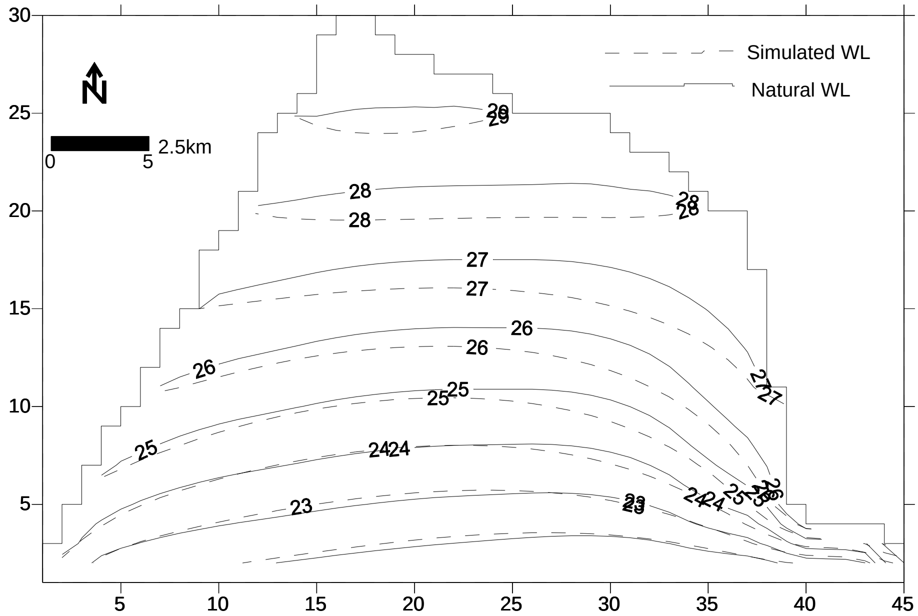


Fig.(3) Comparison between the Natural & simulated WL of the Study Aquifer, m.a.s.l

Simulation of the Aquifer

According to (Forhlich & Kelly, 1988) in the case of data leakage many methods may be followed to compensate the absence of necessary data for modeling process implementation. In the current case study, the specific storage is not exist, therefore according to (Forhlich & Kelly, 1988) it is assumed to be 0.2 and any adjustment should be made through the calibration of the modeling process. This process is the reasonable and unique solution to this problem. The previous value of 0.2 is issued by (Todd, 1980) for the specific yield of rocks similar to those characteristics of the unconfined aquifer (fine-grained, silt, clay, and sandstone). The value 0.2 is specified for any grid inside the modal domain

Usually, the groundwater formation receives its water from the recharge water obtained from precipitation and the losses of surface rainfall. Many values have been assumed for recharge through the process of model calibration. A recharge of 10 cm/year is proved to fit for the considered unconfined aquifer values through the calibration of the model.

The bottom level for each node within the modeling process should be specified depending on the analysis of the geologic formation countered in the area

The model has been operated for sufficient time of 88042.89days. The assumed data is modified in order that the predicted and natural water levels are forced to be coincident. The comparative similarity is presented in Fig.(3).

After the model has been designed and fitted to a specified hydrogeologic area, the predicted water levels as presented in Fig.(3) are set as initial water levels for any future modeling process and then carrying on to achieve any requested scenario. A pumping rate of 2160 m^3/day (25ℓ/s) from a singular-hypothetical well is introduced in the representative location of the mesh A of Fig (2). However, the program is run for a simulated period of .73369.07 days [Table (4), Appendix A]. The resulting drawdown contour map is shown in Fig.(4). From Fig.(4), it can be concluded that the effective distance is about 2 km from of the pumping well of mesh A. For next analysis all pumping wells located inside the cone of depression should be considered.

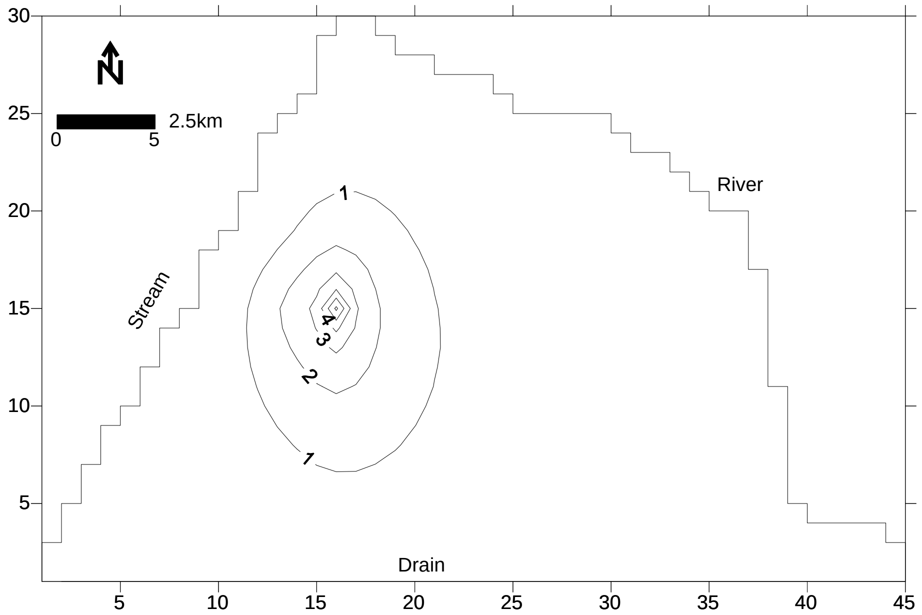


Fig.(4) Drawdown Values(in meters) Contour Map as a Result of Pumping 25 L/s Production Rate for a Period of 182565.7 days for a Signifying Well Location

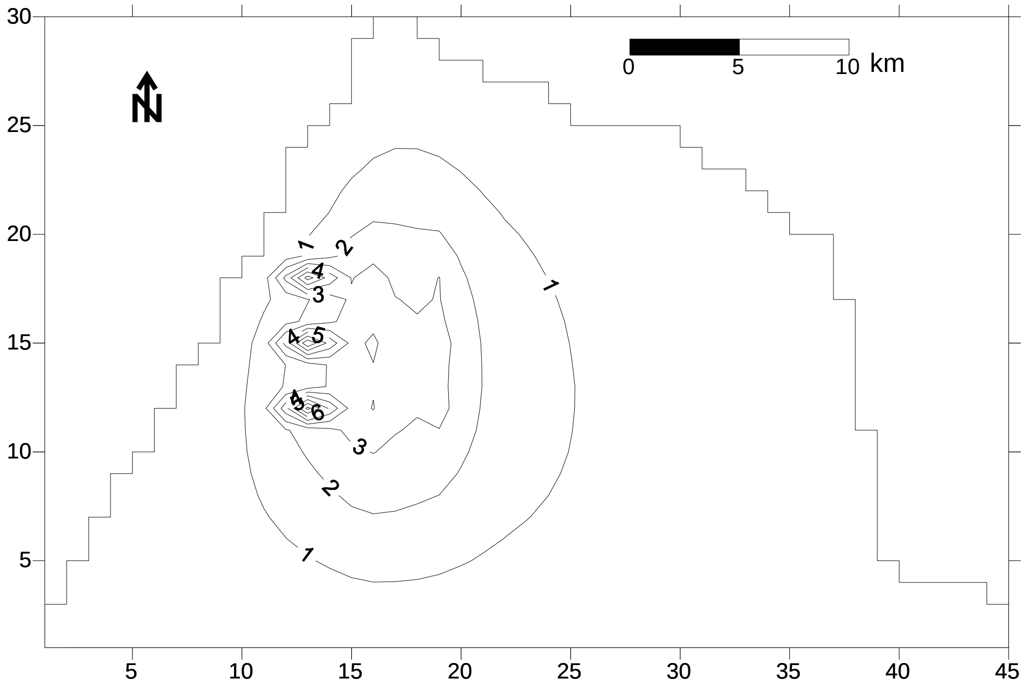
Application of the Theory

Nine pumping wells are adopted within the modeled domain of Fig.(2) with a discharge of (5 L/s) 432 m^3/day is specified for the considered well and zero discharge is allocated for any cell elsewhere as shown in (Fig.(5) , Appendix A) and they are scattered at 1.5 km apart. The model is run for along period (182565.7days) to reach the steady state condition.

The resulted drawdown over the model domain is indicated in Fig.(6) and {Fig (7), Appendix A}. The output results of (Fig. (7), Appendix A) shows that maximum drawdown occurred at the center of the pumping well A.

Conceptualization of the Theory

In the current cenario it is suggested to use the central well (denoted by the symbol A as represented in{ Fig.(5), Appendix A} with yellow color or the point of interest as it is outlined before in the assumptions of the theory. Theoretically, the drawdown-distance curve is estimated by Theis Solution and previous assumptions and the results are compared with numerical drawdown obtained by the current model. The calculations are carried out sumultaneously along the set of cells sequentially denoted by the symbols A, B, C, D, E, F, G, H, I, K, L, M, N, O, P, Q, S, T, U, V see Fig.(2), and{ Fig.(5) & Fig.(7) in Appendix A)}. The estimated results are obtained under the effects of all the pumping well in the vicinity of the effective cone of depression of point of interest. However the comparative results between Theis Solution and numerical output are shown in the Figures (8, 9, 10, 11 & 12).



Fig,(6) Drawdown contour map within the Model Domain, m

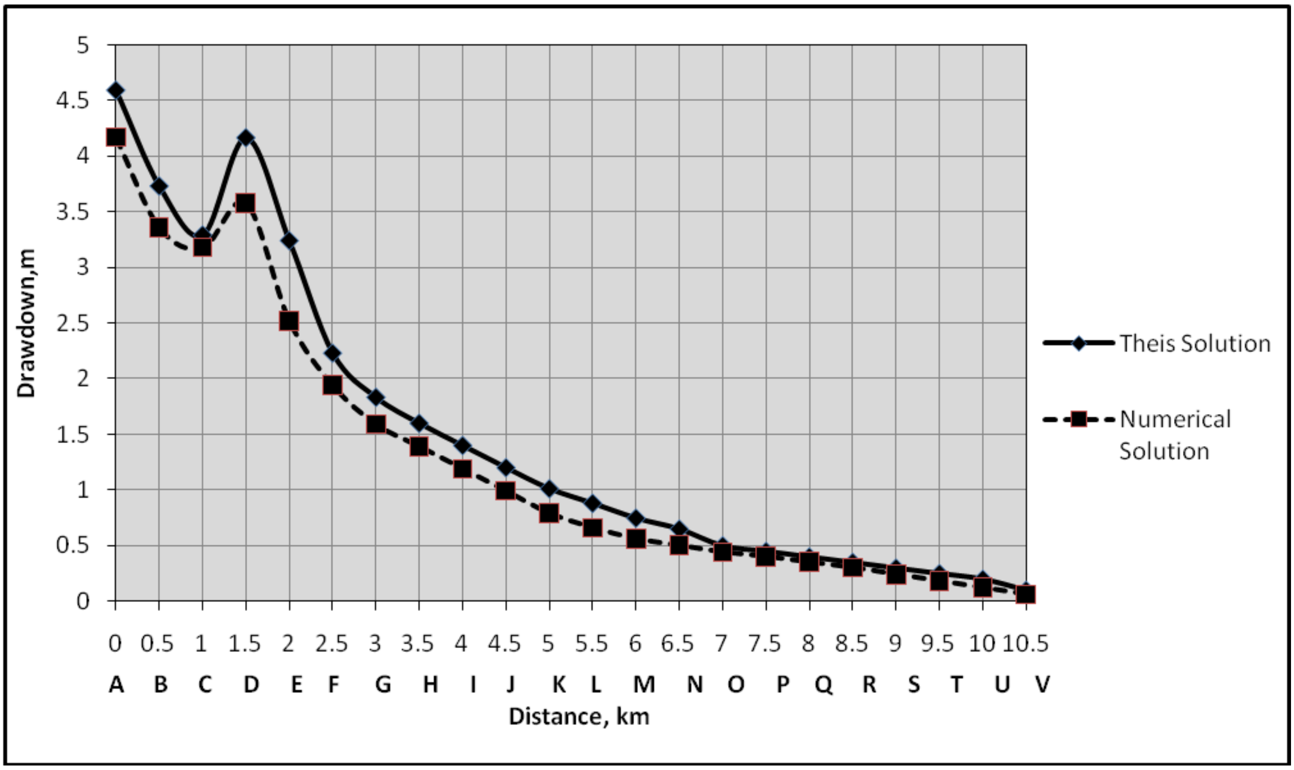


Fig.(8) Distance- Drawdown Curves Showing the Identity between Theis & Numerical Solutions after the Operation Period of 182565.7days

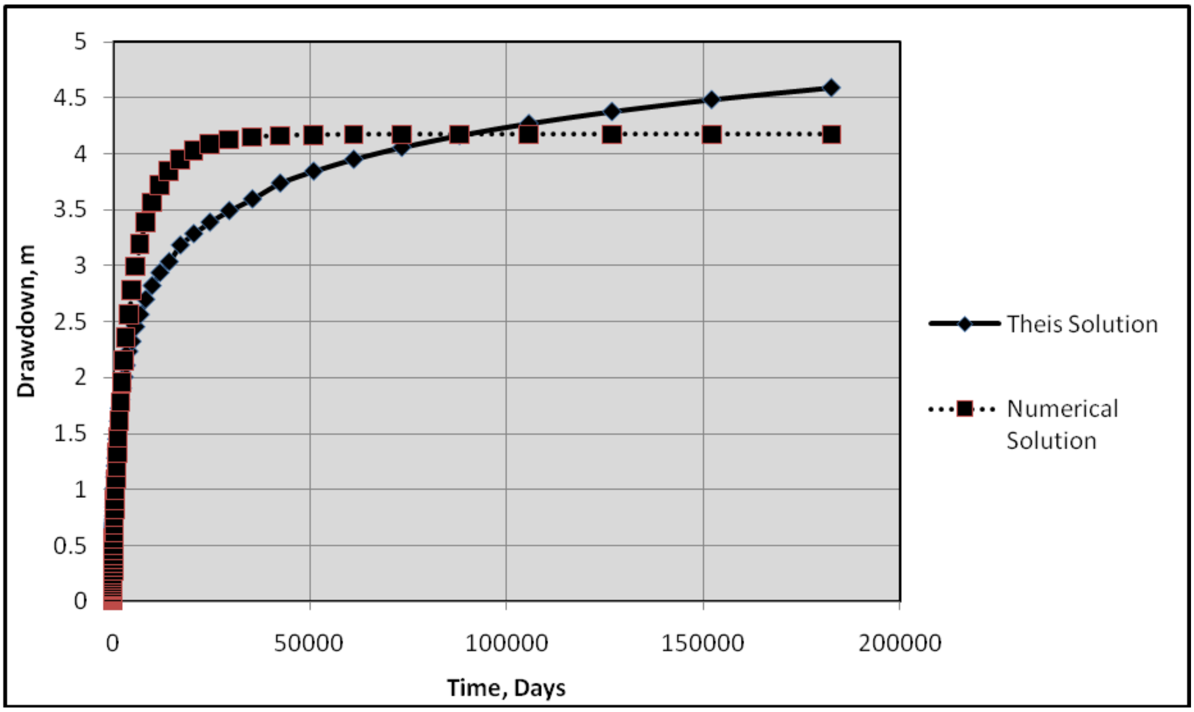


Fig.(9) Time-Drawdown Curves Comparison Between Theis & Numerical Solutions at the Center of Cell A (Point of Interest at 0km)

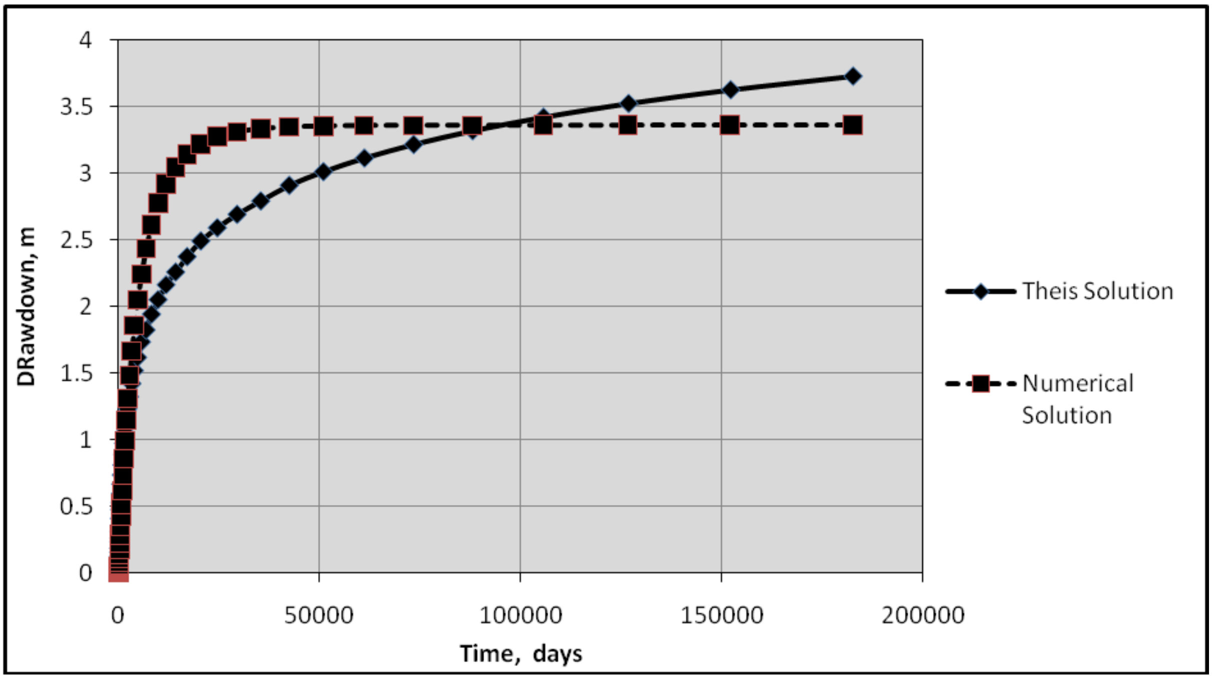


Fig.(10) Time-Drawdown Curves Comparison Between Theis & Numerical Solutions at the Center of Cell B (Point of Interest at 0.5km from point A)

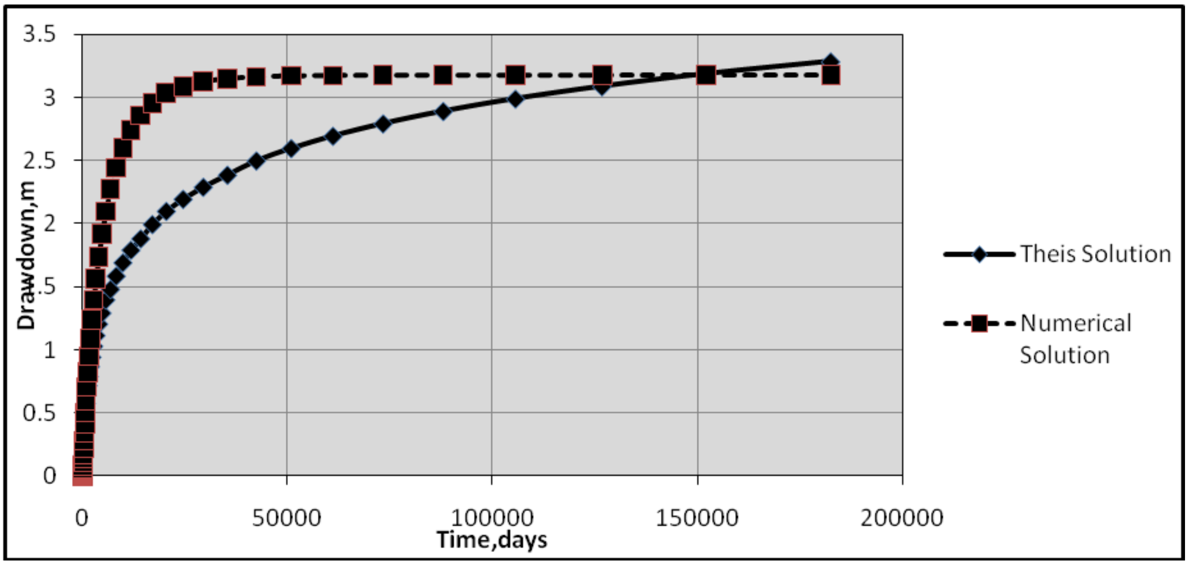


Fig.(11) Time-Drawdown Curves Comparison Between Theis & Numerical Solutions at the Center of Cell C (Point of Interest at 1km from point A)

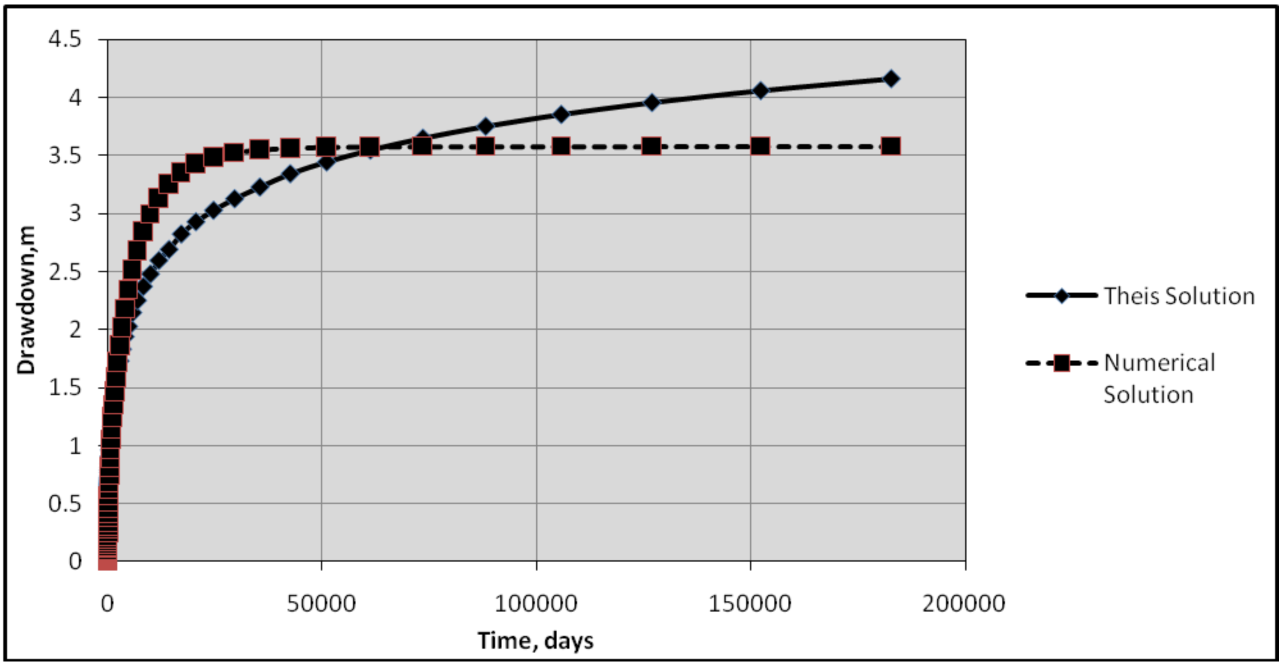


Fig.(12) Time-Drawdown Curves Comparison Between Theis & Numerical Solutions at the Center of Cell D (Point of Interest at 1.5km from point A)

Discussion of the Results

After the model has been calibrated and adjusted by means of matching the natural and simulated hydrologic data of the media under interest as presented in Fig.(3), the new application of the theory in this non-homogeneous media is carried out by comparison the numerical simulated and theoretical drawdown (based on Darcy's Law and Theis Equation).

1- The matching between the natural and simulated W. L. during the calibration of the model gives a good coinciding as shown in Fig. (3).

2- The considered hydrogeologic media seems to be characterized with low safe yield of 5 L/s, since excessive exploitation will cause aquifer drought which mathematically makes the water table equal or less than bottom level and this illogic which in turn terminate the model run. This phenomenon occurred when the recharge of the aquifer is little or neglected.
3- The new application of the superposition theory offers a good identity between the numerical and theoretical drawdown (Theis solution) as shown in Fig.(8). It indicates that drawdown is reduced significantly as we remote from the pumping (well A) except a sudden rise is occurred at distance (mesh C) 1.5 km from point A because there is an existing pumping well in this location.
4- Time-drawdown curves of Fig.(9 to 12) show a good coinciding between Theis and numerical solutions at the initial stages of pumping up to 2968 days and acceptable coinciding beyond this time.

Conclusion

The application of superposition theory under the current assumptions is proven to be a powerful technology. It may be used in unsteady heterogeneous hydrogeologic media. It shows a good coinciding between the time and distance drawdown curves of theoretical (Theis) and numerical solutions for the aquifer.

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Appendix A

Table (1) Base Map Design

Mesh No.	x	y	Mesh No.	x	y	Mesh No.	X	y	Mesh No.	x	y	Mesh No.	x	Y
1	1	1	31	11	21	61	27	25	91	39	7	121	27	1
2	1	2	32	12	21	62	28	25	92	39	6	122	26	1
3	1	3	33	12	22	63	29	25	93	39	5	123	25	1
4	2	3	34	12	23	64	30	25	94	40	5	124	24	1
5	2	4	35	12	24	65	30	24	95	40	4	125	23	1
6	2	5	36	13	24	66	31	24	96	41	4	126	22	1
7	3	5	37	13	25	67	31	23	97	42	4	127	21	1
8	3	6	38	14	25	68	32	23	98	43	4	128	20	1
9	3	7	39	14	26	69	33	23	99	44	4	129	19	1
10	4	7	40	15	26	70	33	22	100	44	3	130	18	1
11	4	8	41	15	27	71	34	22	101	45	3	131	17	1
12	4	9	42	15	28	72	34	21	102	45	2	132	16	1
13	5	9	43	15	29	73	35	21	103	45	1	133	15	1
14	5	10	44	16	29	74	35	20	104	44	1	134	14	1
15	6	10	45	16	30	75	36	20	105	43	1	135	13	1
16	6	11	46	17	30	76	37	20	106	42	1	136	12	1
17	6	12	47	18	30	77	37	19	107	41	1	137	11	1
18	7	12	48	18	29	78	37	18	108	40	1	138	10	1
19	7	13	49	19	29	79	37	17	109	39	1	139	9	1
20	7	14	50	19	28	80	38	17	110	38	1	140	8	1
21	8	14	51	20	28	81	38	16	111	37	1	141	7	1
22	8	15	52	21	28	82	38	15	112	36	1	142	6	1
23	9	15	53	21	27	83	38	14	113	35	1	143	5	1
24	9	16	54	22	27	84	38	13	114	34	1	144	4	1
25	9	17	55	23	27	85	38	12	115	33	1	145	3	1
26	9	18	56	24	27	86	38	11	116	32	1	146	2	1
27	10	18	57	24	26	87	39	11	117	31	1			
28	10	19	58	25	26	88	39	10	118	30	1			
29	11	19	59	25	25	89	39	9	119	29	1			
30	11	20	60	26	25	90	39	8	120	28	1			

Table (3) Steady State Condition of the Hydrogeologic System Output Obtained at the Node (NC=16, NR=15)

Time, days	No. of Steps	Initial W. L. Masl	Steady State W. L. Masl
0.01	30	30	30
.	.	.	.
.	.	.	.
.	.	.	.
.	.	.	.
.	.	.	.
.	.	.	.
.	.	.	.
17063.30	107	30	26.86935
20475.96	108	30	26.72437
24571.15	109	30	26.61592
29485.38	110	30	26.54026
35382.46	111	30	26.49140
42458.95	112	30	26.46246
50950.74	113	30	26.44687
61140.89	114	30	26.43929
73369.07	115	30	26.43600
88042.89	116	30	26.43473***
105651.47	117	30	26.43430

Table (4) Steady Ground W.L(in meters) as a Result of Pumping 25 L/s Production Rate for a Period of 61140.89 days at the Node (NC=16, NR=15)

Time, days	No. of Steps	Initial W. L. Masl	Steady State W. L. Masl
0.01	25	26.43	26.42977
.	.	.	.
.	.	.	.
.	.	.	.
.	.	.	.
.	.	.	.
.	.	.	.
.	.	.	.
11849.51	105	26.43	20.55517
14219.41	106	26.43	20.46017
17063.30	107	26.43	20.38299
20475.96	108	26.43	20.32315
24571.15	109	26.43	20.27918
29485.38	110	26.43	20.24879
35382.46	111	26.43	20.22920
42458.95	112	26.43	20.21753
50950.74	113	26.43	20.21115
61140.89	114	26.43	20.20799***
73369.07	115	26.43	20.20658

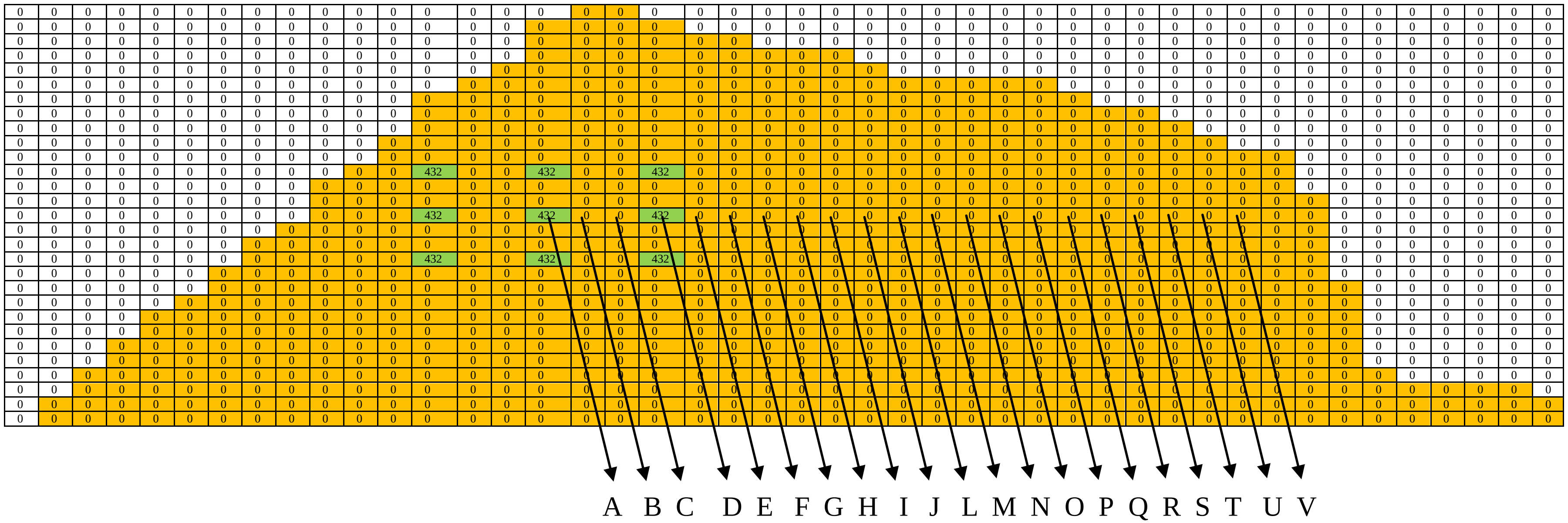

Fig.(5) Scattered Distribution Pumping Well over the Modeled Area Nodes with a Discharge of 5L/s are indicated With a Green color

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.12	0.23	0.17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.25	0.38	0.35	0.22	0.17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.42	0.56	0.54	0.44	0.36	0.2	0.16	0.08	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.28	0.62	0.76	0.75	0.66	0.55	0.4	0.32	0.21	0.1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0.28	0.57	0.86	0.99	0.98	0.9	0.76	0.59	0.48	0.39	0.29	0.2	0.1	0.08	0.07	0.05	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0.28	0.58	0.86	1.14	1.24	1.23	1.14	0.97	0.78	0.64	0.53	0.43	0.31	0.21	0.16	0.14	0.12	0.06	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0.39	0.78	1.18	1.45	1.52	1.49	1.41	1.2	0.97	0.79	0.68	0.55	0.43	0.31	0.24	0.2	0.17	0.12	0.06	0.05	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0.5	1.01	1.53	1.82	1.83	1.78	1.71	1.43	1.15	0.95	0.81	0.67	0.54	0.4	0.32	0.26	0.22	0.17	0.11	0.09	0.05	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0.47	0.94	1.44	1.95	2.25	2.15	2.08	2.05	1.66	1.33	1.1	0.95	0.79	0.64	0.49	0.39	0.32	0.27	0.22	0.17	0.13	0.09	0.05	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0.57	1.17	1.78	2.41	2.76	2.48	2.39	2.47	1.91	1.5	1.23	1.06	0.9	0.73	0.57	0.46	0.38	0.32	0.27	0.21	0.18	0.14	0.1	0.06	0.04	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0.57	3.27	6.44	4.6	2.95	3.42	2.79	2.67	3.02	2.13	1.65	1.35	1.17	0.99	0.81	0.64	0.52	0.44	0.37	0.31	0.27	0.23	0.18	0.14	0.08	0.04	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0.24	0.83	1.39	1.95	2.54	3.15	3.51	3.03	2.89	3.05	2.3	1.77	1.45	1.26	1.07	0.89	0.7	0.57	0.48	0.41	0.36	0.31	0.27	0.22	0.17	0.11	0.06	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0.45	1.13	1.67	2.23	2.81	3.41	3.77	3.23	3.06	3.22	2.42	1.88	1.54	1.34	1.14	0.95	0.75	0.62	0.53	0.46	0.4	0.35	0.31	0.26	0.2	0.14	0.08	0.02	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0.62	1.47	4.33	7.83	5.65	3.72	4.17	3.36	3.18	3.58	2.52	1.94	1.59	1.39	1.19	0.99	0.79	0.66	0.56	0.5	0.44	0.4	0.35	0.3	0.24	0.18	0.12	0.06	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0.18	0.75	1.5	2.01	2.54	3.08	3.65	3.98	3.4	3.21	3.37	2.54	1.97	1.62	1.42	1.22	1.02	0.83	0.69	0.59	0.52	0.48	0.44	0.39	0.35	0.28	0.22	0.15	0.09	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0.09	0.32	0.84	1.59	2.07	2.56	3.09	3.62	3.93	3.36	3.18	3.34	2.52	1.97	1.62	1.42	1.23	1.04	0.84	0.7	0.62	0.55	0.5	0.47	0.43	0.39	0.32	0.26	0.19	0.13	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0.18	0.42	0.9	1.71	4.66	8.33	5.8	3.63	4.04	3.26	3.08	3.45	2.46	1.93	1.6	1.42	1.22	1.03	0.85	0.72	0.63	0.57	0.52	0.49	0.46	0.42	0.36	0.3	0.24	0.17	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0.06	0.23	0.48	0.91	1.52	1.93	2.34	2.77	3.22	3.48	3.05	2.88	2.96	2.32	1.86	1.56	1.37	1.19	1.02	0.84	0.72	0.63	0.58	0.54	0.51	0.49	0.46	0.4	0.35	0.29	0.23	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0.12	0.28	0.51	0.89	1.36	1.72	2.09	2.46	2.85	3.03	2.79	2.64	2.59	2.13	1.75	1.48	1.31	1.15	0.98	0.82	0.7	0.62	0.57	0.54	0.53	0.51	0.49	0.44	0.39	0.34	0.28	0.07	0	0	0	0	0	0	0
0	0	0	0	0	0.04	0.16	0.3	0.52	0.84	1.22	1.52	1.84	2.16	2.49	2.64	2.51	2.38	2.28	1.94	1.63	1.38	1.23	1.09	0.93	0.78	0.67	0.6	0.56	0.54	0.53	0.53	0.51	0.47	0.43	0.39	0.35	0.16	0	0	0	0	0	0	0
0	0	0	0	0.03	0.08	0.18	0.32	0.51	0.77	1.09	1.34	1.6	1.88	2.16	2.28	2.21	2.11	1.99	1.73	1.47	1.28	1.14	1	0.86	0.73	0.64	0.57	0.54	0.52	0.52	0.53	0.53	0.51	0.47	0.45	0.42	0.25	0	0	0	0	0	0	0
0	0	0	0	0.07	0.12	0.2	0.31	0.48	0.7	0.95	1.17	1.39	1.61	1.85	1.95	1.92	1.83	1.71	1.52	1.31	1.14	1.02	0.9	0.79	0.67	0.58	0.54	0.5	0.5	0.51	0.52	0.53	0.52	0.52	0.5	0.5	0.34	0	0	0	0	0	0	0
0	0	0	0.03	0.1	0.14	0.2	0.3	0.43	0.61	0.81	0.99	1.16	1.35	1.53	1.62	1.61	1.55	1.44	1.29	1.13	0.99	0.89	0.79	0.69	0.6	0.53	0.48	0.46	0.45	0.47	0.49	0.51	0.52	0.54	0.55	0.55	0.43	0	0	0	0	0	0	0
0	0	0	0.07	0.11	0.14	0.19	0.27	0.38	0.51	0.67	0.81	0.95	1.09	1.23	1.31	1.3	1.26	1.18	1.06	0.93	0.82	0.74	0.67	0.59	0.51	0.45	0.41	0.4	0.4	0.41	0.44	0.47	0.5	0.53	0.56	0.58	0.49	0	0	0	0	0	0	0
0	0	0.05	0.1	0.12	0.13	0.16	0.22	0.3	0.41	0.52	0.62	0.73	0.83	0.93	1	0.99	0.96	0.89	0.81	0.72	0.64	0.58	0.52	0.46	0.4	0.36	0.33	0.32	0.32	0.34	0.37	0.4	0.44	0.47	0.51	0.53	0.47	0.12	0	0	0	0	0	0
0	0	0.06	0.11	0.11	0.11	0.12	0.16	0.21	0.28	0.36	0.42	0.49	0.57	0.64	0.67	0.67	0.65	0.61	0.55	0.49	0.44	0.4	0.36	0.32	0.29	0.26	0.24	0.23	0.23	0.25	0.27	0.3	0.34	0.37	0.4	0.43	0.38	0.16	0.03	0.07	0.13	0.21	0	0
0	0.03	0.05	0.08	0.07	0.06	0.07	0.09	0.11	0.15	0.18	0.22	0.26	0.29	0.33	0.34	0.35	0.33	0.31	0.28	0.26	0.23	0.21	0.19	0.17	0.14	0.14	0.12	0.13	0.13	0.14	0.15	0.17	0.19	0.22	0.23	0.26	0.23	0.13	0.06	0.1	0.17	0.28	-0.32	0
0	0.01	0.01	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-0.71	-0.71

A B C D E F G H I J K L M N O P Q R S T U V

Fig. (7) Drawdown values produced (m) overall the modeled area nodes