

## Two-Dimensional Numerical Model for Thermal Pollution of Single-Point Sources in Rivers at Different Discharge Depths

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### Abstract

A two-dimensional numerical model has been developed depending on finite-difference techniques to simulate the temperature and flow patterns resulting from unsteady continuous thermal discharge applied to river stream as a single-point source at different thermal discharge locations.

In this study three cases of thermal discharge to the river reach stream are adopted, first case is at the water surface of the river, second case is as submerged outfall lying at 0.42 from the river depth, while the third case is a submerged outfall at 0.83 from the river depth. The latest discharge case is considered the basic contribution in this study. The governing equations in the present model are momentum conservation equation and thermal-energy equation. The turbulent dynamic viscosity and diffusion coefficient were calculated by using the modified (k - ε) turbulence model. The partial differential equations were converted into finite difference form using (ADI) finite difference technique with application the upwinding scheme and then the set of the linear algebraic equations were solved by Gauss-Seidel Point-by-Point Method. (Shati, 2007)

The results of the system were verified by using a numerical model presented by Al-Chalabi, M.A (1993).

A sensitivity analysis was carried out to study the effect of variations of different parameters on the temperature distribution resulting from the application of the model at and beneath the water surface.

:

0.42 ) ( ) ( 0.83 ) ( (K . ε) - ) Up Winding (ADI) ( (point-by-point Gauss – Siedel ) - (1993

### Introduction

The thermal pollution can be defined as the excessive change in the natural or ambient water temperature by the addition or removal of heat through mans' activities.

The major waste-heat producing industries are: steam-electric generation plants, petroleum refineries, steel mill, chemical plants (Mathur, 1976).

Heating up a river to a relatively high temperature, can have considerable consequences on the river water system as follows:

1. Higher temperature reduces the solubility of river oxygen and increased the rivers' chemical reactions and the river water becomes anaerobic with disastrous effects on its odor and appearance.
2. The changing in river water temperature due to the hot wastewater can harm fish and the river water organisms which adapted to a practical water temperature regime.

3. Thermal pollution affecting ecosystem composition: the metabolic rate of aquatic animals, as enzyme activity, is increased by heat meaning that the river organisms will consume more food in a shorter time than if their environment was not changed.

Some of the most related researches of thermal pollution in water bodies are presented below:

**Daws** (1990) presented an analytical and numerical analysis of the cooling process of hot water discharge from a steam power plant into a rectangular pond.

**Al-Challabi** (1993) developed a two- dimensional numerical model for the simulation of the spread and mixing of thermally polluted water disposed into the flow.

**Li-Renyu and Righetto A.M** (1998) presented unsteady state two dimensional models to simulate the velocity and temperature field in the estuary of the Yangtze River in Brazil.

**Reddy.G.s** (1998) developed three-dimensional numerical model to predict the flow and pollutant transport induced by surface discharges in rivers, estuaries and seas.

**Joody** (2001) developed one and two dimensions numerical model for the simulation of the spread and mixing of thermally polluted water disposed at the river surface released from the AL-Daura Power Station.

**Nasir** (2005) developed a two-dimensional numerical model to simulate the mixing and the spreading of thermally polluted water released from single-point source at the river water surface starting from the outfall reign.

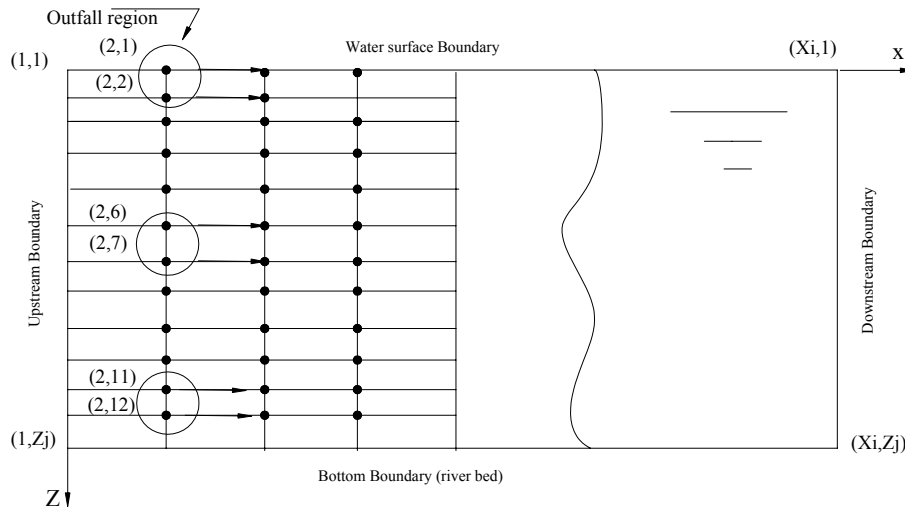
### Research Contributions

This study contributes to the water pollution modeling field in the following areas:

1. The buoyancy effects are considered in the present study by using the modified  $k-\varepsilon$  which is differs from the standard  $(k-\varepsilon)$  model by the buoyancy term  $(-g \frac{\mu}{\sigma_b \rho} \frac{\partial \rho}{\partial Z})$ .

The buoyancy term increase the level of  $k$  and its rate of dissipation ( $\varepsilon$ ), which increase the turbulent dynamic viscosity, and enhance the ability of the jet to entrain ambient fluid.

2. Three limiting schemes are used for the discharge of heated water to a theoretical river with a proposed depth equal to a 3m (Fig. (1)). First scheme is the discharge of heated water at the river surface resulting in minimizing of initial dilution, while maximizing the rate of heat loss to the atmosphere. Second is a single point source submerged at depth equal to 0.42 of the river water depth to promote much initial dilution of the temperature. The third scheme is a single point source submerged at 0.83 from the river water depth to promote more initial dilution for the temperature. The third scheme is considered the basic contribution for this study.
3. Derivation of a dilution factor formula from the temperature distribution resulting from the thermal discharge applied to the river stream .This dilution factor is a dimensionless scale ranges from 0.0 to 1.0 which describes the dilution level of the river reach subjected to any thermal discharge.



**Fig. (1): River Reach Domain for the Three Thermal Discharge Cases**

### Governing Equations:

The main equations which established the numerical model are:

1. Momentum Conservation Equations (Navier-Stokes Equation).

A. Vertical momentum conservation equation:

$$\rho \frac{\partial w}{\partial t} + u \frac{\partial \rho w}{\partial X} + w \frac{\partial \rho w}{\partial Z} = \frac{\partial}{\partial X} \left( \mu \frac{\partial w}{\partial X} \right) + \frac{\partial}{\partial Z} \left( \mu \frac{\partial w}{\partial Z} \right) - \frac{\partial P}{\partial Z} + \rho g \quad \dots (1)$$

B. Horizontal momentum equation:

$$\rho \frac{\partial u}{\partial t} + u_i \frac{\partial \rho u}{\partial X} + w \frac{\partial \rho u}{\partial Z} = \frac{\partial}{\partial X} \left( \mu \frac{\partial u}{\partial X} \right) + \frac{\partial}{\partial Z} \left( \mu \frac{\partial u}{\partial Z} \right) \quad \dots (2)$$

2. Thermal-Energy Equation.

$$\rho \frac{\partial T}{\partial t} + u \frac{\partial \rho T}{\partial X} + w \frac{\partial \rho T}{\partial Z} = \frac{\partial}{\partial X} \left[ \frac{\mu}{\sigma} \frac{\partial T}{\partial X} \right] + \frac{\partial}{\partial Z} \left[ \frac{\mu}{\sigma} \frac{\partial T}{\partial Z} \right] \quad \dots (3)$$

3. Modified K-ε turbulence model.

a. k-equation :

$$\rho \frac{\partial K}{\partial t} + u \frac{\partial \rho K}{\partial X} + w \frac{\partial \rho K}{\partial Z} = \frac{\partial}{\partial X} \left[ \frac{\mu}{\sigma_K} \frac{\partial K}{\partial X} \right] + \frac{\partial}{\partial Z} \left[ \frac{\mu}{\sigma_K} \frac{\partial K}{\partial Z} \right] + Gk + Gb - \rho \varepsilon \quad \dots (4)$$

b. ε-equation

$$\rho \frac{\partial \varepsilon}{\partial t} + u_i \frac{\partial \rho \varepsilon}{\partial X_i} + w_j \frac{\partial \rho \varepsilon}{\partial Z_j} = \frac{\partial}{\partial X_i} \left[ \frac{\mu}{\sigma_\varepsilon} \frac{\partial \varepsilon}{\partial X_i} \right] + \frac{\partial}{\partial Z_j} \left[ \frac{\mu}{\sigma_\varepsilon} \frac{\partial \varepsilon}{\partial Z_j} \right] + C_1 \frac{\varepsilon}{K} (Gk + Gb) - C_2 \rho \frac{\varepsilon^2}{K} \quad \dots (5)$$

Where:

$$Gk = \mu \left[ 2 \left( \frac{\partial u}{\partial X} \right)^2 + 2 \left( \frac{\partial w}{\partial Z} \right)^2 + \left( \frac{\partial w}{\partial X} + \frac{\partial u}{\partial Z} \right)^2 \right] \quad \dots (6)$$

$$Gb = -g \frac{\mu}{\rho \sigma_b} \frac{\partial \rho}{\partial Z}$$

... (7)

## Additional Auxiliary Equations:

### 1. Pressure Distribution:

$$\frac{\partial P}{\partial z} = \rho g \longrightarrow \frac{P_{i,j} - P_{i,j-1}}{\Delta z} = 9.806 \left( \frac{\rho_{i,j} + \rho_{i,j-1}}{2} \right) \longrightarrow P_{i,j} = P_{i,j-1} + 4.703 \Delta z (\rho_{i,j} + \rho_{i,j-1}) \quad \dots (8)$$

### 2. Density-Temperature Relationship:

$$\rho = -0.0055T^2 + 0.0182T + 1000.1 \quad \dots (9)$$

Where:

$\rho$ : Local water density (kg/m<sup>3</sup>)

T: local water temperature (°C)

### 3. Turbulent Dynamic Viscosity:

The distribution of turbulent dynamic viscosity can be obtained from the following equation described by Launder and Spalding (1972).

$$\mu = C_\mu * \rho * \frac{K^2}{\varepsilon} \quad \dots (10)$$

From this equation the turbulent dynamic viscosity ( $\mu$ ) is a function of the turbulence kinetic energy (K) and the rate of dissipation ( $\varepsilon$ ). (Babarutsi and Chu, 1998).

Where:

$C_\mu$ : Empirical constant represented by the dissipation coefficient in the (K- $\varepsilon$ ) turbulence model.

K: Turbulence kinetic energy per unit mass (m<sup>2</sup>/s<sup>2</sup>)

$\varepsilon$ : Dissipation rate of turbulence kinetic energy per unit mass (m<sup>2</sup>/s<sup>3</sup>)

## Initial and Boundary Conditions:

The initial and boundary conditions of the present problem are used to solve the differential equation numerically to determine the velocity and temperature distributions along the flow field.

### 1. Initial Conditions:-

1. In the thermal discharge, the flow domain was initialized with a temperature equal to the ambient water temperature in river except with in the outfall region where it is taken to be constant and equal to the temperature of the heated water at the outlet location.
2. The initial condition for the horizontal velocity component ( $u$ ) is constant in the vertical plane, while the vertical velocity component ( $w$ ) was set equal to zero at every point in the vertical plane.
3. Turbulent viscosity ( $\mu$ ), and ( $\varepsilon$ ) dissipation rate of turbulence kinetic energy in the river are obtained from the following equations: (Babarutsi and Chu, 1998):

$$\mu = 0.077 \rho U_f h \quad \dots (11)$$

$$\varepsilon = S * g * u \quad \dots (12)$$

Where:  $h$  = water depth (m) measured at z-axis.

$S$  = water surface slope (m/m).

$u$  = longitudinal velocity components (m/s).

$U_f$  = friction velocity (m/s).

$g$  = acceleration due to gravity (m/s<sup>2</sup>).

Using initial conditions of ( $\mu$ ) and ( $\varepsilon$ ) as calculated from above equation is to find the initial conditions for the turbulence kinetic energy (K) in river from equation (10). At the outfall region, the initial condition for (K) and ( $\varepsilon$ ) are obtained from the following equations (Babarutsi and chu, 1998)

$$\frac{K_d}{K_r} = \left( \frac{u_d}{u_r} \right)^2 \quad \dots (13)$$

$$\frac{\varepsilon_d}{\varepsilon_r} = \left( \frac{u_d}{u_r} \right)^2 \quad \dots (14)$$

The turbulent dynamic viscosity at the outfall region is obtained from equation (10).

## 2. Boundary Conditions:

The boundary conditions for the present model can represent as follows:-

### ▪ At discharge region:

$w$ ,  $u$ ,  $T$ ,  $K$ , and  $\varepsilon$  equal to their initial condition at each discharge location.

### ▪ At upstream boundary :

where  $X = 0$  and  $0 < Z < D$  due to the assumption of no reverse flow,  $w$ ,  $u$ ,  $T$ ,  $K$  and  $\varepsilon$  are constants and equal to their initial profiles.

### ▪ At the down stream boundary:

Where  $X = L$  and  $0 < Z < D$ . The gradient for  $w$ ,  $u$ ,  $T$ ,  $K$ ,  $\varepsilon$  are

assumed equal to zero  $\frac{\partial}{\partial X_i} (w, u, T, K, \varepsilon) = 0$

### ▪ At the surface boundary :

Where:  $0 < X < L$  and  $Z = 0$  temperature at the image point (point located outside the water surface) is equal to the surface temperature. (Oberikamp and Crow, 1996) and  $P = 0$ ,  $w = 0$ . Gradient of horizontal velocity, turbulent kinetic energy (K) and dissipation rate ( $\varepsilon$ ) are assumed equal to zero.  $\frac{\partial}{\partial Z_j} (u, K, \varepsilon) = 0$

### ▪ At the bottom boundary:

Where:  $0 < X < L$ ,  $Z = D$  and  $w = 0$ , the gradient of  $u$ ,  $T$ ,  $K$ , and  $\varepsilon$  are equal to zero  $\frac{\partial}{\partial Z_j} (u, T, K, \varepsilon) = 0$

## Dilution Factor Formula:

A dilution factor formula was conducted to obtain a dimensionless index for the dilution state to a river stream suffering from any thermal pollution.

The temperature distribution for a river stream along the flow field was transformed to the dilution factor formula as follows:

$$d_f = \frac{(td - tr) - (t_{i,j} - tr)}{(td - tr)} \quad \dots (15)$$

By simplifying the equation to:

$$d_f = \frac{(td - t_{i,j})}{(td - tr)} \quad \dots(16)$$

Where:

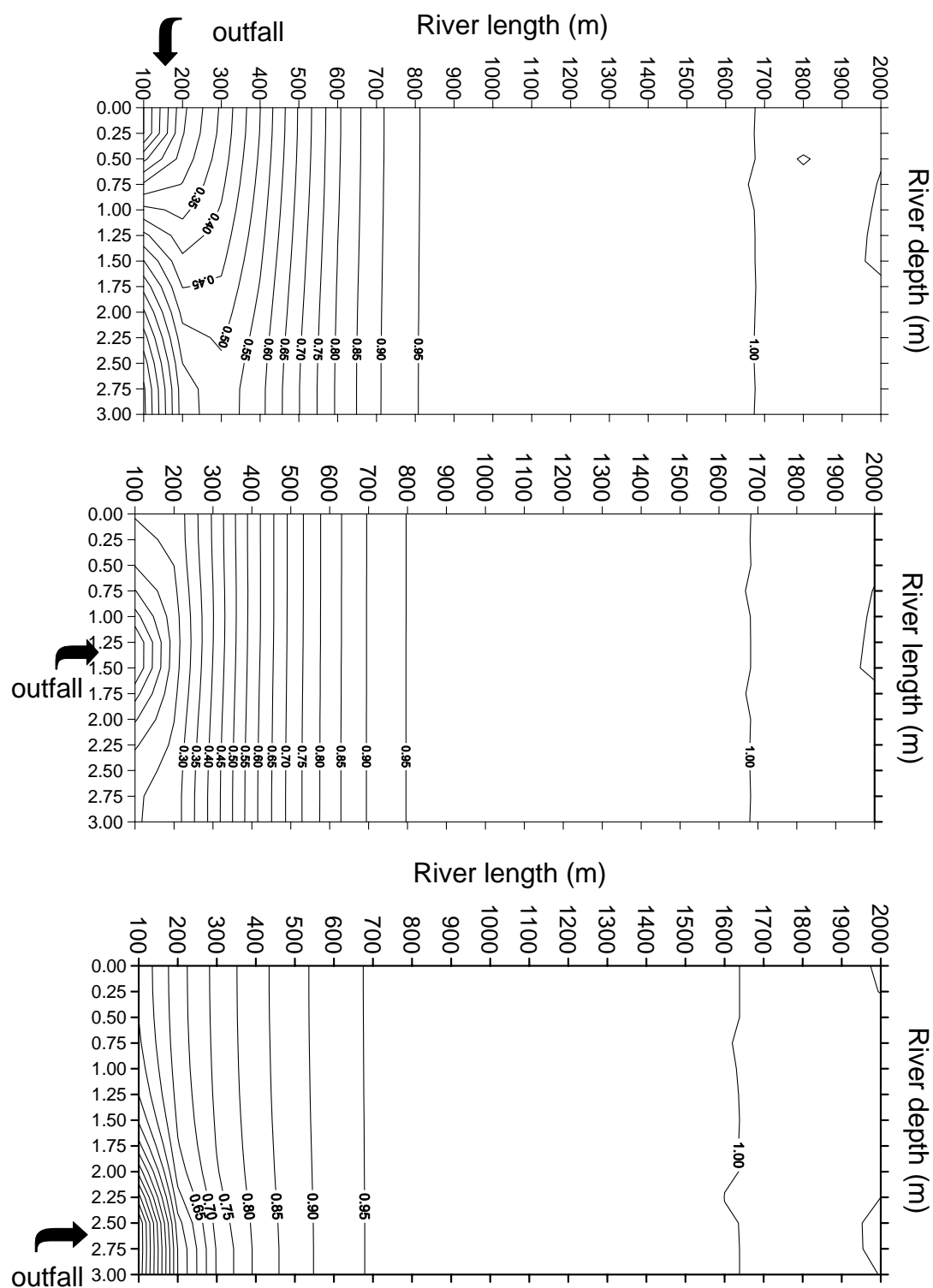
$d_f$  : Dilution factor.

$td$  : Thermal discharge water temperature.

$t_{i,j}$  : Temperature of a river in the selected node.

$tr$  : River water temperature.

The value of the dilution factor range from 0.0 to 1.0 the 0.0 value denoted that there is no dilution yet at the discharge region. While the 1.0 value means that the river stream recovers its original temperature before the thermal discharge. The dilution factor values for the three thermal discharge cases are as follows:



**Fig. (2): Dilution factor distribution of temperature isotherms (for the three thermal discharge cases (surface and two submerged discharges)  $t_r = 20^\circ\text{C}$ ,  $t_d = 40^\circ\text{C}$ , Surface water slope  $= 0.00006\text{ m/m}$ ,  $U = 0.8\text{ m/sec}$ , Time  $= 500\text{ sec}$**

## Numerical Solution

The partial differential equations were formalized and simplified in order to be solved using Alternative Direction Implicit-Explicit finite difference method (ADI) with up-winding technique. The resulting system of linear simultaneous equations was then solved using Gauss-Siedel point-by-point method. In this formulation method the dependent variables (e.g.  $w$ ,  $u$ ,  $T$ ,  $\varepsilon$ ,  $K$ ) at a grid point are connected to their values at the neighboring grid points. The mesh spacing in the (X, Z) direction are  $\Delta X$  and  $\Delta Z$  respectively and the time is segmented into equal intervals ( $\Delta t$ ) subscripts ( $i$ ,  $j$ , and  $t$ ) are associated with each mesh point.

The (ADI) method advances the solution from time level ( $t$ ) to time level ( $t + \Delta t$ ) in two steps:

- The first step advances to the solution implicit in the Z – direction and explicit in the X-direction to intermediate level, usually termed ( $t+0.5 \Delta t$ ).
- The second step advances the solution implicit in the X – direction and explicit in the Z – direction to time level ( $t+ \Delta t$ ).

The two steps are repeated till reaching the specified time of the study, where the temperature is printed at the final time step.

A computer program was written in quick basic 4.5 languages for Ms-Dos Microsoft computers because of large memory and speed required to carryout the numerical calculations in the present work. The input data used to operate the model and run the computer programs are the length, depth, and slope of the river, time interval and final time of the study, water river characteristics (e.g. temperature , density , kinetic viscosity....etc), temperature of heated water, longitudinal and vertical increment. The flow chart of the computer program is shown in Fig. (3)

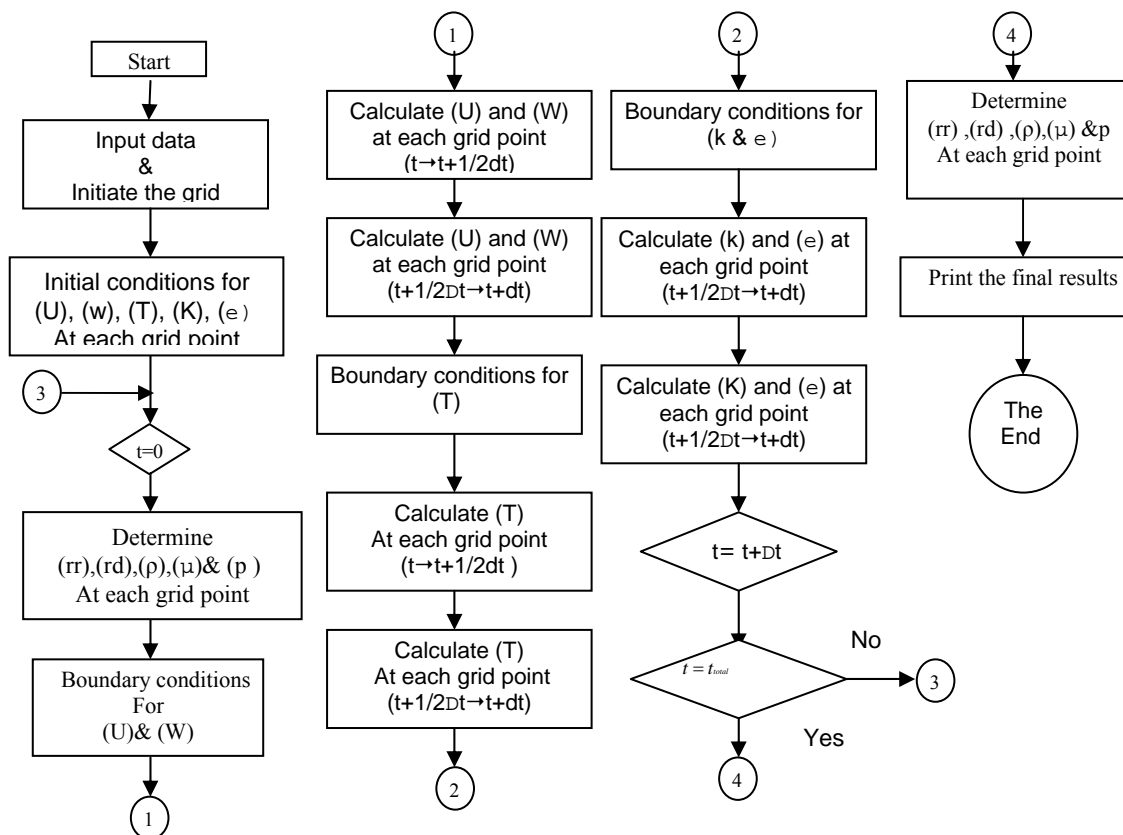


Fig. (3): Flow chart of the computer program



## Results and Discussion

In order to test the most efficient method of disposal for thermal discharges, comparisons between the three discharge locations (surface and submerged at 0.42 (first case), 0.83 (second case) from the river depth) according to the maximum dilution factor achieved for each outfall are established. The maximum dilution factor used to conduct this comparison is (0.95). The dilution factor isotherms for the three discharge cases are adopted, with initial river velocity equal to 0.8 m/sec and river surface slope equal to 0.00006 m/m. Total time of thermal disposal = 500 sec. From Fig. (2) one can find:

1. Maximum dilution factor for the surface disposal case (0.95) is obtained at distance 805 m from the discharge region, while this distance becomes (795 m., 680 m) for the first and second case of submerged disposal.
2. The dilution level near the discharge region at distance 300 m for example is (0.35, 0.4, and 0.7) for the surface and submerged thermal discharges (first and second case) respectively.

The above results indicate that the outfall location near the river bed (0.83 from the river depth) achieves the maximum dilution factor value (0.95) with smaller distance than from outfall lies at the surface and outfall locates at 0.42 from the river depth. Therefore, the submerged discharges near the river bed enhance the river ability to recover fast from the thermal shock.

## Conclusions

The following conclusions are conducted from the present model development with all its assumptions, the conclusions are:

1. In cases where non-unidirectional vertical velocity components take place, it is necessary to apply the up-winding scheme (upstream differencing) for the solution of the convective terms in the governing differential equations in order to obtain a positive values for the coefficients a, b, and c in this equations and the solution become more stable.
2. The submerged thermal discharge for outfall lies near the river bed (0.83 from the river depth) is the best for the river environment causes an earlier formation of uniform vertical temperature faster from that in the outfall at 0.42 from the river depth.
3. The submerged outfall at 0.42 from the river depth is not a favorable option for discharging heat because of low dilution levels achieved compared to the surface disposal scheme and navigation purposes for the rivers.
4. The dilution factor distribution gives the specific temperature value for any node inside the flow domain straightly from the contour map according to equation no. (15), which avoid the computer programming disturbance.
5. An assumption of constant turbulent dynamic viscosity / diffusivity in the numerical model over the whole flow field causes a longitudinal retardation of isotherm at and near the water surface, but the vertical advanced of these isotherms near the bottom of the river when the thermal discharge disposed at the river surface. While it causes a longitudinal advancing of isotherms towards the water surface, and vertically advancing towards the river bottom in case of thermal discharge dispose below the water surface (submerged discharges).
6. The model is insensitive to the variations of water surface slope in case of surface water discharge. While the model is sensitive to the variations in water surface slope.

7. The numerical model is sensitive to the variations in time of disposal, for surface and submerged discharges.
8. The model is sensitive to the increase in initial river water velocity which leads to a distinguished increase in temperature levels at the longitudinal stations and a decrease in the vertical spreading at the same time.

### Recommendations

The following recommendations are suggested for the future studies:

1. Experimental laboratory work /and or field work are suggested to support the theoretical investigation.
2. The model can be extended to study the effect of multi-point sources of thermal discharge on the bed material resistance and sediment transport.
3. Apply the present model by using a multi-port tee diffuser at the river bed for thermal discharge as submerged disposal.
4. Development of a three-dimensional numerical model of single-point source disposal for thermal pollution in river.

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