Free Vibration of Symmetric Laminated Fiber Reinforced Plate with Center Circular Hole

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Abstract

The investigate of the free vibration of fiber reinforced (FR) plate has been studied. The natural frequency of an FR plate depends on a variety of variables, including aspect ratio, size of hole, fiber orientation of the laminae that make up the laminate, and the boundary conditions. These variables using related to the natural frequancy of laminated plates by analyzing a number of laminated plates using the commercially available (ANSYS 9.0) finite element software. From the results, it was found that the natural freqauncy of the FR plate of fixed–free boundary condition increase but simply support-free boundary condition decrease with incraseing the hole size for all values of aspect ratio, where as the hole size increase the composite plates lose more materail and consequently lose more stiffness.

Keywords: Free vibration, fiber reinforced, plate with circular hole, finite element.

الخلاصة

دراسة الاهتزاز الحر للصفائح المقواة بالألياف. إن التردد الطبيعي لتلك الصفائح يعتمد على مجموعة من المتغيرات و المتـضمنة نسبة الواجهة، حجم الثقب، اتجاه ألياف الصفائح الرقيقة و المكونة للصفيحة الكلية و تأثير الظروف الحدوديـة. تـم ربـط هـذه المتغيرات مع التردد الطبيعي للصفيحة وذلك بتحليل عدد من الصفائح باستخدام برنامج العناصر المحددة (Ansys 9.0). من خلال البحث وجد بان التردد الطبيعي لصفيحة مقواة بالألياف ذات إسناد مثبت – حر يزداد ولصفيحة مقواة بالألياف ذات إسناد بسيط – حر يقل بزيادة حجم الثقب لكل قيم نسبة الواجهة حيث بزيادة حجم الثقب تفقد الصحيفة من مادتها وبالتالي تفقد أكثر من جساءتها.

Introduction

The increasing use of fiber reinforced laminates plates with holes are extensively used in space vehicles, aircrafts, automobiles, ships, chemical vessels, mechanical and civil structures in order to obtain the convenient connection of structural members and have necessitated the rational analysis of structures for their mechanical response. In addition, the anisotropy, nonhomogeneity and larger ratio of longitudinal to transverse modulii of these new materials demand improvement in the existing analytical tools. As a result, the analysis of laminated composite structures has attracted many research workers, and has been considerably improved to achieve realistic results. In the design of modern high-speed aircraft and missile structures, swept wing and tail surfaces are extensively employed. Moreover some of the structural elements are provided with cutouts of different shapes to meet the functional requirements like (i) for the passage of various cables, (ii) for undertaking maintenance work and (iii) for fitting auxiliary equipment. Depending upon nature of application, these structural elements are acted upon by mechanical and thermal loads of varied nature. Usually, the anisotropy in laminated composite structures causes complicated responses under different loading conditions by creating complex couplings between extensions, bending, and shears deformation modes. **Huang**

Srinivas and Rao presented a set of complete analytical analyses on bending, buckling and free vibration of plates with both isotropic and orthotropic materials. And the dynamic characteristics have been studied for many years most previous investigations have been confined to isotropic plates with holes. The study of composite plates with holes is rather limited. **Reddy** studied the large amplitude free vibration of layered composite plates with rectangular cutouts by finite element method. Frequencies corresponding to linear and nonlinear situations were presented for thin and thick orthotropic and laminated composite plates. **Prasad and Shuart** presented a closed form solution for the moment distributions around holes in symmetric laminates subjected to bending moments. **Avalos, Larrondo and Laura** obtained the frequency parameters form anisotropic rectangular plates with free-edge holes by using the Rayleigh-Ritz method. The effects of aspect ratio, hole side to plate side ratio and the position of the hole on the frequencies were investigated. However, in these studies the effect of the variation of the thickness on frequencies was not considered. **Ukadgaonker et. al**. gave a general solution for bending of symmetric laminates with holes. **Karami et.al**. has applied Differential Quadrature Method (DQM) for static, free vibration and stability analysis of skewed.

Setoodeh and Karami employed a three-dimensional elasticity based layer-wise finite element method (FEM) to study the static, free vibration and buckling responses of general laminated thick composite plates.

Huang studied the free vibration of orthotropic square plates with a hole. By considering the hole as an extremely thin part of a plate, the free vibration problem of a plate with a hole can be transformed into the free vibration problem of its equivalent square plates with non-uniform thickness.

Niraangin studied the theory elastic behavior of composite skew plates with elliptical cutouts subject to non-linearly varying temperature loading.

This work describes the free vibration analysis of laminated composite plates with circular holes. A finite elements method by using Ansys 9.0 was used t study the effect of plates, aspect ratio, hole size, plate boundary conditions and fiber orientation angles.

Description of Problem

ANSYS was used to analyze the free vibration of various laminated plates in order to see how changes in the laminated plate would affect the natural frequency. The changes to the laminated plate were based on four variables: hole size, boundary condition, aspect ratio and orientation of the stitched mat layers used in FR laminates. The laminated plates were analyzed under two different boundary conditions: simple-simple-free-free, fixed-fixed-free-free. The plate thicknesses, t, was used (1 mm). three different aspect ratios (B/A) were considered: 0.25,0.5 and1. The width "A" was held constant at 0.1 m and the length "B" was varied between 0.4, 0.2 and .1 m. and "D" the diameter of circular hole.

The mat orientation of the (0,90,90,0), (45,-45,-45,45) and (0,45,45,0). Combinations of each of these variables were analyzed for a laminated reinforced plate consist of 4 layers using ANSYS

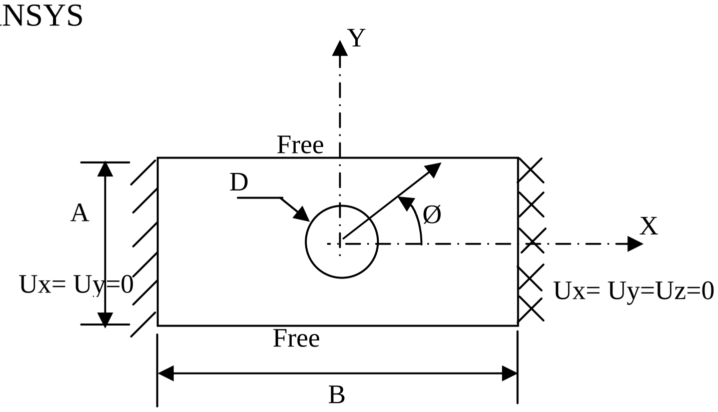


Fig (1): Basic Model (A plate with circular hole)

Free Vibration of Laminate Plate

This section dealt with the analytical determination of the natural frequency of various types of plate, by using finite element program Ansys 9.0. In using Ansys to determine the natural frequency for laminate plates, the effect of hole size, aspect ratio, boundary condition and fiber orientation of laminate plate is taken into consideration.

A general plate is shown in fig (1). The aspect ratio is defined as length "B" divided by width "A" and the hole size defined as diameter "D". The boundary condition notation used (e.g. fixed-fixed-free-free) and (simply-simply-free-free) refers to the boundary condition along edge (x=0)-(x=A)-(y=0)-(y=B)

Macromechanics of a Lamina

The goal of macromechanics of a lamina is to determine the stress-strain behavior of an individual lamina. Since a laminate is made up of laminae with various fiber orientations, the stress-strain relationships for a lamina is first expressed in terms of the lamina coordinate system and then transformed to the global coordinate system of the laminate. This is necessary in order to determine the stiffness of a laminate in terms of the global coordinate system.

Stress-Strain Relationship in a Lamina

Using contracted notation, the generalized Hooke's law relating stresses to strains is

$$\{\sigma_i\} = [C_{ij}]\{\varepsilon_j\} \quad (1)$$

where, σ_i are the stress components, C_{ij} is the 6 * 6 constitutive matrix, and ε_j are the strain components. The stiffness matrix has 36 constants, but by using energy methods it can be shown that the stiffness matrix is symmetric (C_{ij}=C_{ji}) and therefore only 21 of the constants are independent (Jones, 1999). The relationship in Eq.(1) characterizes an anisotropic material, which has no planes of symmetry for the material properties. For a lamina, which is considered to be orthotropic, the stiffness matrix has only nine independent constants.

Lamina Coordinate System.

Assuming a state of plane stress in the 1-2 material plane gives:

$$\sigma_{33} = \sigma_{23} = \sigma_{31} = 0 \quad (2)$$

which reduces Hooke's law to:

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix} \quad (3)$$

where [Q] is the reduced stiffness matrix. The components of the reduced stiffness matrix are defined in terms of the in-plane mechanical properties of the lamina and are

$$
\begin{aligned}
Q_{11} &= \frac{E_1}{1-\upsilon_{12}\upsilon_{21}} \\
Q_{12} &= \frac{\upsilon_{12}E_2}{1-\upsilon_{12}\upsilon_{21}} = \frac{\upsilon_{21}E_1}{1-\upsilon_{12}\upsilon_{21}} \\
Q_{22} &= \frac{E_2}{1-\upsilon_{12}\upsilon_{21}} \\
Q_{66} &= G_{12}
\end{aligned}
\tag{4}
$$

Global Coordinate System.

The response of a laminate to loading in the global coordinate system is found using the stress-strain relationships, determined in terms of the global coordinate system, of each lamina. Generally, Eq. 3 must be transformed to reflect rotated fiber orientation angles. The following relationship reflects this transformation [Brian,1998]:

$$
\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} \tag{5}
$$

where [$\overline{Q}$] is the transformed reduced stiffness matrix, which is found using the relation

$$
[\overline{Q}] = [T]^{-1}[Q][T]^{-T} \tag{6}
$$

where the superscript T denotes the matrix transpose and [T] is the transformation matrix, which is

$$
[T] = \begin{bmatrix} m^2 & n^2 & 2mn \\ n^2 & m^2 & -2mn \\ -mn & mn & m^2-n^2 \end{bmatrix} \tag{7}
$$

where $m = \cos\theta$, $n = \sin\theta$ and θ is the angle between the lamina's coordinate system and the global coordinate system as shown in Fig. 1.

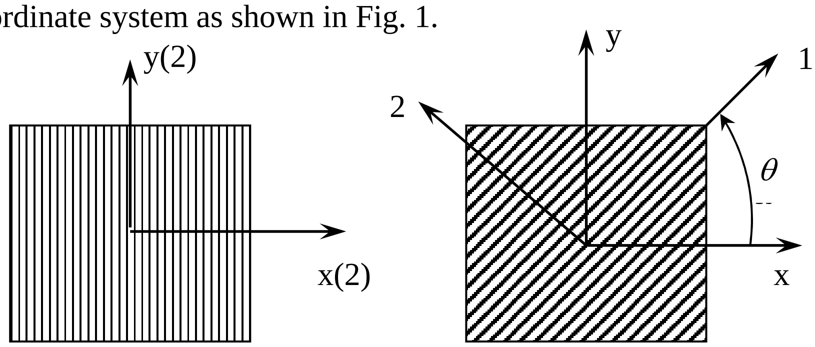


Figure 2: Lamina On- and Off-axis Configurations (Staab, 1999)

Using Eq. (6) and Eq. (7), the components of the transformed reduced stiffness matrix are

$$\begin{aligned}
\overline{Q}_{11} &= Q_{11}m^4 + 2(Q_{12} + 2Q_{66})m^2n^2 + Q_{22}n^4 \\
\overline{Q}_{12} &= (Q_{11} + Q_{22} - 4Q_{66})m^2n^2 + Q_{12}(m^4 + n^4) \\
\overline{Q}_{22} &= Q_{11}n^4 + 2(Q_{12} + 2Q_{66})m^2n^2 + Q_{22}m^4 \\
\overline{Q}_{16} &= (Q_{11} - Q_{12} - 2Q_{66})m^3n + (Q_{12} - Q_{22} + 2Q_{66})mn^3 \\
\overline{Q}_{26} &= (Q_{11} - Q_{12} - 2Q_{66})mn^3 + (Q_{12} - Q_{22} + 2Q_{66})m^3n \\
\overline{Q}_{66} &= (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66})m^2n^2 + Q_{66}(m^4 + n^4)
\end{aligned} \tag{8}$$

Note that the transformed reduced stiffness matrix, [$\overline{Q}$], has terms in all positions in the matrix as opposed to the presence of zeros in the reduced stiffness matrix, [Q]. Therefore, in terms of the global coordinate system, a generally orthotropic lamina appears to be anisotropic, since shear-extension coupling exists (Jones, 1999).

Variation of Strain and Stress in a Laminate

The strain of any point in a laminate that has undergone deformation can be determined by considering the geometry of the undeformed and deformed cross section shown in Fig. 3. Point B in this figure is located at the mid-plane and in going from the undeformed to the deformed shape Point B undergoes a displacement in the x-direction of u_o. (Note that the symbol ‘nought’ (o) designates mid-plane values of a variable) Since, due to Kirchhoff’s hypothesis, line ABCD remains straight under deformation of the laminate, the displacement of arbitrary point C is

$$u_c = u_o - z_c\beta \tag{9}$$

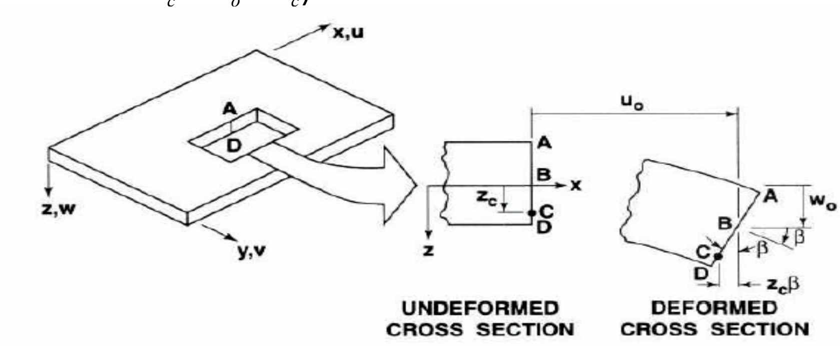


Figure 3: Geometry of Deformation (Jones, 1999)

Based on Kirchhoff’s hypothesis, under deformation, line ABCD remains perpendicular to the mid-plane; therefore, β is the slope of the laminate mid-plane in the x-direction, that is,

$$\beta = \frac{\partial w_o}{\partial x} \tag{10}$$

The displacement, u, at any point z through the thickness of the laminate is

$$u = u_o - z\frac{\partial w_o}{\partial x} \tag{11}$$

Similarly, the displacement, v, in the y-direction is

$$v = v_o - z\frac{\partial w_o}{\partial y} \tag{12}$$

According to Kirchhoff's hypothesis $\varepsilon_z = \gamma_{xz} = \gamma_{yz} = 0$, therefore the remaining non-zero laminate strains are ε_x, ε_y, and γ_{xy}. Combining these relationships with Eq. 5 gives the following expression for the k^{th} layer:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}_k = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix}_k \left[\begin{Bmatrix} \varepsilon_x^{o} \\ \varepsilon_y^{o} \\ \gamma_{xy}^{o} \end{Bmatrix} + z \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \right] \quad (13)$$

Even though the strain variation is linear through the thickness of a laminate, the stress variation is not necessarily linear through the thickness of a laminate because the transformed reduced stiffness matrix, [$\bar{Q}$], can be different for each lamina in a laminate.

By integrating through the thickness of the laminate, the net force resultants and moment resultants can be calculated.

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \int_{-t/2}^{t/2} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} dz = \sum_{k=1}^{N} \int_{z_{k-1}}^{z_k} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}_k dz \quad (14)$$

and

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \int_{-t/2}^{t/2} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} z\,dz = \sum_{k=1}^{N} \int_{z_{k-1}}^{z_k} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}_k z\,dz \quad (15)$$

where z_k and z_{k-1} are defined in the geometry of an N- layered laminate, which is depicted in Fig. 4.

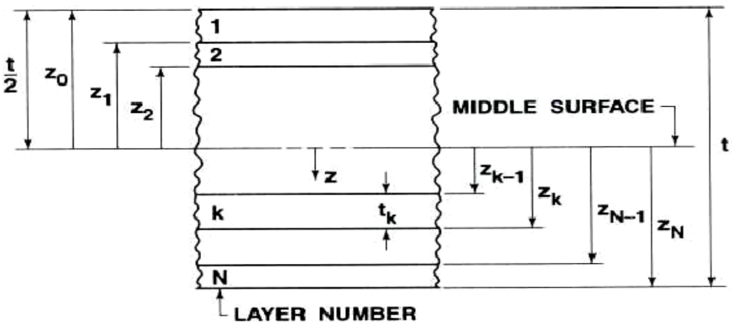


Figure 4: Geometry of an N-Layered Laminate (Jones, 1999)

Combining these relationships with Eq. 13 gives:

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^{o} \\ \varepsilon_y^{o} \\ \gamma_{xy}^{o} \end{Bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \quad (16)$$

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^{o} \\ \varepsilon_y^{o} \\ \gamma_{xy}^{o} \end{Bmatrix} + \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \quad (17)$$

where

$$A_{ij} = \sum_{k=1}^{N} (\overline{Q}_{ij})_k (z_k - z_{k-1})$$

$$B_{ij} = \frac{1}{2} \sum_{k=1}^{N} (\overline{Q}_{ij})_k (z_k^2 - z_{k-1}^2) \quad (18)$$

$$D_{ij} = \frac{1}{3} \sum_{k=1}^{N} (\overline{Q}_{ij})_k (z_k^3 - z_{k-1}^3)$$

The extensional stiffness matrix is [A], the bending-extension coupling stiffness matrix is [B], and the bending stiffness matrix is [D]. The presence of matrix [B] implies that there is a coupling between bending and extension, therefore if a laminate has B_{ij} terms, pulling on the laminate will cause bending and/or twisting of the laminate. The terms A_{16} and A_{26} represent shear-extension coupling, which means coupling exist between shear stress and normal strains and between normal stresses and shear strain, in a laminate. The terms D_{16} and D_{26} represent bending-twisting coupling in a laminate. The [A], [B], and [D] matrices are very useful in understanding the behavior of a laminate under given loading conditions and are used frequently in the analysis of composites.

The potintial energy is represented as

$$\delta P.E = \int_A (\{\delta\varepsilon\}^T \{N\} + \{\delta\kappa\}^T \{M\}) dA = \{\delta q\}^T [K]\{q\} \quad (19)$$

where $\{q\} = \{u, v, \frac{\partial w}{\partial x}, \frac{\partial w}{\partial y}\}$ and [k] stiffness matrix.

The Kinitic eneryg is represented as

$$\delta K.E = \int_A (-I_0 (u''\delta u + v''\delta v) - I_1 (\beta_x{}'' \delta u + u''\delta\beta_x + \beta_y{}'' \delta v v''\delta\beta_y) - I_2 (\beta_x{}'' \delta\beta_y + \beta_y{}'' \delta\beta_y)) dA$$

$$= \{q\}^T [M]\{q\} \quad (20)$$

where the moment of inertia is defined as $(I\ , I_1, I_2) = \int_{-h/2}^{h/2} \rho(1, z, z^2) dz$, and [M] is the mass matrix.

The equation of motion can be obtained as

$$[M]\{q^{\bullet\bullet}\} + [K]\{q\} = 0 \quad (21)$$

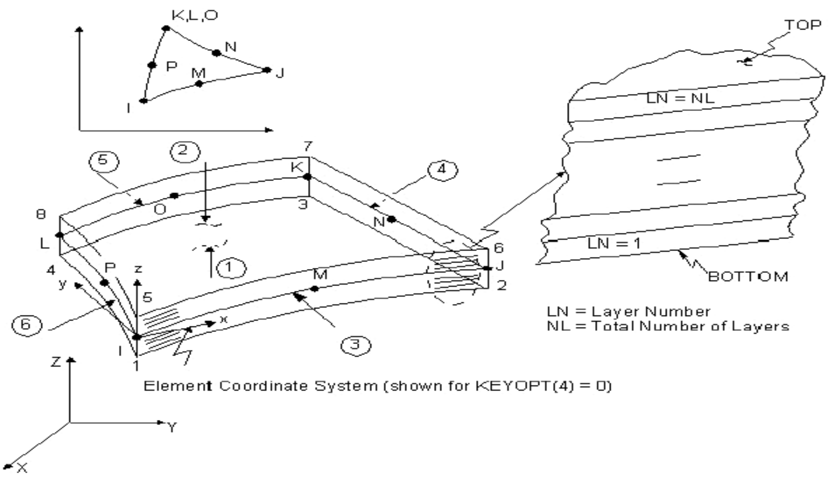
The eiganvalue problem for the vibration analysis be obtained as

$$[k] - \omega^2 [M]\{\Theta\} = \{0\} \quad (22)$$

where (ω, Θ) the natural frequancy and mode shape.

Finite Element Analysis

The element used for the laminated plates was Shell99, which is an 8- node linear layered structural shell element (See Fig. 5). The element has six degrees of freedom at each node: translations in the x, y, and z directions and rotations about the nodal x, y, and z-axes. The Shell 99 element is perfectly suited for composites materials because it allows entry of up to 250 layers. Each layer has its own thickness, material property, and orientation. For laminated FR composites, the direction of the fibers determines the layer orientation. For each layer, the layer material properties (E_1, E_2, ν_{12}, G_{12}, G_{13}, and G_{12} listed in table 1), the orientation (angle between the layer and global coordinate system, θ, as shown in the off-axis configuration of Fig. 2)



Figure(5): Shell 99 Element (ANSYS Element Reference)

Table1:Micromechanical Properties of Stiffened Layers in a Laminate Carbon/ epoxy [Barbero, 1999]

E_1 GPa	E_2 GPa	E_3 GPa	G_{12} GPa	G_{13}=G_{23} GPa	ν_{12}	ν_{13}= ν_{23}
207	5	5	7	8.5	0.21	0.09

Results and Discussion

Table (2) shown the fundamental natural frequency (Hz) for simply supported composite plates compared with the revious work to check the validity of the results, composite plate (for which results are available in literature) are analyzed and the fundamental frequency obtained are given in this table. It can be seen that the results agree fairly will with those available in literature.

The effect of various parameters like hole size, aspect ratio, boundary conditions and fiber orientation on the fundamental natural frequency of vibration is studied by considering fiber reinforced composite plates with central circular hole under boundary conditions fixed and simply supported for two edges and free from other edges.

Table (2): Natural frequency (Hz) of simply supported laminated plates with aspect ratio (A/B=1)

Type of Laminated	Simply support four edges (D/A=0)		Simply support two edge (D/A=0.5)	
	Present study	Latheswary	Present study	Huang
(0,90,90,0)	29.898	31.018	232.82	225.92
(45,-45,-45,45)	27.321	29.214	237.22	229.63

Effect of hole size

To better illustrate the effect of the hole size on the natural frequency of fixed-free and simply supported-free, the variation of the natural frequency with (D/A) ratio is shown in figures (3) to (8) it can be seen that the frequencies decrease with the increase of the (D/A) ratio for both boundary conditions.

Table (3) shown the results of the natural frequency of composite plates with varying hole size, boundary conditions, aspect ratio and fiber orientation. It can be noted that the natural frequency decrease a little first and then increase with fixed-free boundary orientation with hole size increase , while it decrease with simply support boundary condition with hole size increase. To explain the phenomenon, two effects in traduced by a hole are considered. The first one is a reduction in the strain energy of the plate which will decrease the frequency of the plates. The second one is a reduction in the mass which will increase the frequency, for the composite plates with a larger hole the first effect might be still the dominant effect due to its high ratio E1/E2 and the frequency would continues to decrease with D/A < 0.5 for simply supported-free boundary condition.

Effect of boundary conditions

The variation of the natural frequency with hole size for fixed and simply supported edges conditions as shown in figures (6)-(11). In the case of simply supported-free laminate there is a asudden decrease in the natural frequency when the hole size increase becuase loss in area of material this cause decrease in strain energy of strecture but for lamenite fixed-free edge the value of the natural freqauancy increase when hole size incearse becuase the fixed edge increase the strain energy.

Effect of aspect ratio

The effect of the aspect ratio on the natural frequency of vibration is studied by analyzing four-layer laminates (0,90)s,(45,-45)s and (0,45)s by varying A/B ratio, keeping the value of "A" constant in all cases, the natural frequency is found to increase gradually with increase aspect ratio and the variation is show in figures (12)to (17). The increase is due to the incrase in stiffness of the plates with increasing aspect ratio. Where the aspect ratio increase cause to incrase in area and small in hole size with respect to total area for that the strain energy increase and natural freqauncy increasing with varying of fiber orientation of composite plate.

Effect of fiber orientation

Four-layer symmetric (0,90)s,(45,-45)s and (0,45) with hole size (D/A) from 0 to 0.5. A change in fiber orientation angles from 0 to 90^0 leads to an increase in the natural frequency of vibration in the case of fixed-free and simply supported-free for all value of aspect ratio and hole size, also figures (12-17) it can be seen that the natural frequency of vibration of arrangement (0,90)s is higher than from (0,45)s and (45,-45)s, becuase thr fiber orientation effect on the displacement of lamenite plate thi cause to increase in (0,90) and decrease in (45,45) and(45,-45) becuase resolve the displacement in two components this cause decrease strain energy of stracture.

Table (3): Variation the natural frequencies with hole size, aspect ratio, boundary condition and fiber orientation.

Aspect ratio(B/A)	Hole size (D/A) ratio	Fixed –Free boundary condition			Simply support – Free boundary condition		
		(0,90)s	(45,-45)s	(0,45)s	(0,90)s	(45,-45)s	(0,45)s
0.25	0	63.841	27.746	64.707	51.261	22.865	57.787
	0.1	36.376	36.685	29.724	29.843	34.566	21.46
	0.2	39.299	37.723	36.344	35.684	34.131	32.558
	0.3	36.855	35.08	33.199	33.613	31.03	29.306
	0.4	41.03	35.143	38.793	35.821	31.64	30.532
	0.5	42.3	36.615	40.282	34.32	30.98	31.709
0.5	0	255.06	120.26	259.01	104.5	91.773	114.88
	0.1	183.05	136.89	169.74	135.45	130.91	115.55
	0.2	176.78	135.71	162.00	131.44	120.36	111.32
	0.3	181.2	136.36	165.1	126.28	117.56	106.8
	0.4	181.9	183.81	166.35	121.13	110.4	104.12
	0.5	182.28	136.31	168.21	108.84	103.02	92.759
1	0	1016.8	520.75	1033.9	208.24	314.99	190.89
	0.1	597.44	593.86	500.15	302.58	321.85	259.76
	0.2	586.85	555.00	477.6	286.17	301.5	239.48
	0.3	622.24	538.37	519.91	264.59	290.64	219.26
	0.4	626.85	549.42	503.89	242.17	271.4	210.42
	0.5	603.2	610.7	496.01	232.82	237.22	197.79

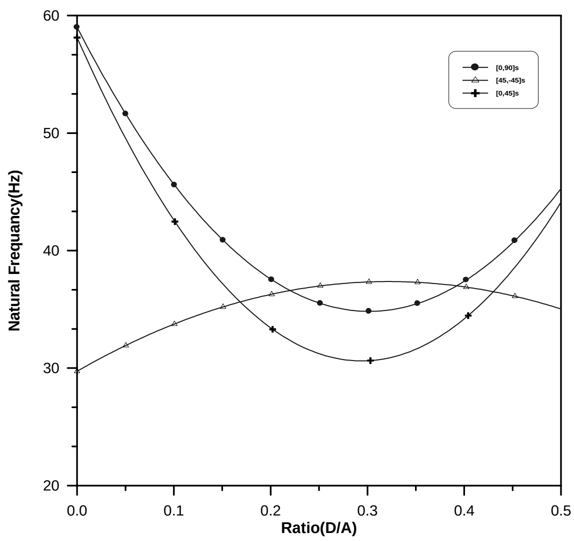


Fig (6): Variation of Natural Frequency with hole size for Fixed-free boundary condition with aspect ratio (0.25)

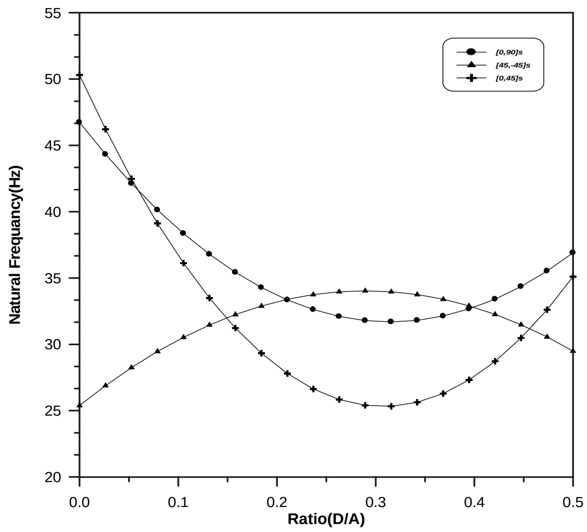
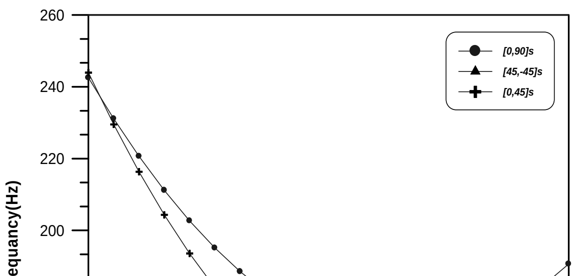


Fig (7): Variation of Natural Frequency with hole size for Simply-Free boundary condition with aspect ratio (0.25)



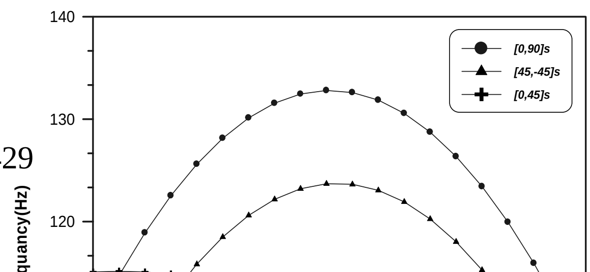
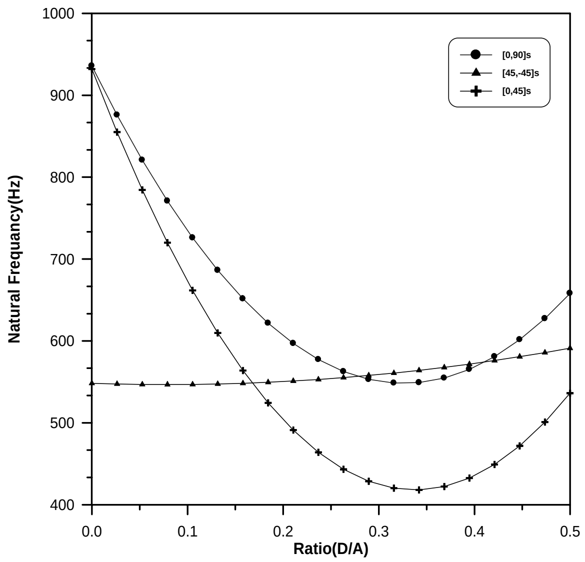


Fig (8): Variation of Natural Frequency with hole size for Fixed-free boundary condition with aspect ratio (0.5)

Fig (9): Variation of Natural Frequency with hole size for Simply-free boundary condition with aspect ratio (0.5)



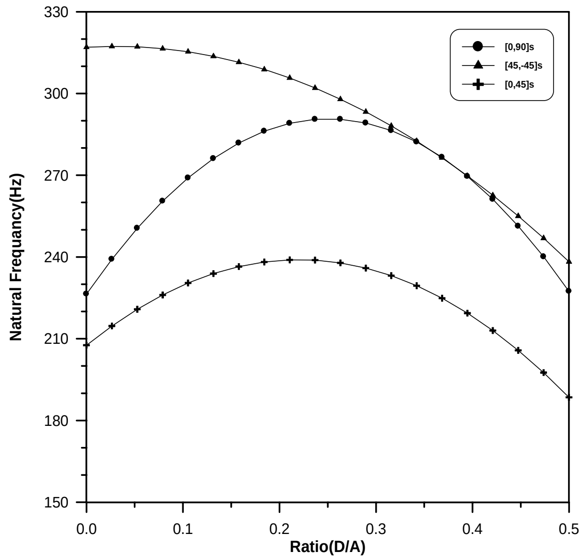
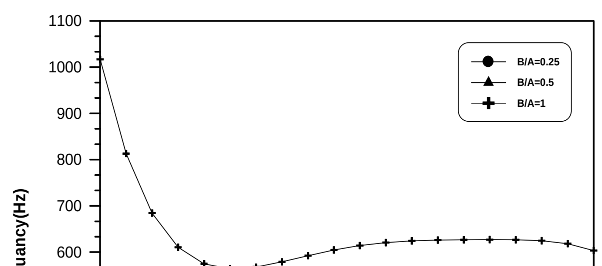


Fig (10): Variation of Natural Frequency with hole size for fixed-free boundary condition with aspect ratio (1)

Fig (11): Variation of Natural Frequency with hole size for Simply-free boundary condition with aspect ratio (1)



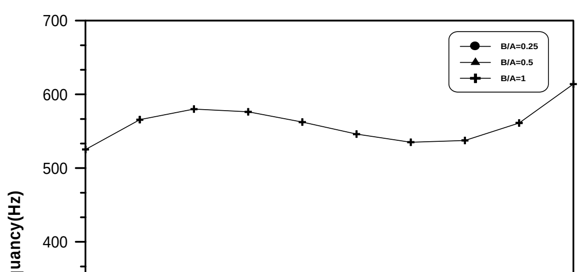
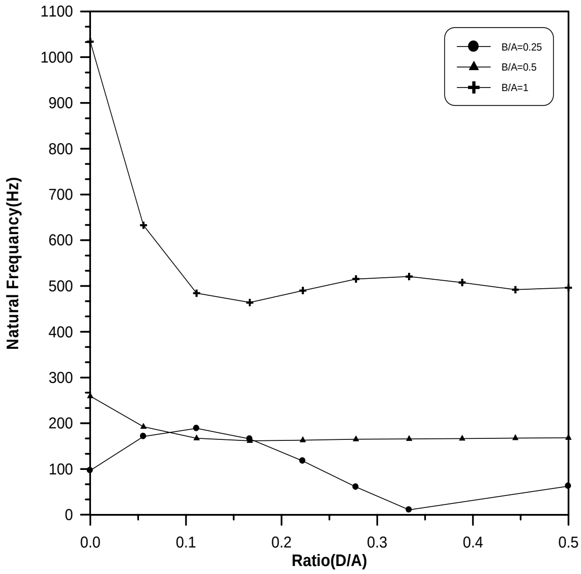


Fig (12): Variation of Natural Frequency with hole size for Fixed-free boundary condition with (45,-45,-45,45)

Fig (13): Variation of Natural Frequency with hole size for Simply-free boundary condition with (45,-45,-45,45)



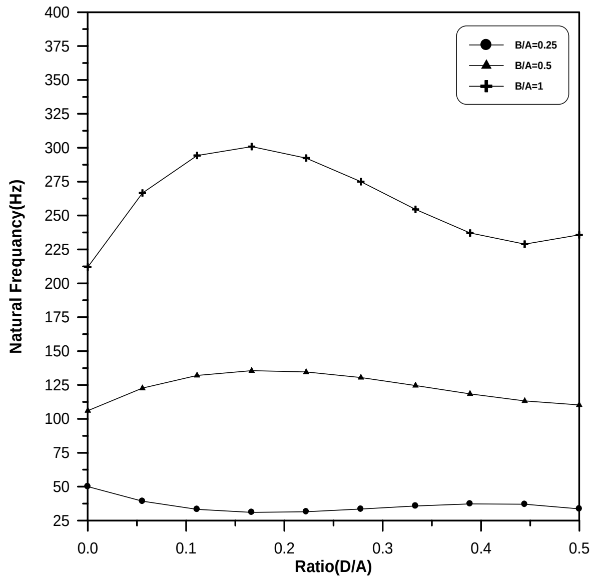
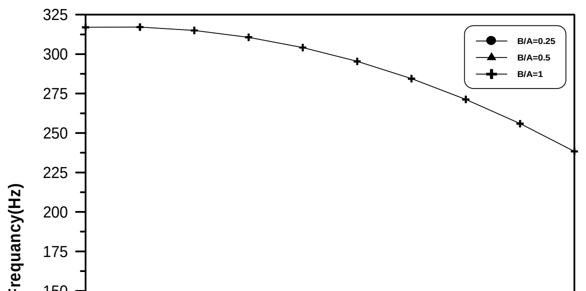


Fig (14): Variation of Natural Frequency with hole size for Fixed-free boundary condition with (0,90,90,0)

Fig (15): Variation of Natural Frequency with hole size for Simply-free boundary condition with (0,90,90,0))



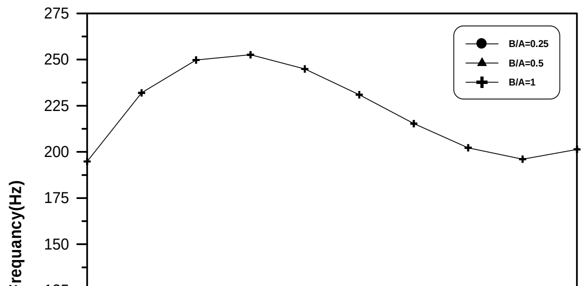


Fig (16): Variation of Natural Frequency with hole size for Fixed-free boundary condition with (0,45,45,0)

Fig (17): Variation of Natural Frequency with hole size for Simply-free boundary condition with (0,45,45,0)

Conclusion

On the basic of present study, which has deal with the free vibration of rectangular composite plates containing central circular holes. The effect of plates aspect ratio, hole size, plate boundary condition and fiber orientation have been studied and the following conclusion can be made:

1-The natural frequency of composite plates containing a circular cutouts increase by the increment of cutout diameter for fixed-free boundary condition but decrease for simply support-free boundary condition.

2- Increasing hole size dose not necessarily reduces the natural frequency of composite plates. For certain plate aspect ratio and boundary condition, the natural frequency strength could increase at large hole size.

3- The natural frequency of composite plates with (0,90,90,0) lamination could be higher or lower than these with (45,-45,-45,45) lamination, depending on the plate aspect ratio, plate boundary condition and hole size.

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