

Analysis and Estimation of Downward Seepage from Lining And Unlining Triangular Open Channel

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ABSTRACT:

This research concerns to study the quantity of seepage running downward from triangular open channel with lining and unlining using computer program SEEP/W (which is a sub-program of Geo-Studio).

Using SEEP/W experiments two groups were carried out with three different downstream slopes of triangular open channel, three different upstream slopes, three variable free board, three different height of triangular open channel, for first group using three different permeability for soil around the triangular open channel and for second group using three different permeability for lining layer and for soil around triangular open channel. For each run the quantity of seepage were determined. Dimensional analysis was used with helping of the theoretical results to develop empirical equations in order to determine the quantity of seepage running downward the triangular open channel for lining and unlining. Also, Verify the SEEP/W results with analytical method of Vedernikov's equation. Results show good agreement for triangular open channel unlining layer.

Keywords: Discharge, SEEP/W, Vedernikov's Equation, Dimensional Analysis, Lining Triangular Open Channel, Unlining Triangular Open Channel.

تحليل وتقدير التسرب باتجاه الاسفل من القنوات المثثة المبطنة وغير المبطنة

الخلاصة:

يهدف هذا البحث لدراسة كمية التسرب من القنوات المثثة باتجاه الاسفل للقنوات المثثة المبطنة وغير المبطنة باستخدام البرنامج الحاسوبي SEEP/W (وهو البرنامج الفرعي للبرنامج الحاسوبي Geo-Studio). باستخدام SEEP/W تم اجراء مجموعتين من التجارب بالاعتماد على استخدام ثلاث ميول لمؤخرة القناة المثثة وثلاث ميول مختلفة لمقدمة القناة المثثة وثلاث قيم لشحنة الارتفاع للماء داخل القناة المثثة وثلاث ارتفاعات مختلفة للقناة المثثة، المجموعة الاولى بالإضافة للمتغيرات اعلاه تم استخدام ثلاث قيم مختلفة لنفاذية التربة المحيطة بالقناة المثثة، اما المجموعة الثانية فتم استخدام ثلاث قيم مختلفة لنفاذية بطانة القناة وثلاث قيم مختلفة لنفاذية التربة المحيطة بالقناة بالإضافة للمتغيرات اعلاه. لكل تجربة من التجارب تم قياس مقدار التصريف المتسرب. كما تم اجراء التحليل البعدي وذلك لإيجاد معادلة رياضية لاستخراج مقدار التسرب الحاصل من باتجاه الاسفل للقنوات المثثة المبطنة وغير المبطنة.

كما تم التحقق من نتائج (SEEP/W) بالمقارنة مع معادلة (Vedernikov) والتي اظهرت تقارب جيد في النتائج للقنوات المثلثة الغير المبطنه.

Nomenclature

- q = Discharge ($L^3/T/L$)
 B = Top width of open channel (L).
 H = Height of open channel (L).
 h = head of open channel (L).
 k_s = Hydraulic conductivity of soil (L/T).
 k_c = Hydraulic conductivity of lining (L/T).
 F_b = Free board of open channel (L).
 α = Angel of downstream slope.
 θ = Angel of upstream slope.

INTRODUCTION

Channel seepage has been identified as a significant loss from the irrigation channels from both water quantity and environmental degradation perspectives. Recent studies have indicated that estimates of channel seepage are an essential component in the management of earthen channel systems.

The seepage in an irrigation canal refers to the water that percolates into the soil strata through the wetted perimeter of a canal [1]. Seepage losses affect the operation and maintenance of the canals in the sense that part of the water diverted for the users is lost from the conveyance system, and at the same time this water might produce piping, erode the bank of the canals whether they are lined or not, produce excessive saturation, uplift pressure, which might produce failures of the canal and other structures [1]. For instance, some canals are lined in order to reduce the seepage loss; however, according to British researchers the seepage losses in a concrete lined canal might be the same as an unlined canal if 0.01% of the lined area consists of cracks [2].

The importance of seepage in the design of channels depending on the direction of the groundwater flow, seepage normal to the boundary can be either in the form of injection (seepage through a channel in the upward direction) or suction (seepage through a channel in the downward direction) [3].

Xiao et al. (2014) studied the effect of upward seepage on bed load transport rate to generalized empirical formula presented the prediction of the bed load transport rate with occurrence of injection based on the bed load transport rate directly measured from experiments [4].

Noorduijn et al. (2014) Estimating seepage flux from ephemeral stream channels using surface water and groundwater level data this approach using a controlled flow event along a 1387 m reach of artificial stream channel [5].

Moavenshahidi et al. (2014) studied the factors affecting the estimation of Seepage Rates from channel automation data; including rainfall and evaporation during the pond age period, number of water level measurements in the pond age, and duration of gate shutdown and also including surface water elevation at the start of the pond age condition and its relation to supply level of the channel at each gauge, seasonal variations in seepage rate, and suspected leakage through macropores in banks of the channels [6].

Aghvami et al. (2013) studied behavior of the seepage in channels and estimate trapezoidal channels Seepage Using Seep/w and Evolutionary Polynomial Regression (EPR) Modelling for Qazvin and Isfahan Channels [7].

Mohammed (2013) investigates the effect of the longitudinal channel slope on the discharge coefficient of cylindrical weirs. The methodology applied is by adopting three different diameters of the cylindrical weir and at three different slopes of the channel [8].

Carabineanu (2012) presented an inverse method for studying the seepage from soil channels unlining for various symmetric or asymmetric channels, with smooth contours or with angular points [9].

Liqiang et al. (2012) studied effects of canal lining and multi-layered soil structure on canal seepage, and soil water dynamics included the effect of soil layering at the experimental site. Also he studied the combination of canal lining and a low-permeability layer below the canal which has effect in reducing canal seepage [10].

Adrian (2011) studied of the seepage from symmetrical earthen open channels; calculate numerically the contour of the channel, the phreatic lines, the seepage loss, the velocity field, the streamlines and the equipotential lines [11].

Shaimaa et al. (2011) studied the changes of the chemical properties of the soil which has been irrigated for a long period with polluted river water, and the effect of each property on the other properties to Army Canal in Baghdad for a Long Period [12].

Rao et al. (2009) design of channel at incipient motion to develop a resistance equation which will consider the seepage effect at incipient motion of the channel [4].

Charles (2008) studied seepage reduction in canal by soil compaction, seepage reduction was obtained about 86% when the sides and bottom were compacted and reductions of 12-31% were when only sides were compacted [13].

Francalanci et al. (2008) studied the effect of seepage induced Non-hydrostatic pressure distribution on bed load transport and bed Morph dynamics [14].

Chahar (2007) Analysis of Seepage from Polygon Channels at a shallow depth to obtain the solutions for the quantity of seepage, location of the phreatic line and width of seepage at the drainage layer contain improper integrals which can only be evaluated by numerical integration [15].

Khan (2007) studied the quantitative assessment of channel seepage using the artificial neural network (ANN) [16].

Subhasish (2003) generalized theoretical equations of Reynolds stress and bed shear stress for steady-nonuniform flow in open channels, having upward seepage [17].

Bhagu (2001) studied an extension of vedernikov's graph for seepage from canals exact analytical from a slit and a strip have been presented from triangular rectangular and trapezoidal canals of any side slope can be computed easily [18].

Mowafy (2001) evaluated seepage losses at different critical sections and comparing between different empirical, analytical and field measured results, then generalized a formula to compute seepage losses in the future [19].

Chahar (2000) and Swamee et al. (2001) obtained an analytical solution for seepage from a rectangular channel in a soil layer of finite depth overlying a drainage layer using inversion of hodograph and conformal mapping techniques [20]-[21].

Prabhata et al. (2000) design the optimal trapezoidal section has the least seepage loss and cross-sectional area by applying nonlinear optimization technique [22].

Youngs (1987) studied seepage from channels and ponds with impervious boundary walls extending below floor level [23].

Roland (1968) studied seepage from channels through layered porous mediums for steady-state free surface problems consisting of layers of different permeability are obtained by using the methods of finite differences seepage from a trapezoidal channel through homogeneous isotropic

mediums in each layer and the case of anisotropic mediums in which the ratio of horizontal to vertical permeability is different for each layer [24].

Vedernikov (1934) proposed a direct method to solve for the seepage from open channel by proposed the following equation for calculating the flow

$$q = k(B+AH) \quad (1)$$

k = Hydraulic conductivity of soil (L/T).

B = Effective top width of open channel (L).

H = Effective height of open channel (L).

α = Angel of downstream slope.

Where A is a function of B/H and $\cot \alpha$ 'see figures (1)-(2) [25].

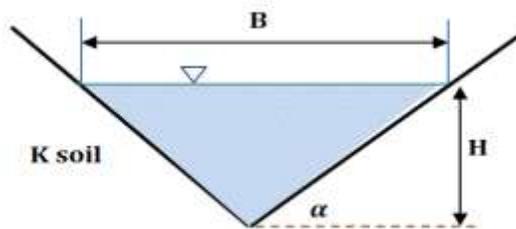


Figure 1. The general section of triangular open channel [25].

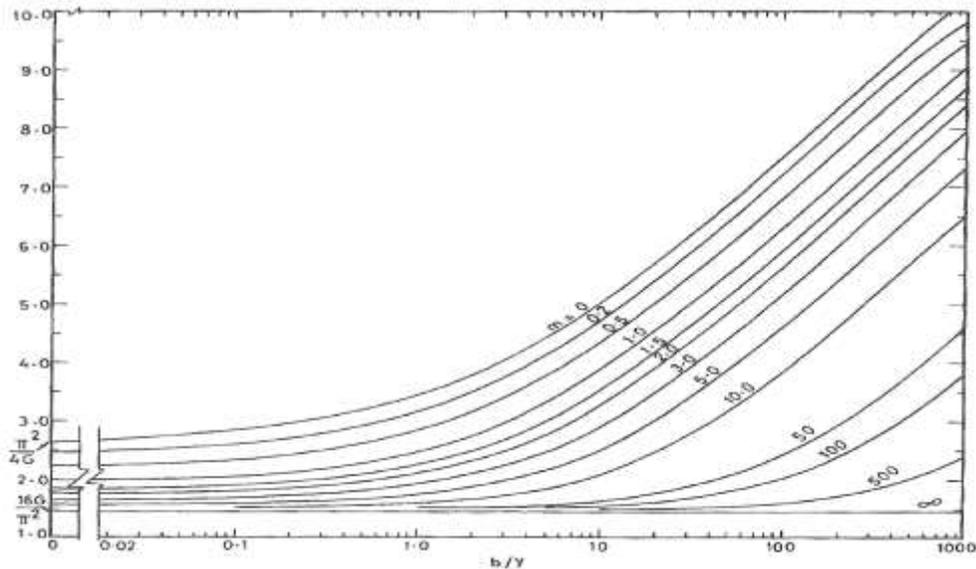


Figure 2. Triangular open channel Parameter [25].

The present paper analysis triangular open channel order to determine the downward quantity of seepage from lining and unlining triangular open channel, by application of dimensional analysis to the setup experiment of SEEP/W results, and using software program SPSS-19 Statistics, these equations will provide information on the amount of drainage running downward triangular open channel for lining and unlining layer Also the continuity of seepage from SEEP/W model verified using analytical formula for Vedernikov's solutions. Equations that will determine the values of seepage running downward the channel for asymmetrical triangular shape and for two cases; first

unlined channel and a second lined channel, which did not have equations to account directly in the previous studies.

Procedure of Experimental Setup

For the purpose of running SEEP/W model tests, it was using two groups of parameters, the first group which represent the triangular open channel unlining consist of three different values for each variable, which are upstream slopes (1:1, 1.91:1 and 1.43:1), downstream slopes (1:1, 1.43:1 and 1.73:1), freeboard (Fb) (0.5, 0.6, 0.7)m, height of open channel (H) (4, 4.5, 5)m and permeability (ks) (for soil around the triangular open channel, so the overall runs were carried out for the first group are (243) runs. The second group which represent the lining triangular open channel consist of three different values for each variable, which are upstream slopes (1:1, 1.91:1 and 1.43:1), downstream slopes (1:1, 1.43:1 and 1.73:1), freeboard (Fb) (0.5, 0.6, 0.7)m, height of open channel (H) (4, 4.5, 5)m, permeability for soil (ks) (1.5*10-3, 10-4, 10-5) m/sec, and permeability for the lining of triangular open channel (kc) (10-6, 10-7, 10-8)m/sec, so the overall runs were carried out for the second group are (729) runs. For each run determine the amount of the seepage running out downward to the soil from the triangular open channel.

Dimensional Analysis

Dimensional analysis proves to be a generally valid method to recognize the information structure in the relationships between physical quantities in a precise and clear way. It starts from the fact that in quantitative natural science the descriptive quantities have dimensions and can be divided correspondingly into *basic quantities* and *derived quantities*. [6].

A dimensional analysis is applied in order to develop empirical equations to determine the seepage running downward from the triangular open channel for lining and unlining.

As shown in Figure (3), the possible variables that can effect on the quantity of seepage running out triangular lining open channel:

$$q = f(\tan\theta, \tan\alpha, K_s, K_c, H, F_b) \quad (2)$$

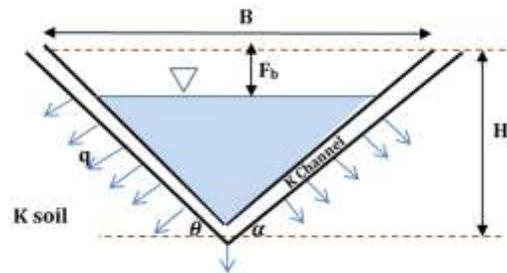


Figure. 3. The general section of triangular open channel with lining

Using π theorem, the following dimensionless terms may be obtained from the above equation.

$$(q/K_s H) = f(\tan\theta, \tan\alpha, K_c / K_s, F_b / H) \quad (3)$$

As show in figure (4) the possible variables that can be effect on the quantity of seepage running out triangular open channel unlining:

$$q = f(\tan\theta, \tan\alpha, K_s, H, F_b, B) \quad (4)$$

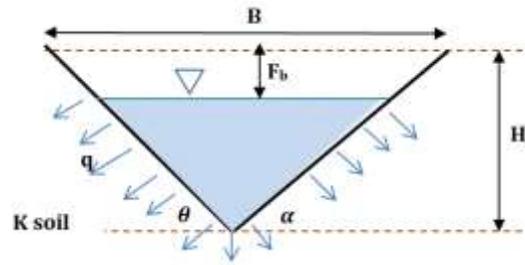


Figure. 4. The general section of triangular open channel unlining.

Using π theorem, the following dimensionless terms may be obtained from the above equation.

$$(q/KsH) = f(\tan\theta, \tan\alpha, Fb/H, B/H) \quad (5)$$

Results and Discussion

Relations between the Variables of Lining Triangular Open Channel

Using SEEP/W data, the following relations between the variables shown in the left side of the equation (2) with each one of the variable in the right side of this equation were obtained.

Figure (5) shows the relationship between the permeability of lining channel (kc) with the discharge seep from lining channel (q) for some models of triangular lining open channel with boundary conditions of constant upstream slope, permeability of soil and free board, with three different downstream slope, height of open channel. From this figure it can be shown that the discharge increases with increasing the channel lining permeability, the discharge increases with increasing the height of open channel. Also, the figure show that the discharge increases with decreasing the downstream slope of open channel. This result shows the dimension of it and permeability that effect of seepage running downward the open channel.

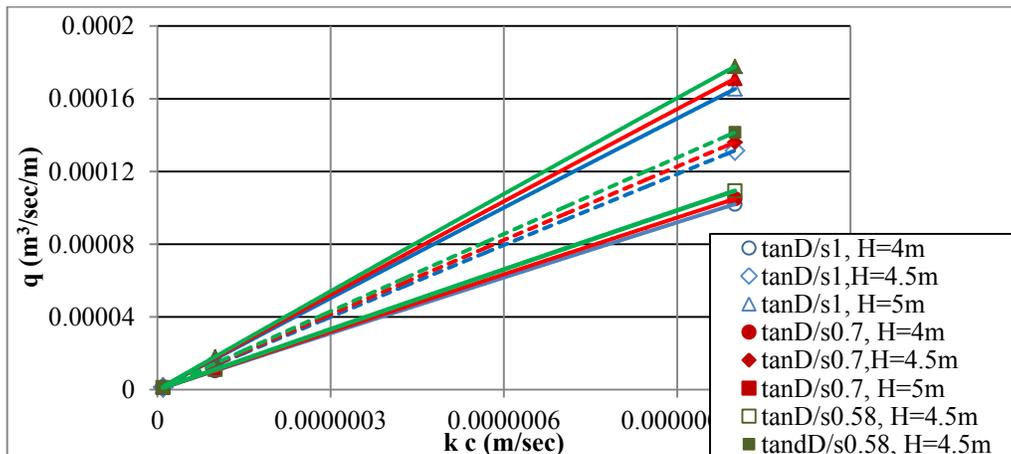


Figure.5. Relationship between (kc) and (q) of triangular lining open channel with upstream slope 1:1, Fb=0.5m.

Figure (6) shows the relationship between the permeability of canal (kc) with the discharge seep from triangular lining open channel (q) for some models triangular lining open channel with boundary conditions of constant downstream slope, permeability of soil and free board, with three different upstream slope and height of open channel. From this figure it can be shown that discharge

behave the same behavior of the previous case. Also, show that the discharge increases with decreasing the upstream slope.

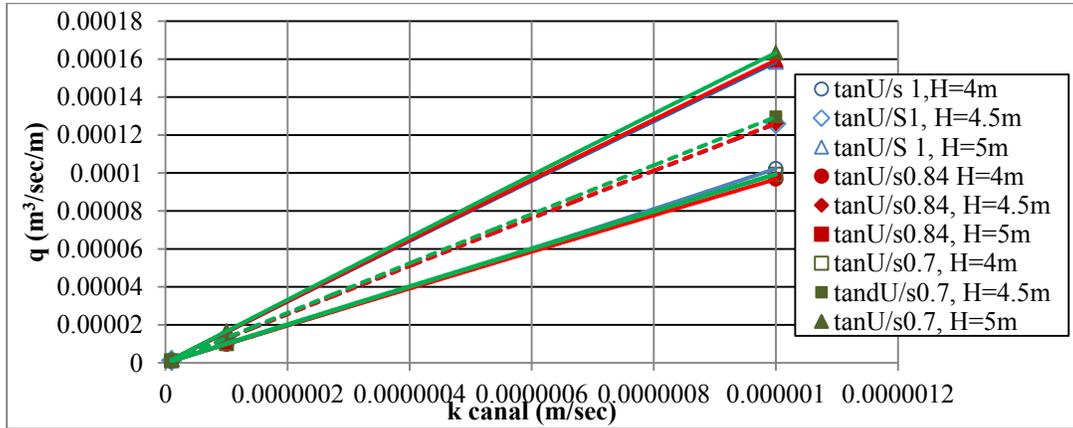


Figure.6. Relationship between (k_{canal}) and (q) of lining open channel with Downstream slope 1:1, $F_b=0.5m$.

Figure (7) shows the relationship between the permeability of soil around lining triangular open channel with the discharge seep from it (q) for some models of triangular lining open channel with boundary conditions of constant upstream slope and free board, height of open channel, with three different downstream slope and permeability of lining channel. From this figure it can be shown that the discharge increases with increasing the soil permeability, the discharge increases with increasing the lining permeability of channel. Also, show the low effect of changing soil permeability in most cases on quantity of seepage running downward from lining triangular open channel which it duo to the homogenous soil around the open channel.

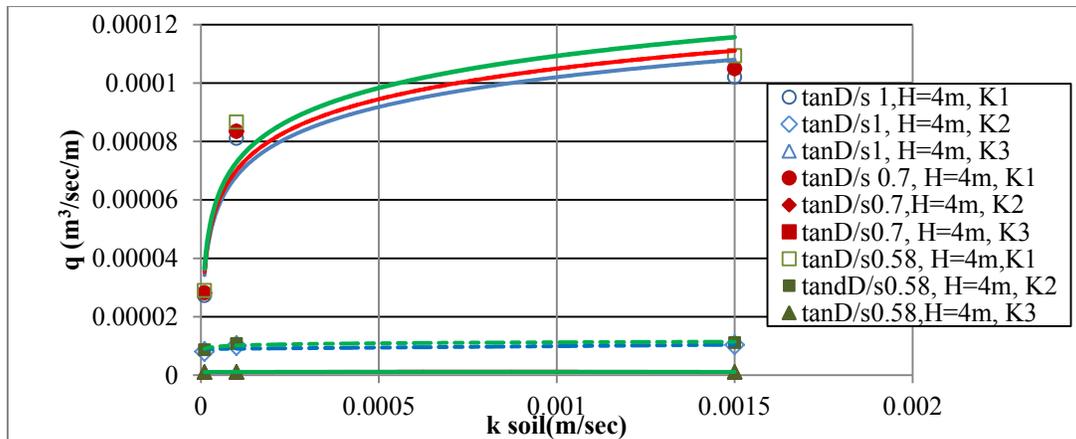


Figure.7. Relationship between (k_{soil}) and (q) of triangular lining open channel with Upstream slope 1:1, $F_b=0.5m$, $H=4m$.

Figure (8) shows the relationship between the permeability of soil around lining triangular open channel with the discharge seep from it (q) for some models of triangular lining open channel with boundary conditions of constant downstream slope, height of open channel and free board, with

three different upstream slope and permeability of lining channel. From this figure it can be shown discharge behaves the same behavior of the previous case, and the discharge increases with decreasing upstream slope.

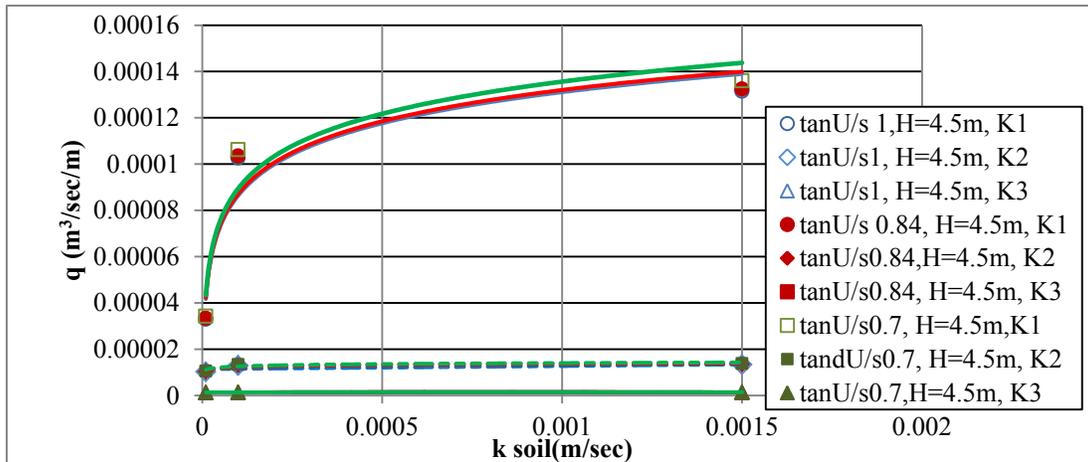


Figure.8. Relationship between (k soil) and (q) of lining open channel with Upstream slope 1:1, Fb=0.5m, H=4m.

Figure (9) shows the relationship between the Upstream Slope ($\tan u/s$) with discharge seep downward (q) for some models of triangular lining open channel with boundary conditions of constant freeboard, soil permeability and lining channel permeability, with three different high of open channel and downstream slope. From this figure, it can be shown the discharge increase with increasing the height of open channel. Also, the figure show that discharge increases with decreasing downstream slope and upstream slope which it a result for increasing the volume of open channel contain the water.

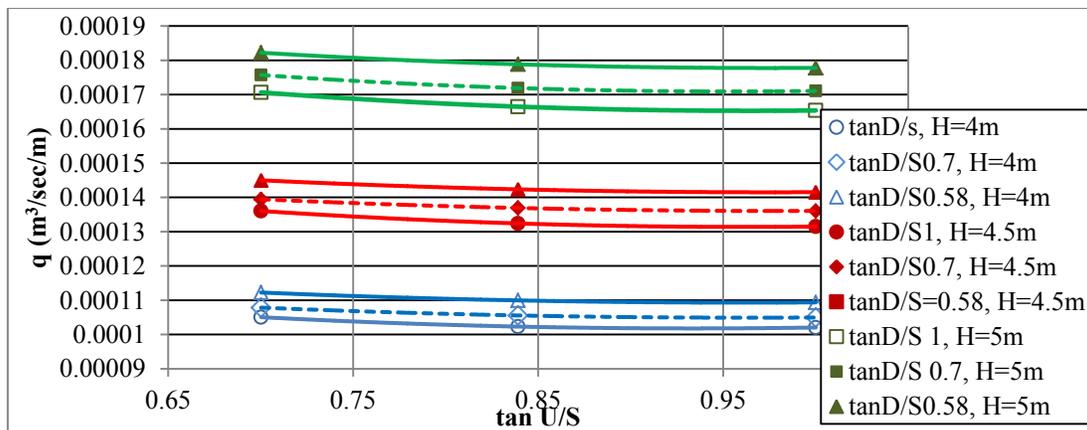


Figure.9. Relationship between ($\tan U/S$) and (q) of triangular lining channel, Fb=0.5m, k canal 10-6 m/sec, k soil 0.0015m/sec.

Figure (10) shows the relationship between the free board (Fb) with the discharge seep from the open channel (q) for some models of triangular lining open channel with boundary conditions of constant upstream slope, soil permeability and the lining channel permeability, with three different downstream slope and height of open channel. From this figure, it can be shown that the discharge increase with decreasing the free board. Also, the figure show that the discharge increases with decreasing the downstream slope and the discharge increase with increasing the height of channel.

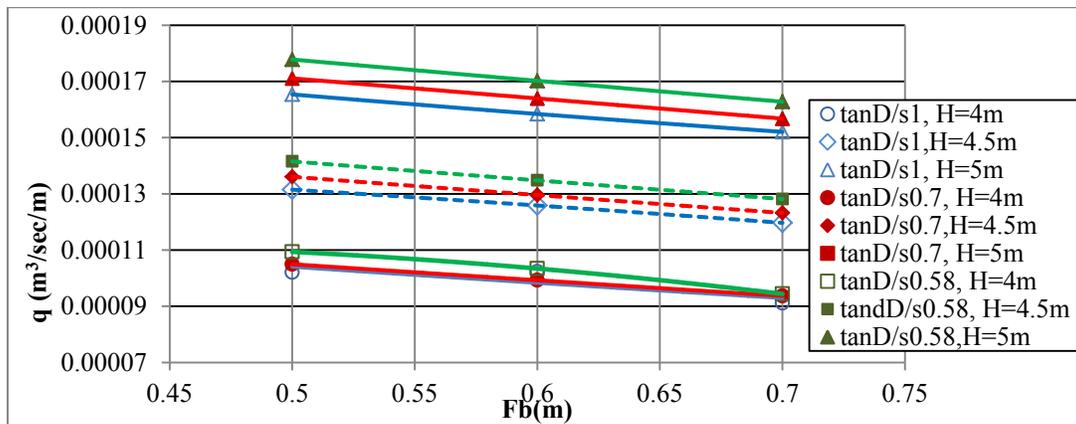


Figure.10. Relationship between (Fb) and (q) of lining open channel with upstream slope 1:1, k soil 0.0015m/sec, k canal 10-6m/sec.

Relations between the Variables of Triangular Open Channel unlining

Figure (11) shows the relationship between the permeability of soil (ks) with the discharge seep from open channel (q) for some models triangular open channel unlining with boundary conditions of constant upstream slope, downstream slope, with three different height of open channel and free board. From this figure it can be shown that the discharge increases with increasing the soil permeability, the discharge increases with increasing the height of open channel. Also, the figure show that the discharge increases with decreasing the free board of open channel.

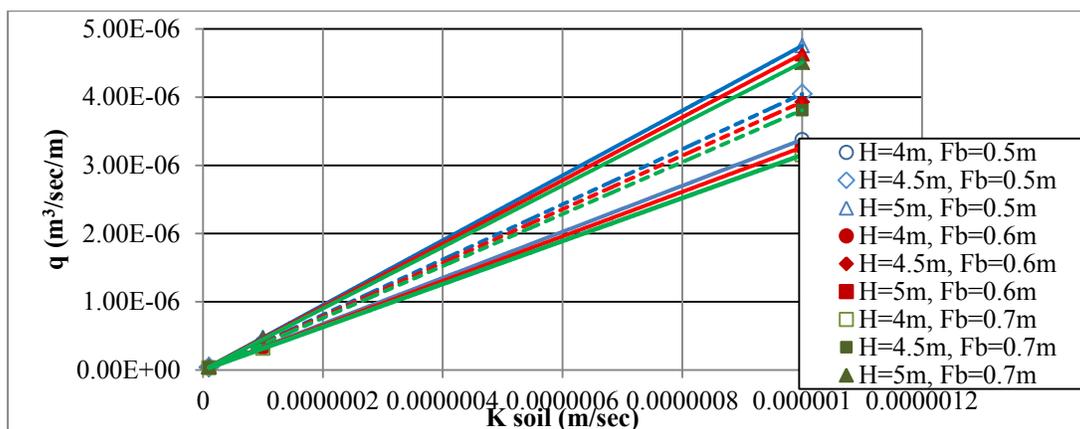


Figure.11. Relationship between (ksoil) and (q) of open channel with Downstream slope 1:1, Upstream slope1:1.

Figure (12) shows the relationship between the free board (Fb) with the discharge seep from the open channel (q) for some models of triangular open channel unlining with boundary conditions of constant upstream slope and soil permeability, with three different downstream slope and height of open channel. From this figure it can be shown that the discharge increases with decreasing the free board. Also, the figure show that the discharge increases with decreasing the downstream slope and the discharge increase with increasing the height of channel.

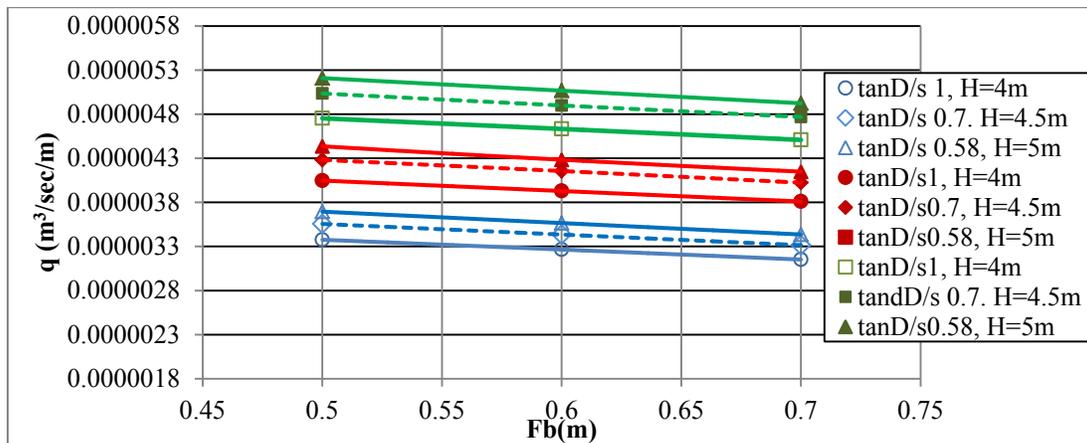


Figure.12. Relationship between (Fb) and (q) of open channel with upstream slope 1:1, k soil 10-6 m/sec.

Figure (13) shows the relationship between the free board (Fb) with the discharge seep from the open channel (q) for some models of triangular open channel unlining with boundary conditions of constant Downstream slope and soil permeability, with three different downstream slope and height of open channel. From this figure it can be shown that the discharge increases with decreasing the free board. Also, the figure show that the discharge increases with decreasing the Upstream slope and the discharge increase with increasing the height of channel.

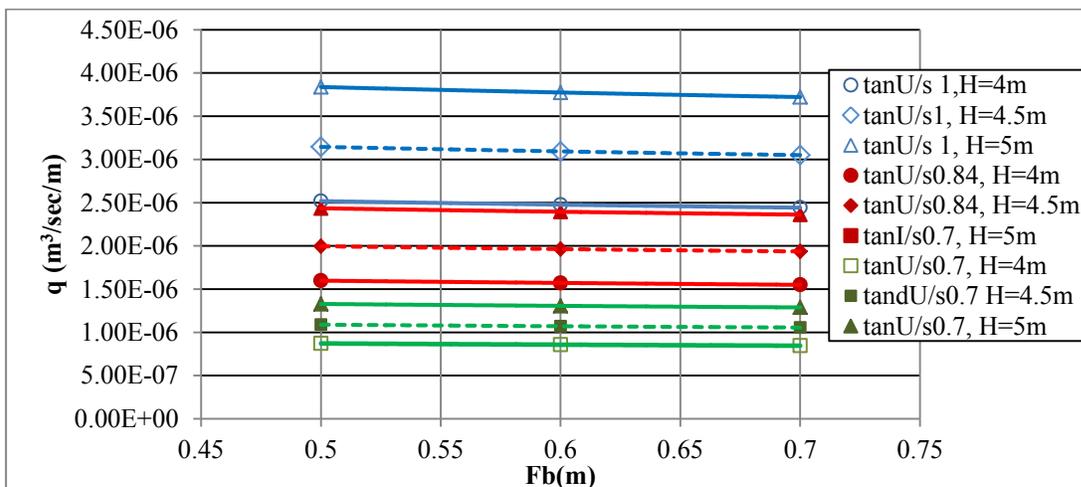


Figure.13. Relationship between (Fb) and (q) of open channel with Downstream slope 1:1, k soil 10-6 m/sec.

Equations For Computing The Discharge Seep from lining and unlining open channels

By substituting approximately two thirds of the SEEP/W results in software program SPSS-19 Statistics, it will be getting the following equation which used to determine the quantity of seepage running downward from triangular open channel for lining and unlining:

$$q = 9073.245 * (\tan u/s)^{-6.351} (\tan D/s)^{-10.248} (fb)^{-0.091} (H)^{19.025} (ks)^{0.927} (b)^{-17.137}$$

..... (6) R2=0.8 for lining triangular open channel

$$q = 5.36 * (\tan U/s)^{0.003} (\tan D/s)^{-0.115} (fb)^{-0.279} (H)^{2.09} (kc)^{0.916}$$

R2=0.952 for triangular open channel unlining

Figures (14, 15) shows the comparison between the remaining one third results of the discharge running downward from triangular open channels by SEEP/W and those that which calculated from the equations (6 and 7) using the same characteristics and geometry boundary conditions. The figures below show good agreement between the calculated discharge from the equations (6 and 7) and measuring from SEEP/W model.

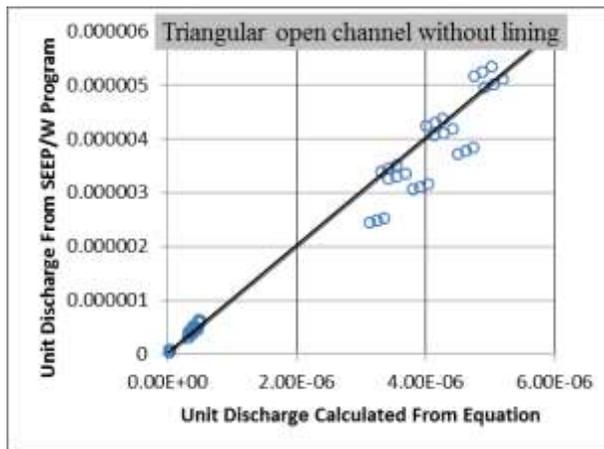


Figure 14. Comparison between the calculated discharge from the equation (7) and measuring from SEEP/W model.

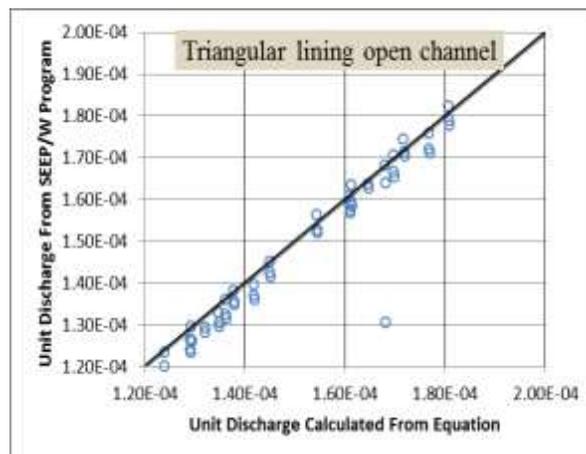


Figure 15. Comparison between the calculated discharge from the equation (6) and measuring from SEEP/W model.

Comparison Discharge with Other Method

Figure (16) compares the results from the suggested equation (7), SEEP/W results and the results obtained from the equation recommended by previous researcher Vedernikov (1934).

The figure below shows good agreement between SEEP/W, equation (7) and Vedernikov's results. Also, show the effect of changing the upstream slope, free board on the magnitude of seepage which was not taken on Vedernikov's equation.

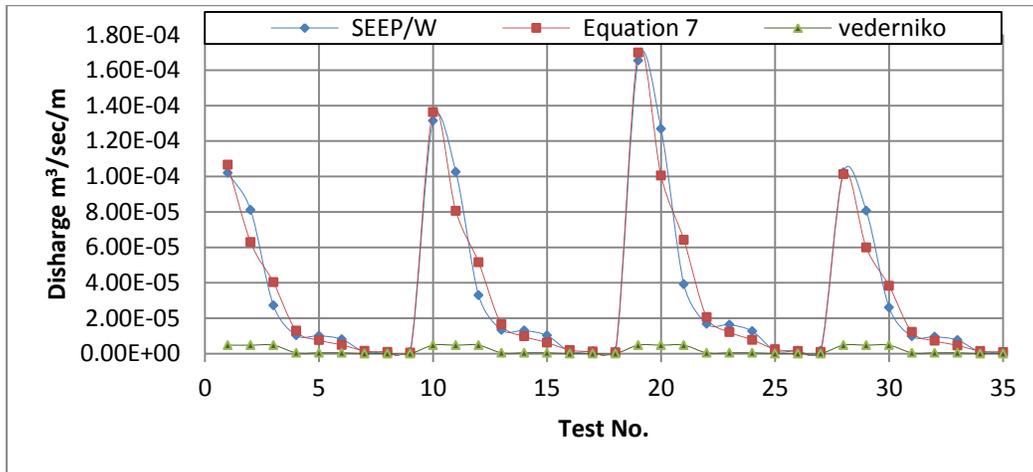


Figure. 16. Comparison between the calculated discharge for thirty tests with different method.

Also, by compare the discharge for triangular open channel by suggested equation (6) with Vedernikov's equation (See figure 17), it's shown different in results duo to neglect of lining layer for the open channel, upstream slope and free board in Vedernikov's equation.

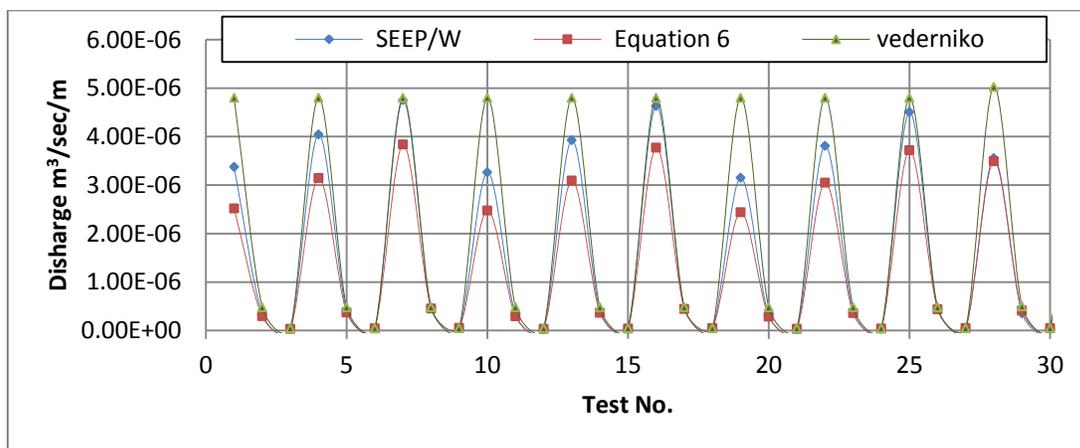


Figure. 17. Comparison between the calculated discharge for thirty five tests with different method.

CONCLUSIONS

In the current research, the SEEP/W model was used to simulate the seepage flow running downward the triangular open channel for two cases, first case with lining it can be conclude the following: the discharge running downward the open channel increases with increasing the channel

lining permeability, the height of open channel and the soil permeability. Also, the discharge increases with decreasing the downstream slope of open channel, the upstream slope and the free board. Low effect of changing soil permeability is in most cases on quantity of seepage running downward from lining triangular open channel. The discharge increases with decreasing upstream slope.

The second case was triangular open channel unlining, it can be conclude the following: The discharge increases with increasing the soil permeability, the height of open channel. Also, the discharge increases with decreasing the free board of open channel, downstream slope and upstream slope.

When compare the suggested equation for unlining triangular open channel with Vedernikov's equation it's shown good agreement between them, while by compare with suggest equation for triangular open channel with lining show different in result due to neglecting the effect of lining layer, upstream slope and free board in Vedernikov's equation.

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