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Elasto-Plastic Deflection of Continuous Composite Beams

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Abstract

An analytical method is presented for the elasto-plastic analysis of simply supported and continuous composite steel-concrete beams. A general formulation for the calculations of the deflection at elasto-plastic stage is developed. Numerical results are given and compared with a nonlinear finite difference analysis. Acceptable accuracy is found for the presented method.

Keywords: Composite beam; elasto-plastic analysis; deflection

الخلاصة

تناول البحث التحليل المرن- اللدن للاعتاب البسيطة والمستمرة المركبة من المقاطع الحديدية والخرسانة المسلحة وتم تطوير صيغ جديدة لحسابات الهطول في تلك المرحلة من التحميل . تم تقديم نتائج عددية وقورنت مع طرق تحليل غير خطية وقد وجدت دقة مقبولة للطريقة المقترحة.

Notation

M_p	Plastic moment
M_y	Yield moment
M_f	Modified moment
σ_y	Yield stress
α	Relative depth of plastic zone
y	Depth of plastic zone
S_c	Elastic section modulus of the composite section
Z_c	Plastic section modulus of the composite section
t_p	Thickness of cover plate
h	Total depth of steel section
h_1	Depth of web
b	Steel cross-section width
Δ_{el}	Elastic deflection
E_s	Modulus of elasticity of steel
f	Shape factor
Ic	Moment of inertia of composite section
L	Length of composite beam
b_f	Effective width of concrete flange
h_f	Thickness of concrete flange
f'_c	Concrete strength
n	Modular ratio
S_s	Elastic section modulus of the steel beam
S_{eff}	Effective section modulus of composite section

1- Introduction

In recent years composite steel and concrete beams have been used extensively in the construction of buildings and bridges. The most frequently encountered structural form consists of a number of steel I-section beams, on top of which is cast a reinforced concrete slab, composite behavior is achieved by connecting the steel beam to the reinforced concrete slab by shear concretes. The floor slab in composite construction acts as an integral part of the beam. It actually serves as a large cover plate for upper flange of the steel beam, appreciably increasing the beam's strength.

Because of the added compression area furnished by the concrete flange in a composite beam the neutral axis at working loads is near the top of the steel, resulting in a tensile stress that is much higher than the compressive stress. In an attempt to balance these stresses a cover plate is sometimes added to the tension flange for the purpose of increasing the section modulus on this side. For best economy, thicker slabs and cover plated beams are usually of advantage (McCormack, 1981).

Plastic deflections result in a structure collapse, only elastic deflections have any significance, so for this reason deflections are always computed using elastic analytical procedures for both general methods of design (Bowles, 1981).

The main objective of this paper is to determine the flexure displacement of a composite beam for small plastic strain (partially plastic section). When the beam is loaded with a moment greater than yield moment (M_y) but not plastic moment(M_p).

2- Theoretical Consideration

2-1 Simply supported composite beam

For design purpose, a single steel beam is assumed to act compositely with an effective width of the concrete slab, which is limited by the influence of shear lag.

The maximum resisting moment predicted by elastic theory occurs when the stress at the extreme fiber reaches the elastic yield value (σ_y) expressed as

$$M_y=S_c\,\sigma_y \qquad \dots (1)$$

In elasto-plastic stage after formation of yield zone, the stiffness of beam at yield zone becomes variable. This variable stiffness can be found if we assume that process bending is elastic without limit(Chuden, 1962) (the stress-strain relation is still linear), so at the extreme fiber of cross section the stress $\sigma_f > \sigma_y$ (Fig.1) and (Fig. 2).

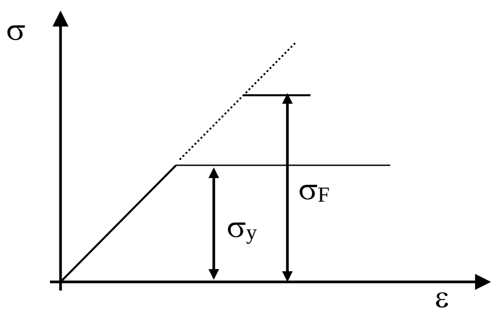

Fig (1)Idealized Stress-strain diagram for mild steel

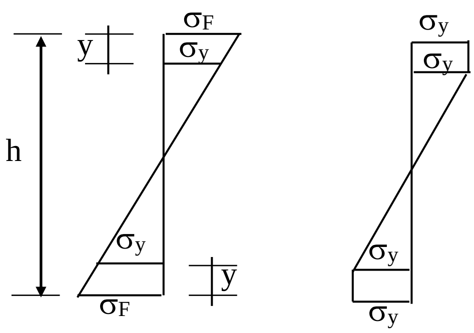

Fig. (2) Flexural stress in elastio-plastic range

Figure (1) represents an idealized form of stress-strain diagram for mild steel. Figure 2 shows flexural stresses at the beam section in elasto-plastic range, from these Figures, the following relation can be obtained:

$$\sigma_F=\frac{\sigma_y}{(1-\alpha)} \qquad \dots(2)$$

$$M_F=\sigma_F S_c=\frac{\sigma_y S_c}{(1-\alpha)}=\frac{f\sigma_y Z_c}{(1-\alpha)} \qquad \dots(3)$$

When M_F is the modified moment.

(α) is the relative depth of plastic zone

$$\alpha = \frac{2y}{h} \qquad \text{...(4)}$$

and (f) is the shape factor for composite section

$$f = \frac{S_c}{Z_c} \qquad \text{...(5)}$$

The moment at any section (x) located within the limits of yielded region (Fig. 3) is obtained by the following formula (Chuden, 1962) .

$$M_x = \sigma_y Z_c \left[1 - \frac{(1-\alpha)^2}{8} \right] \qquad \text{...(6)}$$

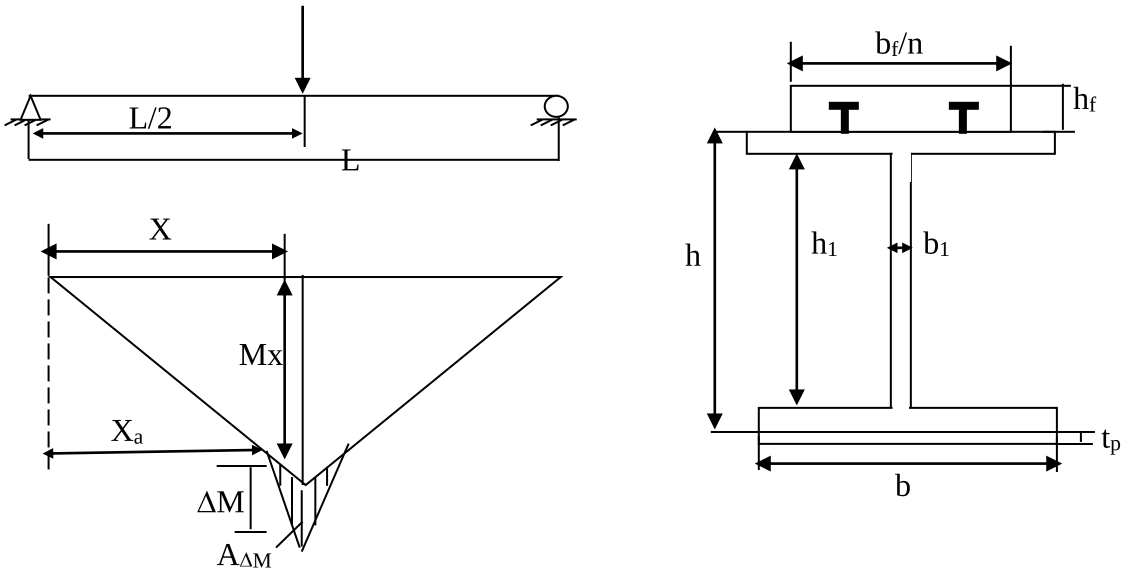

Fig. (3) Simply supported beam with additional moment (ΔM)

Graph analytical method is used to determine the deflection of a beam, the flexural displacement (Δ) is found by taking moment of $\frac{M}{EI_c}$ area between midspan and support about the support point. The total deflection of beam in elasto-plastic range can be expressed as (Dawood *et al.,* 2003):

$$\Delta_{total} = \Delta_{el} + \Delta\delta \qquad \text{...(7)}$$

When Δ_{el} = elastic deflection

$\Delta\delta$= additional deflection due to plastic zone.

Fig. (3) shows that the additional moment due to plastic zone causes an additional deflection ($\Delta\delta$). This deflection may expressed as:

$$\Delta\delta = \int_{xa}^{L/2} \frac{\Delta M_x \, dx}{EIc} \qquad \text{...(8)}$$

$\Delta M = M_F - M_x$ …(9)

Substitution of equation (3) and (6) into equation (9) yields:

$$\Delta M = \sigma_y Z_c \left[\frac{(1-\alpha)^2}{8} + \frac{f}{(1-\alpha)} - 1 \right] \quad \ldots(10)$$

Substituting equation (10) into equation (8) after integrating and simplifying the incremental plastic deformation can be obtained:

$$\Delta\delta = \frac{5\sigma_y Z_c L^2}{72EI_c} \left[\frac{(5\alpha + 3\alpha^2 - \alpha^3 - 7)}{8(1-\alpha)} + \left(\frac{f}{1-\alpha} \right) \right] \quad \ldots(11)$$

Or equation (11) can be written in terms of elastic section modulus of composite section S_c.

$$\Delta\delta = \frac{5\sigma_y S_c L^2}{72EI_c} \left[\frac{(5\alpha + 3\alpha^2 - \alpha^3 - 7)}{8f(1-\alpha)} + \left(\frac{1}{1-\alpha} \right) \right] \quad \ldots(12)$$

For simply supported composite beam loaded by concentrated load the elastic deflection is:

$$\Delta_{el} = \frac{\sigma_y S_c L^2}{12EI_c} \quad \ldots(13)$$

2-2 Continuous composite beam

Composite construction is of particular advantage economically when loads are heavy, spans are long. Composite bridges are generally economical for continuous spans greater than about 18.0m.(McCormack, 1981).

For calculation the properties of composite section in continuous beam, the reinforcing bars in the negative moment regions which are parallel to the steel girders and within the calculated effective width of the slab can be used.

Deflection (live load deflection) of continuous composite beam in elasto-plastic range can be found using the same principles indicated earlier.

$\Delta_{total} = \Delta_{el} + \Delta\delta$ …(7)

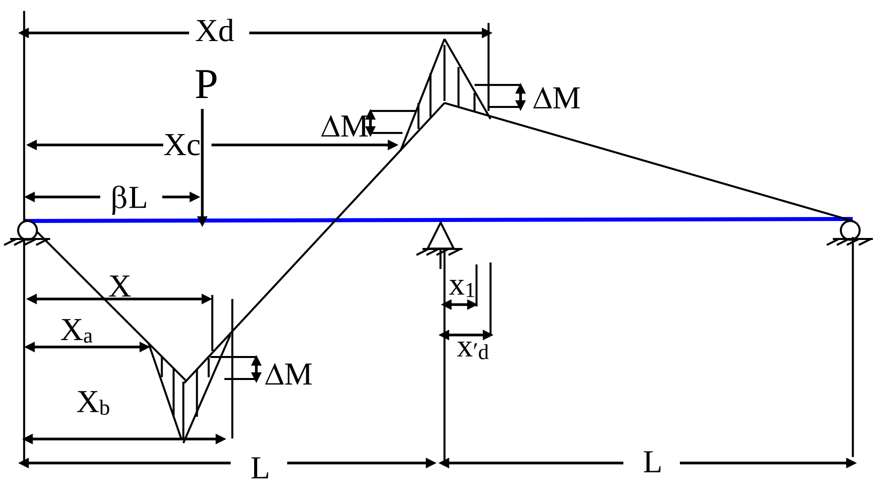

Fig. (4) Continuous beam with additional moment (ΔM)

Fig. (4) shows that the additional moment due to plastic zone causes an additional deflection ($\Delta\delta$). This deflection may expressed as:

$$\Delta\delta = -\frac{1}{EI_c}\int_{\beta L}^{xb}\Delta M(x-\beta L)dx + \frac{1}{EI_c}\int_{xc}^{L}\Delta M(x-\beta L)dx + \frac{(1-\beta)}{EI_c}\int_{0}^{x'd}\Delta M(L-x_1)dx_1 \qquad \ldots(14)$$

Where

$$Xa = \frac{2}{3}\beta L \qquad X_b = (5\beta+1)\frac{L}{6} \qquad X_c = (5+\beta)\frac{L}{6}$$

$$X'd = \frac{L}{3}$$

Substituting equation (10) into equation (14) and integrating for β=0.5 after simplifying, we get

$$\Delta\delta = \frac{0.17\sigma_y Z_c L^2}{EI_c} + \left[\left(\frac{5\alpha+3\alpha^2-\alpha^3-7}{8(1-\alpha)}\right)+\left(\frac{f}{1-\alpha}\right)\right] \qquad \ldots(15)$$

Or equation (15) can be written in terms of elastic section modulus of composite section S_c.

$$\Delta\delta = \frac{0.17\sigma_y S_c L^2}{EI_c} + \left[\left(\frac{5\alpha+3\alpha^2-\alpha^3-7}{8f(1-\alpha)}\right)+\left(\frac{1}{1-\alpha}\right)\right] \qquad \ldots(16)$$

3- The Relationship Between (α) and Applied Load

Most modern codes of practice permit the ultimate moment of resistance of composite beam to be determined from assumed rectangular plastic stress blocks (Al-Amery and Roberts, 1990).

The relationship between applied moment and full plastic moment for composite beam in elasto-plastic range can be found using the conditions of equilibrium. The bending moment at which cross-section is partially plastic is equal and opposite to the internal resisting moment .

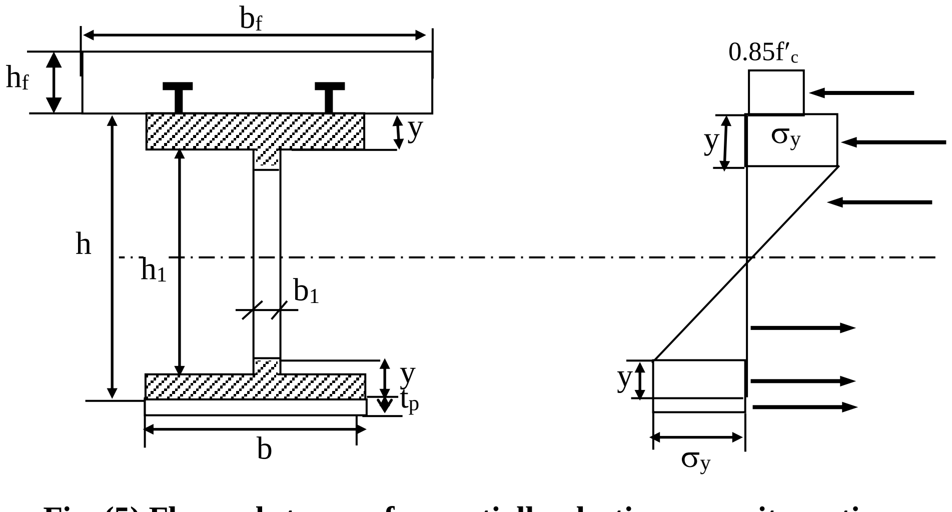

Fig. (5) Flexural stresses for partially plastic composite section.

Corresponding to the stress distribution of Fig. (5). The moment can be computed, if the stress distribution in the partially plastic condition is replaced by six equal and opposite statically equivalent concentrated force. The moment produced by this internal couple is

$$M = \sigma_y . y . b_1 (h - y) + \sigma_y (b - b_1)\left(\frac{h - h_1}{2}\right)\left[h - \frac{1}{2}(h - h_1)\right] +$$
$$\frac{\sigma_y}{2}\left(\frac{h}{2} - y\right) b_1 \frac{4}{3}\left(\frac{h}{2} - y\right) + 0.85 f'c b_f h_f \left(h + \frac{h_f}{2} + \frac{t_p}{2}\right) \quad \ldots(17)$$

Substituting eq. (4) into eq. (17) and simplifying

$$M = \sigma_y \left[\left(\frac{b - b_1}{4}\right)(h^2 - h_1^2) + b_1 h^2 \left(-\frac{1}{12}\alpha^2 + \frac{1}{6}\alpha + \frac{1}{6}\right)\right] +$$
$$+ 0.85 f_c' b_f h_f \left(h + \frac{h_f + t_p}{2}\right) \quad \ldots(18)$$

Full plastic moment of composite section may be expressed as:

$$M_p = \sigma_y \left[\left(\frac{b - b_1}{4}\right)(h^2 - h_1^2) + \frac{b_1 h^2}{4}\right] + 0.85 f_c' b_f h_f \left(h + \frac{h_f + t_p}{2}\right) \quad \ldots(19)$$

The ratio of applied moment to full plastic moments is given by:

$$\frac{M}{M_p} = \frac{\sigma_y \left[\left(\frac{b - b_1}{4}\right)(h^2 - h_1^2) + b_1 h^2 + \left(-\frac{1}{12}\alpha^2 + \frac{1}{6}\alpha + \frac{1}{6}\right)\right]}{\sigma_y \left[\left(\frac{b - b_1}{4}\right)(h^2 - h_1^2) + \frac{b_1 h^2}{4}\right]} \quad \ldots(20)$$

This ratio is shown in Fig. (6) for different value of α and for a given section

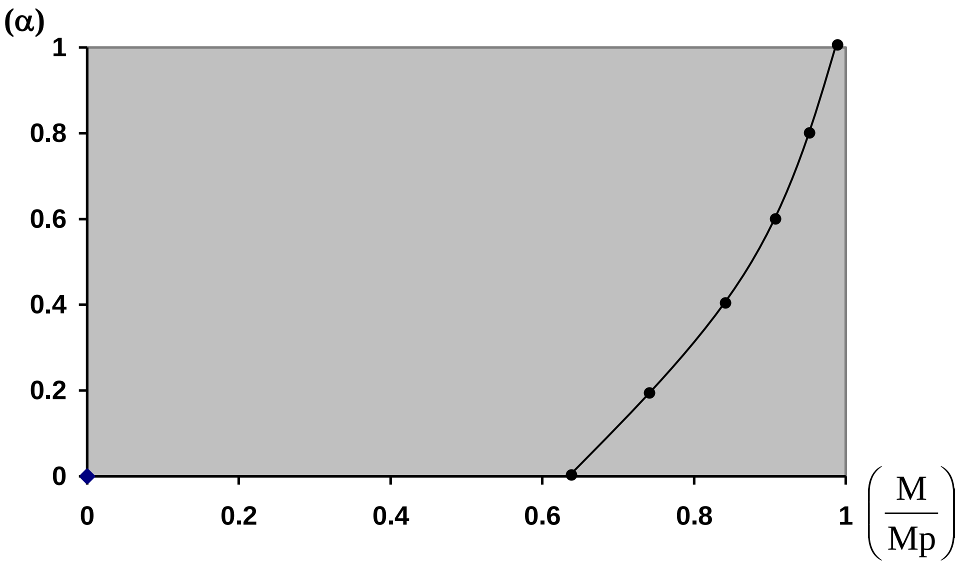

Fig. (6) The relationship between α and applied moment

4- Results and Comparison with F.D.M

To study the efficiency and the accuracy of the presented method a numerical example is selected to be solved using the presented formulas and the results of this example is compared with the "Finite Difference Method". A simply supported composite steel and concrete beam, having the cross-section dimensions shows in Fig. (7) is considered. This example was analyzed by Al-Amery and Roberts (1990) using the nonlinear finite difference method. A 9-m span simply supported composite beam was solved using 25 nodes along the length of the beam, concrete slab was divided into ten equal strips, flange and web of the steel beam were divided into four and ten equal strips respectively. A bilinear stress-strain curve is assumed for the steel as it shown in Fig. (1). The concrete is assumed to have no tensile strength and the ultimate compressive strain is limited to 0.0035. The cube strength of the concrete σ_{cu} and yield stress of the steel σ_y were taken as 30 and 280 N/mm^2 respectively.

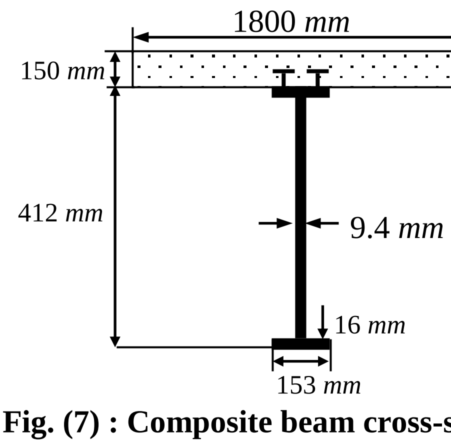

Fig. (7) : Composite beam cross-section dimension

Partial interaction is considered between concrete slab and steel beam by using pairs of 19-mm diameter, 100mm long, headed stud with a spacing of 240mm.

In the present study, the plastic moment (M_p) at collapse can be found from the plastic analysis of the beam. The elasto-plastic deflection is calculated for a loading stage $\frac{P}{P_u}$, for this stage of loading, (α) can be calculated from eq. (20) or from fig. (6). The total deflection will be calculated as follows:

$$\Delta_{total} = \Delta_{el} + \Delta\delta \qquad \ldots(7)$$

The results of the deflection at the beam center are shown in Tab.(1). From this Tab. and by comparing the results, it is clear that the results obtained by the presented method less than that obtained by F.D.M by about (9%) because in the present study the concrete below neutral axis is considered for deflection calculations. The partial composite action may be considered using AISC specifications (which permits an effective section modulus to be determined by the following expression.

$$S_{eff} = S_s + \frac{V'_h}{V_h}(S_c - S_s) \qquad \ldots(21)$$

Where (V_h) is the total shear to be resisted between the point of maximum positive moment in a simple span and the end of the beam.

(V'_h) is the total shear developed between the steel and the concrete on each side of the point of maximum moment.

Table (1). Comparison of (P-Δ) analysis by present method and F.D.M. (Al-Amery & Roberts , 1990)

P/Pu	F.D.M.	Present study
	Δ(mm)	Δ(mm)
0.53	30.2	28.6
0.66	39.7	36.5
0.8	59.4	54.8
0.93	172.7	157.6

Conclusion

A general formulation for the analysis of composite beam in elasto-plastic range has been developed. A comparison of result with the finite difference method indicates that reasonably accurate results can be obtained.

The proposed simple equations leads to less efforts and time in computation of deflection which makes the present equations represents a practical solution, and will be valuable to the design engineer. The numerical examples demonstrate that the yielded zone have extra effect on deflection of the beam, the total deflection of beam in elasto-plastic range increased about (35-45)%.

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