An Algorithm For Optimum Design Of Three Phase Core Type Distribution Transformer

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Abstract

The field of transformer design remains a very active field for researchers because of the large practical applications of transformers not only in Ac bulk power transmission and distribution but also in modern power converters. This research presents an algorithm, which design an optimum core type transformer. The proposed algorithm varies the dimensions of the window of the core by varying window height to window width ratio. For each ratio the program give a complete design and calculates the performance criteria. The algorithm adopts four vital criteria for evaluating each design. The optimum design is selected by compromising between these four criteria. The algorithm was tested with distribution transformers from the Iraqi distribution power system by selecting standard KVA and turn ratio used in this system. The result obtained suggests that the given design has a very high efficiency, an acceptable no load to full load current ratio, a copper to core loss ratio within the specified standard and a reasonable weight. From the results obtained its concluded that all optimum designs are achieved with relatively low window height / width ratio.

الخلاصة

يتناول هذا البحث بناء برنامج للتصميم الامثل للمحولات ثلاثية الأطوار قلبية الشكل. يمتاز مجال تصميم المحولات برغبة واسـعة من قبل الباحثين لما تمتاز به المحولات من تطبيقات واسعة في عدة مجالات . يقوم البرنامج المقترح في هذا البحث بتغيير أبعاد القلب الحديدي للمحولة من خلال تغيير نسبة ارتفاع النافذة إلى عرضها داخل القلب الحديدي للمحولة. لكل قيمة من هذه النسبة يقوم البرنامج بأجراء تصميم للمحولة وحساب معايير الأداء.لتقييم التصميم يقوم البرنامج باحتساب أربعة معايير. تم تطبيـق البرنـامج المقترح على تصميم محولات ذات مقننات قياسية من نظام التوزيع لمنظومة القدرة العراقية. تظهر النتائج بان التصميم المثالي له كفاءة عالية و نسبة تيار اللاحمل إلى تيار الحمل التام ملاءمة وكذلك نسبة الخسائر النحاسية إلى الخسائر الحديدية تقع ضمن الحد المقرر للتصميم الكفوء.

List Of Symbols

nol: - number of legs in the core.
E_t: - voltage per turn.
A_i: - cross sectional area of the core.
f: - Rated frequency.
K:-iron space factor.
K_S:-stacking factor.
B_{Max}: - maximum flux density.
d: - diameter of circumscribing of core.
A_W: - window area.
K_W: - window space factor.
N_{lv} : - number of turns per phase of low voltage winding.
N_{hv} : - number of turns per phase of high voltage winding.
a_{lv} : - cross sectional area of low voltage winding.
a_{hv}: - cross sectional area of high voltage winding.
ρ_1: - resistivity at room temperature.
ρ_2: - resistivity at operating temperature.
t_2: - operating temperature.
R_{lv}: - resistance per phase of low voltage winding.

$R_{hv:}$ - resistance per phase of high voltage winding.
d_{hv} :- diameter of disc conductor for high voltage winding.
P_c: - core loss of transformer.
$V_{L:\,-}$ line voltage.
I_o: -no load current of transformer.
I_C: - working (active) component of no-load current.
$I\mu$: - magnetizing (reactive) component of no-load current.
K_t: - transformation ratio.
$I_{FL/lv}$: - full load current of transformer referred to low voltage side.
$I_{FL/hv}$: - full load current of transformer referred to high voltage side.
$P_{Cu/ph}$: - total copper losses per phase of designed transformer.
P_{Cut}: - total copper losses of designed transformer.
CR: - current ratio.
LR: - loss ratio.
N: - fraction of full load KVA.
S: - rated KVA.
pf: - power factor of load.
η: - efficiency of designed transformer.

1-Introdution

The procedure of designing electrical equipment requires extensive effort .The design procedure is frequently modified or adapted to meet the desired requirements.

The transformer is a device that works up on the theory of electromagnetic induction, namely faraday's law of electromagnetic induction and mutual induction. However, a complete design procedure requires knowledge of not only electromagnetism, magnetic circuit analysis, electric circuit analysis, but also power loss of electric and magnetic circuits and heat transfer. Power and distribution transformers are one of the main elements in bulk ac power transmission and distribution, hence the research of transformer design remain a very interesting and active field.

(Rubaai 1994) describes a single-phase transformer design suitable only for classroom use. The scope of the design is limited to the specification of core and coils of transformer. The paper does not present mathematical relation ships employed in design.

(Basak *et al* 1994) presented a design procedure for a transformer, by computing the core losses. The paper suggests a package that has been developed using finite elements method to compute flux density and hence loss distribution in transformer

(Ward 1990) presents a simple program of single-phase transformer design. The program is used to implement simple laboratory experiments such as open and short tests. The calculations are used in laboratory sessions only.

In this research, the basic equations for the design of power and distribution transformers are presented. The algorithm in this research is intended to give optimum transformer design from the point of view of efficiency, copper loss to iron loss ratio, no load to full load current ratio and weight. In addition, coil particulars of low and high voltage windings are determined.

2-Basic Equation For Design of Transformer

The design of electric transformer is broadly split in to two categories, design of core (all relevant dimension of the core), and the design of low voltage and high voltage windings.

2-1 Core design

The design of the core means finding all relevant dimensions of the core. In general, the core is constructed from thin strips (usually about 0.35 mm). Arrangement of the strips is commonly done in steps to give approximately a round cross sectional area. This will reduce the amount of useful space that is wasted compared to square cores. The number of steps usually chosen is 3, 4,5,6,7, or 9. For large size transformer, more steps may be used. Various steps of core and largest width of the core are shown in fig 1.

The design of the core begins by selecting the maximum flux density (wb /m^2) and current density δ (A/mm^2). Then the voltage per turn is calculated using the following formula ((Deshpande, 1983),

$$Et = \frac{\sqrt{\frac{S \times 1000}{nol}}}{40} \quad \text{-------------------------------------} \quad (1)$$

Normally for three phases, nol is equal to three.

To calculate the cross sectional area of the core (Ai),

$$Ai = \frac{E_t}{4.44 \times B_{max} \times f} \quad \text{-------------------------------------} \quad (2)$$

The diameter of the circumscribing circle for the core (d) is found as follows,

$$Ai = k \times k_s \times \frac{\pi \times d^2}{4} \quad \text{-------------------------------------} \quad (3)$$

The value of K is given in table (1) below.

Particulars of each window in the core is calculated as follows,

For window area (Aw),

$$S = 3.33 \times Ai \times Aw \times K_w \times \delta \times B_{max} \times f \times 10^{-3} \text{----------------} \quad (4)$$

The value of K_w is given in table (2).

$$Aw = \frac{S}{3.33 \times Ai \times Aw \times K_w \times \delta \times B_{max} \times f \times 10^{-3}} \quad \text{-------------------} \quad (5)$$

The ratio of height of window to width of window is taken from 2 to 6.

A complete lay out of the core with various dimensions are shown in fig (2).

Table (1) Iron space factor for typical number of steps in core

frame	Ducts	Core diameter	No of steps	Iron space factor
Single	Nil	Less than 100mm	1	0.64
			2	0.79
			3	0.84
			4	0.87
		150-100mm	5	0.88
		250-300mm	6	0.89
	Longitudinal ducts	350-750mm	6-8	0.86
Double framed	Longitudinal and cross ducts	550-1000mm	7-10	0.88

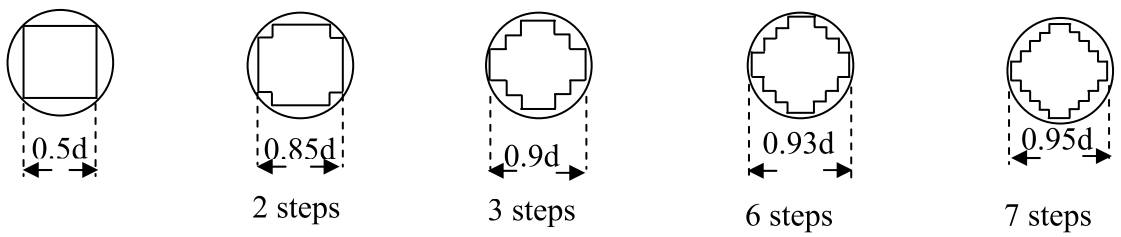

Fig. (1) core of transformer with various steps

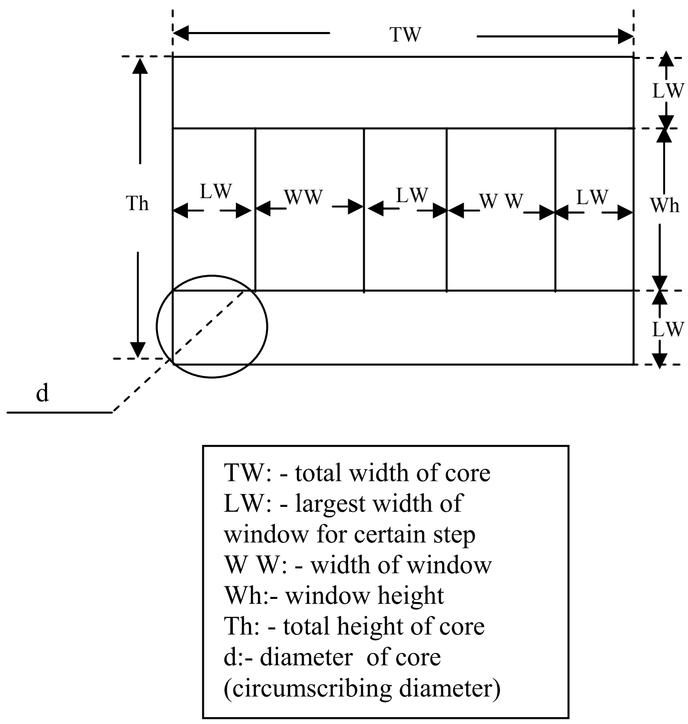

Fig. (2) core lay out of transformer

Table (2) window space factor

KVA	3 KV	10 KV	30 KV	100 KV
100	0.28	0.2	0.14	-
800	0.37	0.27	0.2	0.15
2000	0.4	0.31	0.23	0.16
10000	0.45	0.37	0.28	0.21

2-2 Design of Windings

The design of windings includes low voltage (lv) and high voltage (hv) windings.

2-2-1 Design of low voltage winding

The design begins by calculating the number of turns of lv winding per phase,

$$N_{lv} = \frac{V_{lv}}{E_t} \quad \text{---} \quad (6)$$

Then the cross section area of conductor for the winding is calculated

$$a_{lv} = \frac{I_{FL/lv}}{\delta} \quad \text{---} \quad (7)$$

The number of layers for lv winding is decided according to the type of winding say (n_{lv}). In addition, the designer decides the size of the conductor and shape of conductor rectangular, disc, ---ect. In this research we will select rectangular shape for lv side and disc shape for hv side. These two selections are the most common shapes in practical situations For rectangular conductor of dimension $L_{lv} \times W_{lv}$. Then height of lv winding in window is,

Height of lv in core window = $n_{lv} \times (W_{lv}$+Thickness of paper insulation) ------ (8)

Thickness per layer of lv coil = [2(L_{lv} +Thickness of paper insulation)] -------- (9)

Total thickness of lv coil =2×thickness per layer of lv coil -------------------- (10)

The inside diameter of lv coil =d+ (2×X) ---------------------------------- (11)

Where
X:-distance between core and lv coil.

Out side diameter of lv windings =d+ (2×total thickness of lv coil) ---------- (12)

Mean diameter of lv coil =d+ total thickness of lv coil --------------------- (13)

Mean length of turn of lv coil =π × mean diameter of lv coil ---------------- (14)

The resistance of lv winding, if specific resistance at a room temperature of 20 c^0 (say t_1) and the operating temperature is t_2 then the specific resistance at t_2 is,

$$\rho_2 = \rho_1 \times (1 + \alpha_o \times (t_2 - t_1)) \quad \text{-----------------------------------} \quad (15)$$

Then resistance of lv winding per phase is,

$$R_{lv} = \rho_2 \times \frac{\text{mean length of turn of lv} \times N_{lv}}{a_{lv}} \quad \text{--------------------------------} (16)$$

2-2-2 Design of high voltage winding

Once again, the design begins by calculating the number of turns in high voltage winding per phase,

$$N_{hv} = \frac{V_{hv}}{Et} \quad \text{--} (17)$$

The cross sectional area of hv conductor is calculated as,

$$a_{hv} = \frac{I_{FL/hv}}{\delta} \quad \text{--} (18)$$

The shape of high voltage winding is decided so as the number of layers (n_{hv}). From accommodation of winding point of view, it is better to choose disc coil, with diameter (d_{hv}). Then d_{hv} is calculated as follows,

$$d_{hv} = \frac{\sqrt{4 \times a_{hv}}}{\pi} \quad \text{--} (19)$$

Then

Inside diameter of h v winding =out side diameter of l v winding + (2×Y) ---(20)

Where

Y: - the distance (clearance) between lv and hv windings.

Since paper insulation on conductor is used, then total diameter of hv winding and paper insulation is,

Total diameter of hv winding =d_{hv} + thickness of paper insulation --------- (21)

Thickness of each coil in hv winding =n_{hv} × total diameter of hv winding ---(22)

Out side diameter of hv coil =inside diameter of h v winding + (2×thickness of each coil in h v winding) --- (23)

Mean diameter of hv coil =inside diameter of hv winding +thickness of each coil in hv winding --- (24)

Mean length of turn of hv winding =π× mean diameter of hv coil -------- (25)

Finally, the resistance per phase is calculated,

$$R_{hv} = \rho_2 \times \frac{\text{mean length of turn for hv} \times N_{hv}}{a_{hv}} \quad \text{-----------------------} (26)$$

3-Criteria For Judging The Performance of The Designed Transformer

In this research, three vital criteria are adopted. The proposed algorithm calculates the criteria below to judge the performance of each design. The criteria are given below with relevant calculations:-

3-1 No-load current to full load current ratio

The criteria state that the ratio of no-load to full load current should be about 2% (Rubaai, 1994). The proposed algorithm calculates the no-load current for each design as follows:-
The volume of core and yoke is first calculated,

$$\text{Volume} = A_i\,[2 \times \text{total width of the core} + 3 \times \text{height of window}] \qquad (27)$$

The weight of core and yoke can now be calculated for the designed transformer.

$$\text{Weight of core and yoke} = \text{weight of iron} \times \text{volume} \qquad (28)$$

Where
Weight of iron $= 7.85 \times 1000$ kg/m^3

From magnetic properties of the core material for a certain maximum flux density the core loss per Kg is known then, the core losses (P_c) of the designed transformer can be calculated as follows,

$$P_c = \text{core loss per Kg} \times \text{weight of core and yoke} \qquad (29)$$

Core loss per Kg =2.4 w /Kg (Rubaai, 1994)

The active component of no-load current (Ic) is calculated

$$P_c = \sqrt{3}\,V_L I_o \cos\varphi_o \qquad (30)$$

Moreover, the working component is,

$$I_c = I_o \cos\varphi_o \qquad (31)$$

To calculate the magnetizing component of the no-load current (I_μ), the algorithm must have as an input the magnetizing V A first the VA/Kg can be known from the curve properties of the core for the designed transformer. Then total magnetizing VA is
Magnetizing VA=(VA/Kg)×total weight of core & yoke (32)

Where
VA/Kg = 10 (Deshpande, 1983)
Then

$$I_\mu = \frac{\text{magnitizing VA}}{\sqrt{3} \times V_L} \qquad (33)$$

The no-load current is

$$I_o = \sqrt{I_c + I_\mu} \quad \text{-------} \quad (34)$$

Finally the no-load /full load current ratio is,

$$CR = \frac{I_o}{I_{FL}} \quad \text{-------} \quad (35)$$

3-2 Copper to core loss ratio

The algorithm at this stage calculates the total copper losses of the designed transformer. The copper losses are calculated as follows:-

First, resistance of hv winding is referred to lv side and accumulated with resistance of lv side, so the equivalent resistance per phase is,

$$R_{lveq/ph} = K_t^2 R_{hv} + R_{lv} \quad \text{-------} \quad (36)$$

The total copper losses per phase is,

$$P_{Cu/ph} = I^2_{FL/lv} \times R_{lveq/ph} \quad \text{-------} \quad (37)$$

And total copper losses of the designed transformer is,

$$P_{Cut} = 3 \times P_{Cu/ph} \quad \text{-------} \quad (38)$$

Then the copper /core loss ratio is,

$$LR = \frac{P_{Cut}}{P_C} \quad \text{-------} \quad (39)$$

3-3 Efficiency of the designed transformer

Finally, the proposed algorithm calculates the efficiency of the transformer. The algorithm calculates the efficiency for 50%, 75 %, and 100 % of full load. The efficiency is calculated as follows,

$$\eta_{(N)} = \frac{N \times S \times pf}{N \times S \times pf + N^2 \times P_{CUt} + P_c} \times 100\% \quad \text{-------} \quad (40)$$

4- The Proposed Algorithm

The proposed algorithm, which is used to find optimal design of transformer, can be divided in to the following stages:-

4-1 Stage one

This stage involves entering all required specification and data for the design. This data includes-

1. KVA rating of the designed transformer
2. Rated voltage for hv side
3. Rated voltage for lv side
4. Rated frequency

5. Number of legs for the core of transformer (usually three for three-phase transformer)
6. Watt per Kg loss of core material
7. Magnetizing VA/Kg of core
8. Current density
9. Maximum flux density
10. Iron space factor
11. Stacking factor

4-2 Stage two

This stage involves the calculations of core particulars. These particulars are calculated according to the equations in section (2-1) of this research.

4-3 Stage three

In this stage, the algorithm calculates lv and hv winding particular. The entire variable related to lv and hv winding are determined. These variables are presented in sections (2-2) and (2-3) respectively

4-4 Stage four

At this stage the program evaluates the performance of the designed transformer. The no- load to full load current ratio, copper to core losses ratio and efficiency at different loading conditions is calculated.

4-5 Stage five

The final stage involves calculation of total weight of the designed transformer. The calculations are performed as follows,

Total transformer weight =weight of core and yoke +weight of copper---------

The calculation of weight of core and yoke is as presented in section (3-1)

Weight of copper per phase =weight of lv winding +weight of hv winding----(42)

Weight of lv winding = density of copper $\times a_{lv} \times N_{lv}$ ×mean length of turn for lv winding---(43)

And

Weight of hv winding =density of copper $\times a_{hv} \times N_{hv}$ ×mean length of turn for hv winding --- (44)

Where
density of copper =8.89 g/cm^3

Then

Total weight of copper =3×weight of copper per phase -------------------- (45)

5- Implementation of The Program

The program was implemented using MAT LAB 7.0. The ratio of height to width of the window of the core is varied from 2 to 6 and the design is repeated until an optimal design is found. The program was used to design distribution transformer with standards from the Iraqi power distribution system. The standards for distribution transformer are Delta/star and 11/0.4 KV (voltage rating). The KVA ratings are 250 KVA, 400 KVA, 630 KVA, and 1000 KVA. The number of steps used for the core in this research are seven steps.

The algorithm designs each of the above transformers with a variable window height to window width ratio (2 to 6). Flow chart of the proposed algorithm is shown in fig (3)

6- Results

The output from the program is six possible designs for each KVA. Each design has a different window height to window width ratio. This ratio has values of 2, 3, 4, 5, and 6. The program calculates the current ratio, loss ratio, efficiency, and total weight of the transformer. Tables (3-6) shows the result for each KVA. Fig (4-5) shows variation of current ratio and loss ratio for various window heights to window width ratios for 250 KVA transformer. Moreover fig (6-7) shows the variations for 400 KVA transformer, fig (8-9) are for 630 KVA transformer and fig (10-11) are for 1000 KVA transformer

Its clear that for 250 KVA transformer design No (1) is the optimum design. While for 400 KVA transformer design, No (2) is the optimum. For 630 KVA design No (1) and for 1000 KVA design No (1) is the optimum.

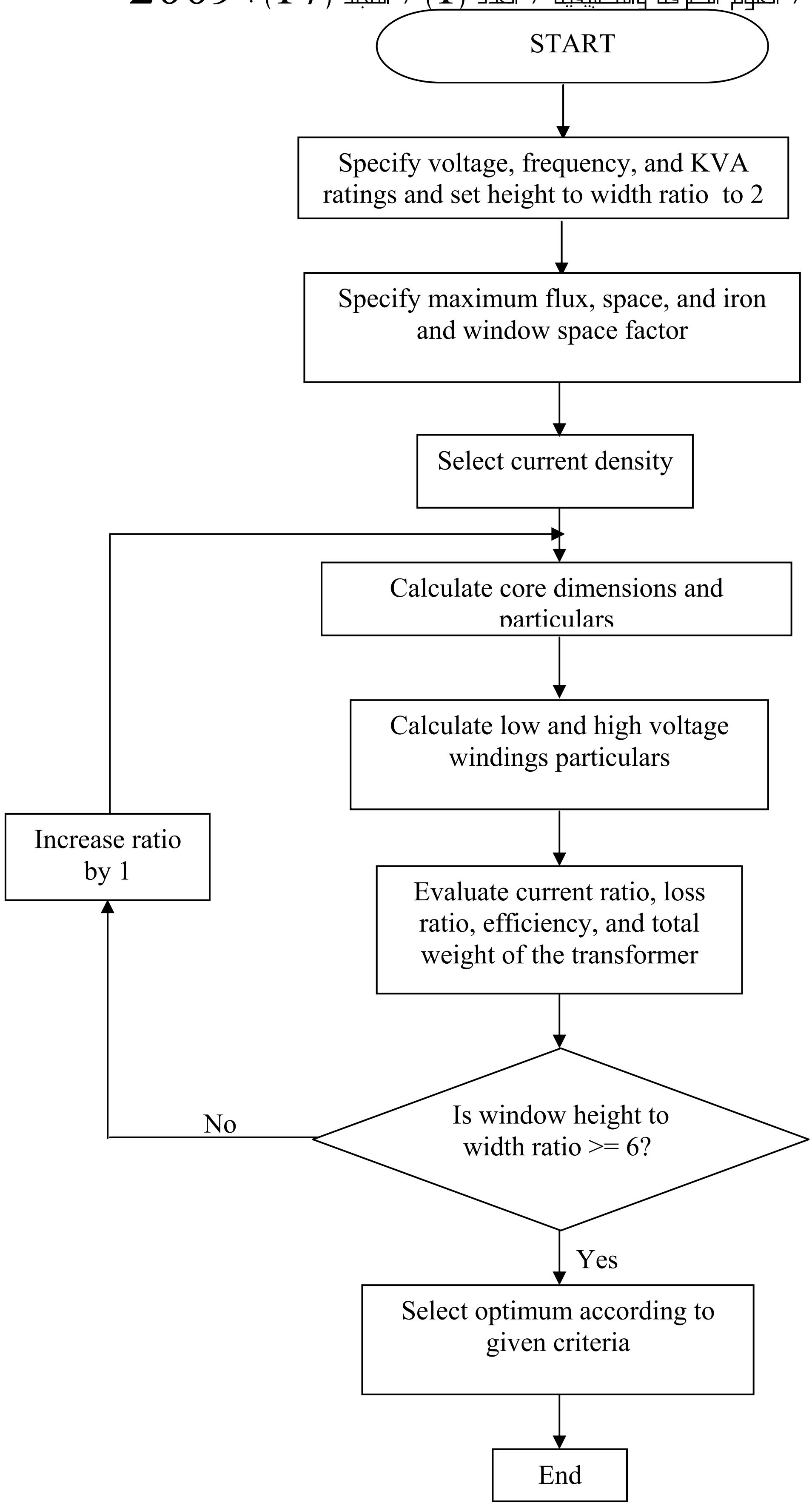

Fig. (3) flow chart of the proposed algorithm

Table (3) Results for 250 KVA

Design No	Window height to width ratio	CR	LR	η (50% FL)	η (75% FL)	η (100% FL)	Total weight (Kg)
1	2	1.7447	1.9363	98.78%	98.84%	98.76%	641.414
2	3	2.066	1.6351	98.63%	98.74%	98.7%	719.53
3	4	2.3873	1.415	98.49%	98.64%	98.62%	797.64
4	5	2.7087	1.2471	98.34%	98.55%	98.55%	875.763
5	6	3.03	1.1149	98.2%	98.45%	98.47%	953.88

Table (4) Results for 400 KVA

Design No	Window height to width ratio	CR	LR	η (50% FL)	η (75% FL)	η (100% FL)	Total weight (Kg)
1	2	1.545	1.8949	98.93%	98.98%	98.92%	902.26
2	3	1.830	1.5992	98.79%	98.89%	98.95%	1013
3	4	2.116	1.3833	98.66%	98.81%	98.79%	1124.5
4	5	2.402	1.2188	98.54%	98.72%	98.72%	1235.7
5	6	2.687	1.0892	98.41%	98.63%	98.66%	1346.8

Table (5) Results for 630 KVA

Design No	Window height to width ratio	CR	LR	η (50% FL)	η (75% FL)	η (100% FL)	Total weight (Kg)
1	2	1.374	1.859	99.05%	99.1%	99.05%	1256.1
2	3	1.629	1.568	98.93%	99.02%	98.99%	1412.4
3	4	1.884	1.3558	98.82%	98.95%	98.93%	1568.6
4	5	2.139	1.1942	98.7%	98.87%	98.87%	1724.8
5	6	2.394	1.067	98.58%	98.79%	98.82%	1881.1

Table (6) Results for 1000 KVA

Design No	Window height to width ratio	CR	LR	η (50% FL)	η (75% FL)	η (100% FL)	Total weight (Kg)
1	2	1.220	1.8262	99.16%	99.21%	99.17%	1760.5
2	3	1.447	1.5396	98.05%	99.14%	99.11%	1981.5
3	4	1.675	1.3308	98.95%	99.07%	99.06%	2202.3
4	5	1.902	1.1718	98.85%	99%	99.01%	2423.3
5	6	2.129	1.0468	98.74%	98.93%	98.96%	2644.2

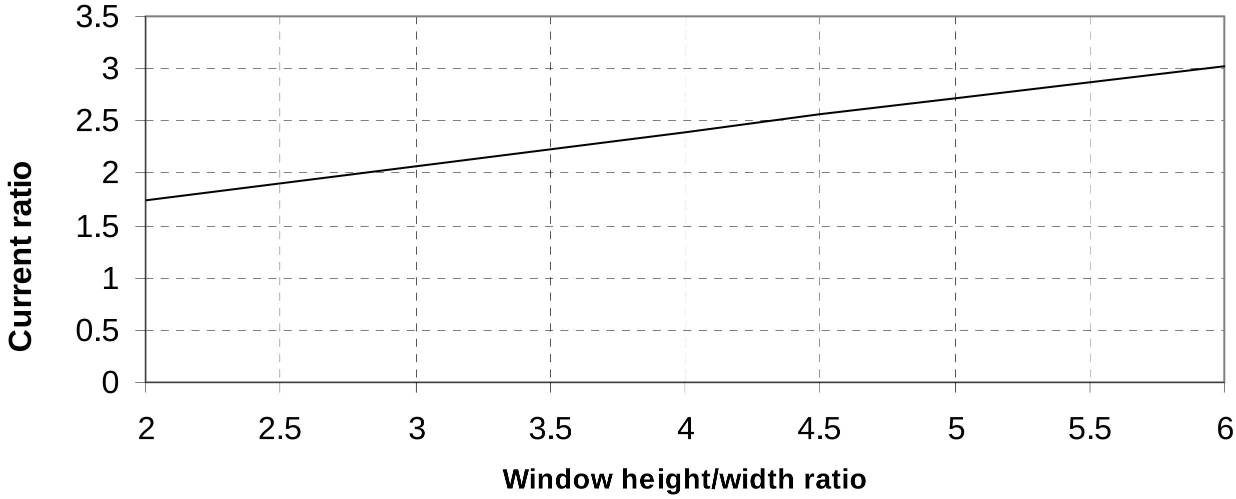

Fig. (4) current ratio verses window height /width ratio for 250 KVA

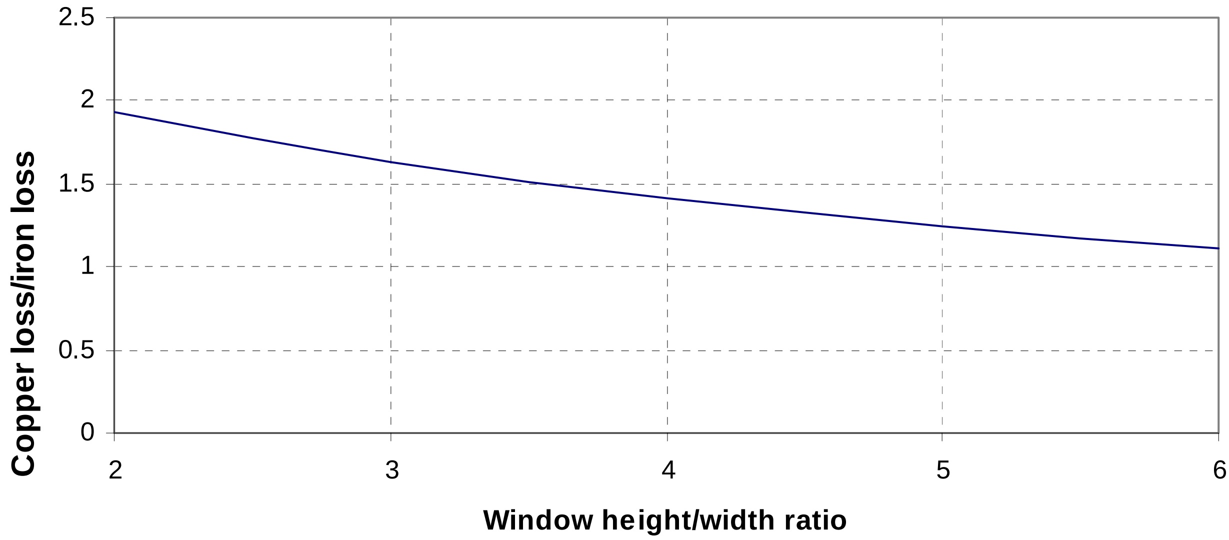

Fig. (5) loss ratio verses window height /width ratio for 250 KVA

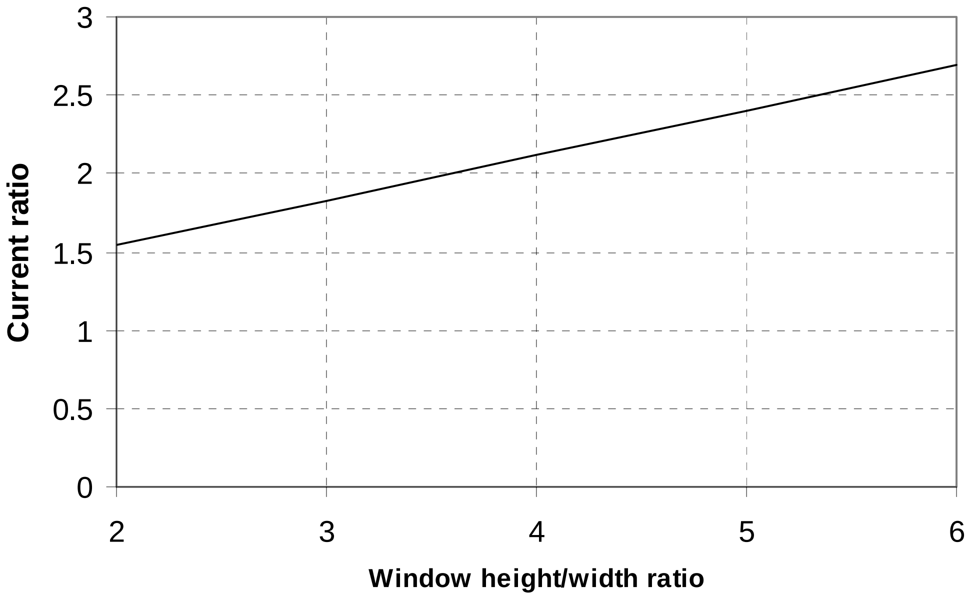

Fig.(6) current ratio verses window height /width ratio for 400 KVA

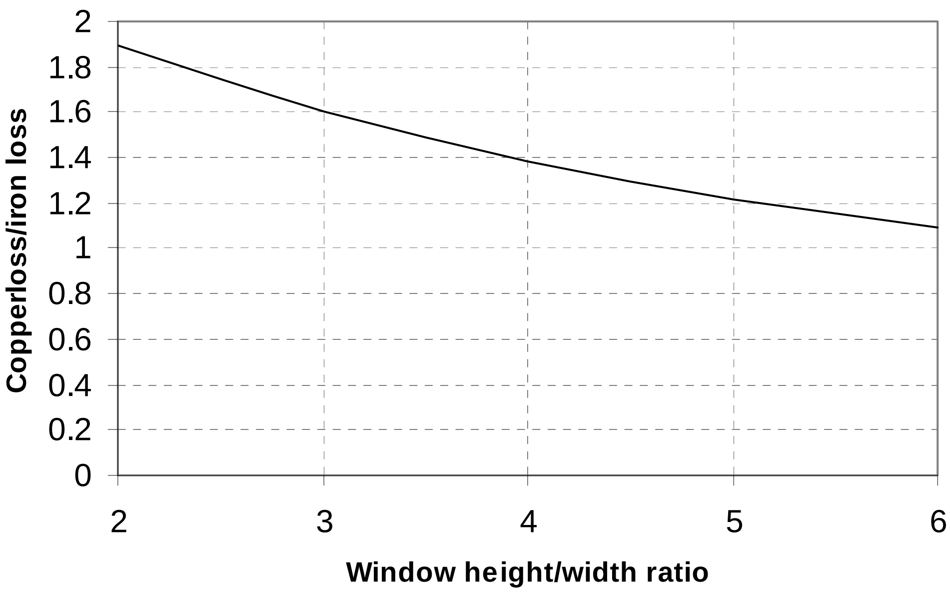

Fig. (7) loss ratio verses window height /width ratio for 400 KVA

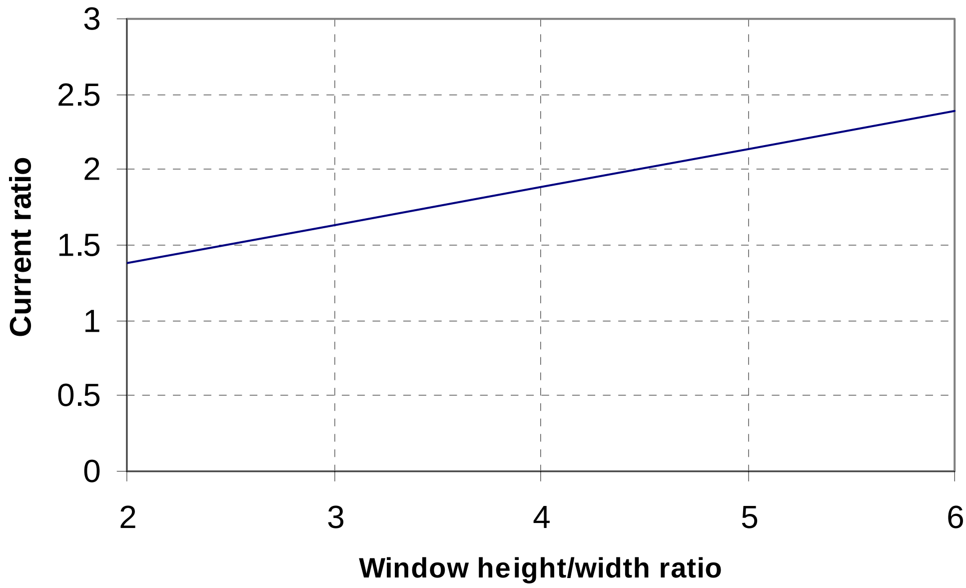

Fig. (8) current ratio verses window height /width ratio for 630 KVA

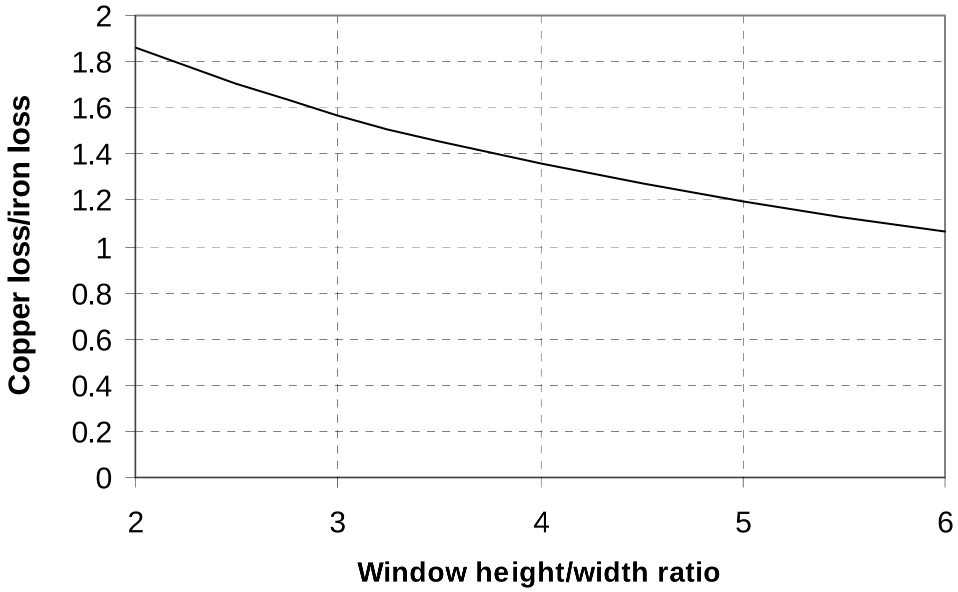

Fig. (9) loss ratio verses window height /width ratio for 630 KVA

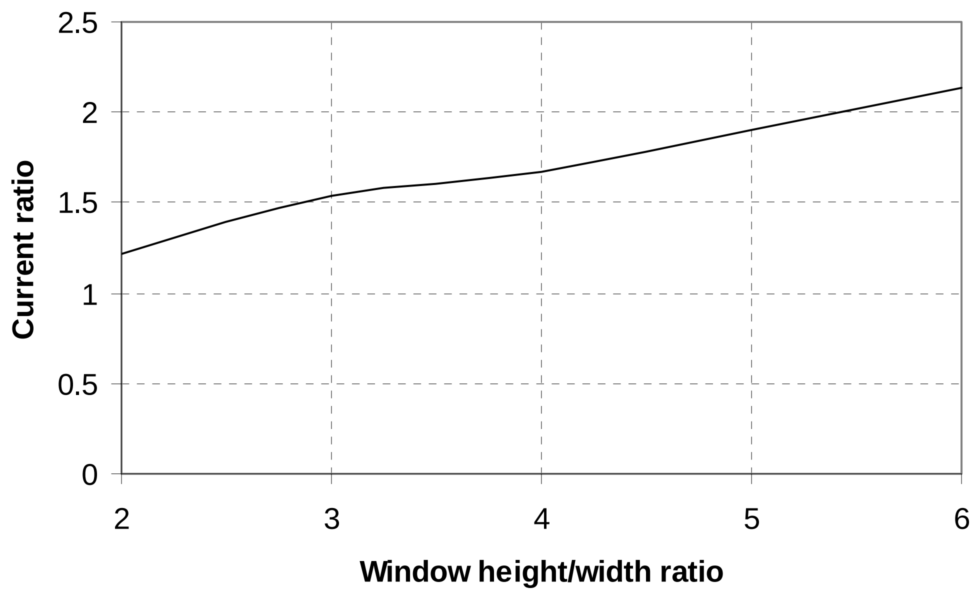

Fig. (10) current ratio verses window height /width ratio for 1000 KVA

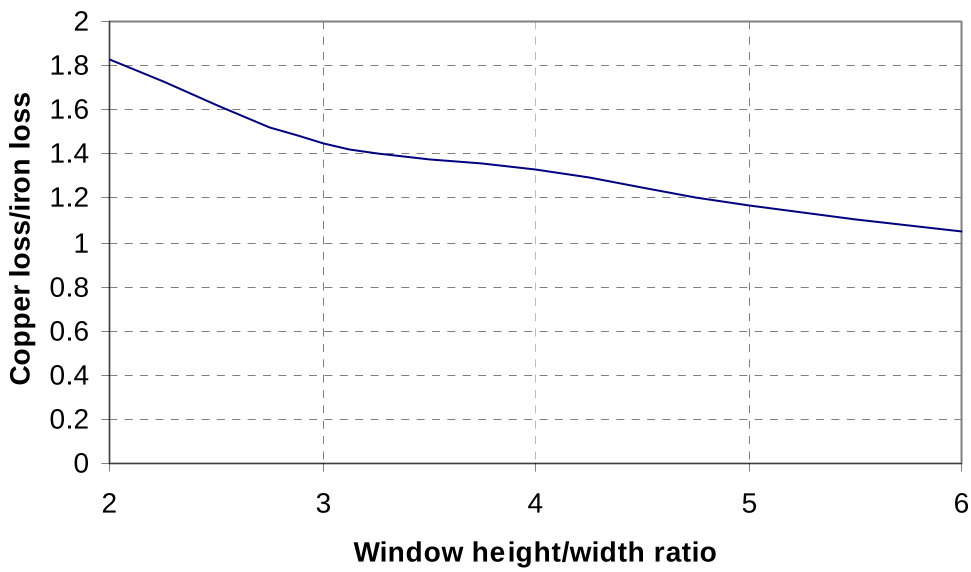

Fig. (11) loss ratio verses window height /width ratio for 1000 KVA

7- Conclusions

From the results obtained in this research, the following conclusions can be made:-

1. An algorithm is proposed to design a complete three-phase distribution transformer.
2. The program gives several alternative designs for each required transformer.
3. The presented algorithm is applied to standard distribution transformers used in the Iraqi power system.
4. The proposed algorithm tests the validity of each design by calculating no load to full load current ratio, copper to core losses ratio, efficiency for various loading conditions and total weight of transformer.
5. Each design, for every given KVA has a very high efficiency, an acceptable current ratio, and a very reasonable loss ratio.
6. Its clear that CR increases with an increase in window height /width ratio while LR decrease.
7. All optimum designs are achieved for values of window height /width ratio ranging from 2 to 3.
8. For values of window height /width ratio from 2 to 3, the corresponding design has a very reasonable weight.
9. The presented algorithm can be used to design power transformers.
10. The algorithm can be adapted to design power and distribution transformers of shell type structure.
11. The proposed program can be used by transformer manufacturers and companies.

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