Separation Axioms and δ- Open Sets in Bitopological Spaces

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Abstract

In this paper we study especial cases of separation axioms in bitopological spaces by considering δ-open sets and prove some results about them comparing with similar cases in topological spaces.

Keywords

Bitopological space (B.S) , δ-open set , pre open set , δ-T_i B.S (i=0,$\frac{1}{2}$,1,2,2$\frac{1}{2}$,3,4) , δ-regular , δ- normal B.S .

الخلاصة

في هذا البحث درسنا حالة خاصة لبديهيات الفصل في الفضاءات ثنائية التوبولوجيا بالاستفادة من المجموعات δ المفتوحة واثبتنا بعض النتائج بالمقارنة مع الحالات المشابهة في الفضاءات التوبولوجية

1.Introduction

The study of bitopological spaces was initiated by (Kelly, 1963).

A triple X,τ,Ω is called bitopological space if X,τ and X,Ω are two topological spaces.

This notion was studied in different senses ,one of these is the δ-open sets ,that suggested by (Jaleel, 2003).We have study especial case of connectedness and especial case of compactness in bitopological spaces in the sense of δ-open sets in (Alhosaini, 2007; Alswidi and Alhosaini, 2007).

Other sense in bitopological spaces ,is the pre open sets that was suggested by Abdul (Raaof , 2005).In this paper we first (in section 2) introduce a comparable studying between

δ-open and pre open sets ,and then (in section 3) we study especial case of separation axioms in bitopological spaces in the sense of δ-open sets.

2.pre open sets and δ- open sets

2.1 Definition : Let X,τ and X,Ω be two topological spaces then X,τ,Ω is called a bitopological space (B.S). A subset A in X,τ,Ω is called pre open set if $A\subseteq\tau$-int(Ω-clA). The set of all pre open sets in X,τ,Ω is denoted by pr-o(X), (Raaof, 2005).
A subset A in X,τ,Ω is called δ-open set if $A\subseteq\tau$-int(Ω-cl(τ-intA)).The set of all δ-open sets in X,τ,Ω is denoted by δ-o(X), (Jaleel, 2003)..

2.2 Remark : In general pr-o(X) and δ-o(X) do not form a topology on X. In fact ,the union of any family of elements in pr-o(X) (δ-o(X)) is an element of pr-o(X) (δ-o(X)),but the intersection of any two elements of pr-o(X) (δ-o(X)) need not be an element of pr-o(X) (δ-o(X)), (Jaleel, 2003; Raaof , 2005).Of course X and ϕ are always elements of pr-o(X) (δ-o(X)).

2.3 Remark : If X, τ, Ω is a B.S, then $\tau \subseteq \delta$-o(X) $\subseteq$ pr-o(X).

proof: A $\in \tau$ implies τ-intA=A,A $\subseteq \Omega$-clA implies τ-intA $\subseteq \tau$-int(Ω-clA) so A $\in \tau$ implies A $\subseteq \tau$-int(Ω-clA)= τ- int(Ω-cl(τ-intA)) i.e $\tau \subseteq \delta$-o(X) and $\tau \subseteq$ pr-o(X).Now since τ-intA $\subseteq$ A so, τ- int(Ω-cl(τ-intA)) $\subseteq \tau$- int(Ω-clA) which implies ,if A $\subseteq \tau$- int(Ω-cl(τ-intA)) then A $\subseteq \tau$- int(Ω-clA) i.e δ-o(X) $\subseteq$ pr-o(X).

2.4 Remark : A necessary condition for a non empty set A to be δ-open set is τ-int A $\neq \phi$,(Jaleel, 2003). This is not true for pr open sets, see the following example.

2.5 Example : X= $\{a,b,c\}$, $\tau = \{X, \phi, \{a\}, \{b\}, \{a,b\}\}$, $\Omega = \{X, \phi, \{c\}\}$ then δ-o(X)= τ but pr-o(X)= $\tau \cup \{\{c\}, \{a,c\}, \{b,c\}\}$ .Not that τ-int $\{c\}$ = ϕ.

2.6 Remark : In X, I, Ω ,where I is the indiscrete and Ω is any topology on X, δ-o(X)=I but pr-o(X) may contain subsets other than X and ϕ.

Proof: The first part follows from Remark 2.4 and the second is shown in the following example.

2.7 Example: X=$\{a,b,c,d\}$,I=$\{X, \phi\}$, $\Omega = \{X, \phi, \{a\}\}$,then δ-o(X)=I but pr-o(X)=$\{X, \phi, \{a\}, \{a,b\} \{a,c\}, \{a,d\}, \{a,b,c\}, \{a,b,d\}\}$.

2.8 Remark : If D is the discrete and Ω is any topology on X, then in X,D,Ω we have δ-o(X)= pr-o(X)=D.

proof: It follows from 2.3.

2.9 Remark : If τ is any topology and D is the discrete topology on X , then in X,τ ,D we have; δ-o(X)= pr-o(X)= τ .

Proof: It follows from the facts 1) D-cl A=A, 2) D-cl(τ-int A) = τ-int A ,and 3) A $\subseteq \tau$-int A if and only if A $\in \tau$.

2.10 Remark : If τ is any topology and I is the indiscrete topology on X ,then in X,τ ,I we have pr-o(X)= P(X) ,set of all subsets of X, but δ-o(X) need not equal P(X).

2.11 Example : X=$\{a,b,c\}$, $\tau = \{X, \phi, \{a\}\}$, I=$\{X, \phi\}$,then pr-o(X)= P(X), where δ-o(X)= $\{X, \phi, \{a\}, \{a,b\}, \{a,c\}\}$.

2.12 Remark : If I is the indiscrete and τ is any topology on X , then in X ,τ ,I we have δ-o(X)=$\{A \subseteq X \mid A \text{ contains some non empty } \tau\text{-open set}\} \cup \{\phi\}$.

2.13 Remark : If I is the indiscrete and Ω any topology on X , then in X , I , Ω we have: a non empty subset A of X is pre open if and only if Ω-cl A=X i.e pr-o(X)= $\{A \subseteq X \mid A \text{ is } \Omega\text{-dense subset of } X\} \cup \{\phi\}$.

The following table summarize the above Remarks;

Case	first topology	second topology	pr-o(X)	δ-o(X)
1	D	Ω	D	D
2	τ	D	τ	τ
3	I	Ω	not fixed	I
4	τ	I	D	not fixed

where I is the indiscrete , D ,the discrete , τ and Ω are any topologies on X.

3. Separation axioms in bitopological spaces

We first recall some definitions and notations from(Jaleel, 2003) ;

A subset F of (X,τ,Ω) is called δ-closed if X-F is δ-open.

δ-cl A$=\cap\{F \mid F \text{ is } \delta-\text{closed and } A\subseteq F\}$ it is called δ-closure of A.

A set A is said to be a δ-neighborhood of a point x if there exists a δ-open set U such that $x\in U\subseteq A$.

3.1 Definition : Let (X,τ,Ω) be a B.S ;two subsets A and B of X are δ-separated if each is disjoint from the other's δ-closure.(i.e $A\cap\delta$-clB$=\phi$ and (δ-clA) $\cap B=\phi$).

Two points x and y in X are δ-distinguishable if they do not have exactly the same δ-neighborhoods (i.e there exists a δ-open set containing x but not containing y or containing y but not containing x).

Two points x and y are δ-separated if the singletons $\{x\}$ and $\{y\}$ are δ-separated.

3.2 Definition : A B.S (X,τ,Ω) is called δ-T_0 if any two distinct points are δ-distinguishable.

3.3 Remark : If (X,τ) is a T_0 space then for any topology Ω on X the B.S (X,τ,Ω) is δ-T_0.

proof: It follows from the fact that any τ-open set is a δ-open set in (X,τ,Ω).

3.4 Remark : The converse of 3.3 is not true ,see the following example.

3.5 Example : $X=\{a,b,c,d\}$, $\tau=\{X,\phi,\{a\},\{a,d\},\{b,c\},\{a,b,c\}\}$, $\Omega=\{X,\phi,\{a\}\}$,then δ-o(X)= $\tau\cup\{\{a,b\},\{a,b,d\},\{a,c\},\{a,c,d\}\}$ and it is clear that (X,τ,Ω) is δ-T_0 but (X,τ) is not T_0 space since the points b and c are not distinguishable.

3.6 Theorem : A B.S (X,τ,Ω) is δ-T_0 if and only if for each two distinct points x and y δ-cl$\{x\}\neq\delta$-cl$\{y\}$.

Proof: Suppose that (X,τ,Ω) is δ-T_0 and let $x\neq y$ be two points of X such that δ-cl$\{x\}=\delta$-cl$\{y\}$,therefore $x\in\delta$-cl$\{y\}$ and $y\in\delta$-cl$\{x\}$ .If U is a δ-open set such that $x\in U$ and $y\notin U$, then $y\in X-U$ (a δ-closed set) so δ-cl$\{y\}\subseteq X-U$ which means δ-cl$\{x\}\subseteq X-U$ and so $x\in X-U$ i.e $x\notin U$ a contradiction!. Similarly the assumption that $x\notin V$ and $y\in V$(for some δ-open set V) leads to a contradiction ,that is (X,τ,Ω) is not a δ-T_0.

On the other hand suppose that for each $x,y\in X$ and $x\neq y$ we have δ-cl$\{x\}\neq\delta$-cl$\{y\}$, therefore either $x\notin\delta$-cl$\{y\}$ and so $x\in X$- cl$\{y\}$ but $y\notin X$- cl$\{y\}$,or $y\notin\delta$-cl$\{x\}$ and so $y\in X-\delta$-cl$\{x\}$ but $x\notin X-\delta$-cl$\{x\}$ (where $X-\delta$-cl$\{x\}$ and X- cl$\{y\}$ are δ-open sets in (X,τ,Ω)) i.e x and y are δ- distinguishable,hence (X,τ,Ω) is δ-T_0.

3.7 Theorem : If (X,τ,Ω) is a δ-T_0 B.S and Y is a subset of X then (Y,τ_Y,Ω_Y) is a δ-T_0 too.

Proof: It follows from the fact that the δ-open sets of (Y,τ_Y,Ω_Y) are the intersections of Y with the δ-open sets of (X,τ,Ω).

3.8 Definition :A subset A of a B.S (X,τ,Ω) is said to be δ-g-closed set if δ-clA $\subseteq$ U whenever $A\subseteq U$ and U is a δ-open set in (X,τ,Ω).

3.9 Definition : A B.S (X,τ,Ω) is said to be δ-$T_{1/2}$ if every δ-g-closed set in (X,τ,Ω) is δ-closed.

3.10 Lemma : A subset A of X,τ,Ω is δ-g-closed set if and only if δ-cl$\{x\}\cap A\neq\phi$ for each $x\in\delta$-clA.

Proof: Suppose that A is a δ-g-closed set, and for some $x\in\delta$-clA, δ-cl$\{x\}\cap A=\phi$,then $A\subseteq X$-(δ-cl$\{x\}$),where X-(δ-cl$\{x\}$)is a δ-open set ,so by definition 3.8 δ-clA$\subseteq$X-(δ-cl$\{x\}$),hence $x\in$ X-(δ-cl$\{x\}$)i.e $x\notin\delta$-cl$\{x\}$ which is a contradiction. Conversely assume that for each $x\in\delta$-clA, δ-cl$\{x\}\cap A\neq\phi$; if there is a δ-open set U such that $A\subseteq U$ but δ-clA$\not\subset$U then there exists $x\in\delta$-clA and $x\notin U$, so $x\in X-U$ which implies δ-cl$\{x\}\subseteq X-U$(since X-U is δ-closed) i.e δ-cl$\{x\}\cap A=\phi$, a contradiction. Therefore A is δ-g-closed set.

3.11 Lemma : If δ-cl$\{x\}\cap A\neq\phi$ for each $x\in\delta$-clA, then(δ-clA)-A does not contain a non empty δ-closed set .

proof: Suppose (δ-clA)-A contains a non empty δ-closed set ,say B, then $x\in B$ implies
δ-cl$\{x\}\subseteq B\subseteq\delta$-clA-A , and δ-cl$\{x\}\cap A=\phi$,which contradicts the hypothesis.

3.12 Theorem : A B.S X,τ,Ω is δ-$T_{1/2}$ if and only if ,for each $x\in X$,$\{x\}$ is δ-closed or δ-open.

Proof: Assume that X,τ,Ω is δ-$T_{1/2}$ and $\{x\}$ is neither δ-closed nor δ-open then X-$\{x\}$ is not δ-closed so δ-cl(X-$\{x\}$)=X$\subseteq$X i.e X-$\{x\}$ is a δ-g-closed set , by definition of δ-$T_{1/2}$,X-$\{x\}$ must be δ-closed, a contradiction with the assumption.

On the other hand suppose that for each x in X,τ,Ω ,$\{x\}$ is δ-closed or δ-open .Let A be a δ-g-closed set in X,τ,Ω , then by 2.10 and 2.11 (δ-clA)-A does not contain a non empty δ-closed set ,so if $x\in$(δ-clA)-A then δ-cl$\{x\}\not\subset$(δ-clA)-A i.e δ-cl$\{x\}\neq\{x\}$ which means $\{x\}$ is not δ-closed, so it must be δ-open , but $\{x\}\cap A=\phi$ implies $x\notin\delta$-clA ,a contradiction ,hence (δ-clA)-A=ϕ .Therefore A is δ-closed ,and so X,τ,Ω is δ-$T_{1/2}$.

3.13 Theorem : If X,τ,Ω is δ-$T_{1/2}$ then it is δ-T_0.

proof: Suppose X,τ,Ω is δ-$T_{1/2}$,by 2.12 every singleton is either δ-closed or δ-open. Let $x\neq y$ (in X),if $\{x\}$ is δ-closed then X-$\{x\}$ is a δ-open set containing y but not containing x ; and if $\{x\}$ δ-open then it is containing x but not containing y. So X,τ,Ω is δ-T_0.

3.14 Remark : The converse of 2.13 is not true , see the following example.

3.15 Example : X=$\{a,b,c\}$, $\tau=\{X,\phi,\{a\},\{a,b\}\}$, Ω =D(the discrete topology on X) then δ-o(X) $=\tau$ and X,τ,Ω is a δ-T_0 but not δ-$T_{1/2}$ (since $\{b\}$ is neither δ-closed nor δ-open).

3.16 Theorem: If X, τ is $T_{1/2}$ space ,then for any topology Ω on X ,the B.S X,τ,Ω is δ-$T_{1/2}$.

proof: It follows the fact that $\tau\subseteq\delta$-o(X) and theorem 3.12.

3.17 Remark : The converse of 3.26 is not true ,see the following example.

3.18 Example : X= $\{a,b,c\}$, $\tau=\{X,\phi,\{a\},\{a,b\}\}$, Ω =I (the indiscrete topology on X) then δ-o(X) = $\tau\cup\{\{a,c\}\}$ and X, τ is not $T_{1/2}$ but X, τ , Ω is δ-$T_{1/2}$.

3.19 Definition : A B.S X, τ , Ω is said to be δ-T_1 if any two distinct points in X are δ-separated.

3.20 Theorem: A B.S X, τ , Ω is δ-T_1 if and only if every singleton of X is δ-closed.

Proof: Suppose X, τ , Ω is δ-T_1 ,and x $\in$ X ,if y $\in$ δ-cl$\{x\}$ but y $\neq$ x then δ-cl$\{y\}\subseteq\delta$-cl$\{x\}$ on the other hand by definition of δ-T_1 we have

$\{y\}\cap\delta$-cl$\{x\}=\phi$

which is a contradiction ,so δ-cl$\{x\}=\{x\}$ i.e $\{x\}$ is a δ-closed set.

Conversely if for each x , $\{x\}$ is δ-closed then δ-cl$\{x\}=\{x\}$ and any two distinct points of X are δ-separated i.e X, τ , Ω is δ-T_1 .

is δ-T_1 .

proof: It follows from the fact that in T_1-space every singleton is a closed set in X, τ , also any closed set in X, τ is a δ-closed set in X, τ , Ω .

3.22 Remark : The converse of 3.21 is not true ,see the following example.

3.23 Example :

X= $\{a,b,c,d\}$, $\tau=\{X,\phi,\{a\},\{d\},\{a,d\},\{b,c\},\{a,b,c\},\{b,c,d\}\}$, $\Omega=\{X,\phi,\{a\},\{d\},\{a,d\}\}$, then δ-o(X) = $\tau\cup\{\{a,b\},\{a,c\},\{b,d\},\{c,d\},\{a,c,d\},\{a,b,d\}\}$,so X, τ , Ω is δ-T_1 since all singletons are δ-closed sets but X, τ is not a T_1-space since $\{b\}$ and $\{c\}$ are not closed sets.

3.24 Theorem: If X, τ , Ω is δ-T_1 and Y is a subset of X then Y, τ_Y , Ω_Y is a δ-T_1 too.

Proof: It follows from theorem 3.20.

3.25 Theorem: If X, τ , Ω is δ-T_1 then it is δ-$T_{1/2}$.

proof: It follows from theorems 3.20 and 3.12 .

3.26 Remark : The converse of 3.25 is not true ,see the following example.

3.27 Example :X = $\{a,b\}$, $\tau=\{X,\phi,\{a\}\}$, Ω =D (the discrete topology on X),then δ-o(X)= τ (for X, τ , Ω),and X, τ , Ω is δ-$T_{1/2}$ (since $\{a\}$ is δ-open and $\{b\}$ is δ-closed),but not δ-T_1 since $\{a\}$ is not δ-closed .

3.28 Definition : A B.S X, τ , Ω is called δ-T_2 or δ-Hausdorff if any two distinct points in X are separated by δ-neighborhoods (i.e if for each x ,y $\in$ X , x $\neq$ y there is δ-open sets U and V such that x $\in$ U , y $\in$ V and U $\cap$ V= ϕ).

3.29 Theorem: If X, τ is a T_2 space ,then for any topology Ω on X, the B.S X, τ , Ω is δ-T_2 .

proof: It follows from the fact that $\tau\subseteq\delta$-o(X).

3.30 Remark : The converse of 3.29 is not true ,see the following example.

3.31 Example : X= $\{a,b,c,d\}$, $\tau=\{X,\phi,\{a\},\{b\},\{c\},\{a,b\},\{a,c\},\{b,c\},\{a,b,c\}\}$, Ω =I(the indiscrete topology on X) ,then

δ-o(X)= $\tau\cup\{\{a,d\},\{b,d\},\{c,d\},\{a,b,d\},\{a,c,d\},\{b,c,d\}\}$ and X, τ is not T_2 space but X, τ , Ω is δ-T_2 .

3.32 Remark : If (X,τ,Ω) is δ-T_2 then it is δ-T_1 too. But the converse is not true see the following example.

3.33 Example : X= $\{a,b,c,d\}$, $\tau=\{X,\phi,\{c\},\{a,b,d\}\}$, $\Omega=\{X,\phi,\{c\}\}$ then δ-o(X)= $\tau\cup\{\{a,b,c\},\{a,c\},\{b,c\},\{a,c,d\},\{b,c,d\},\{c,d\}\}$ and it is clear that (X,τ,Ω) is δ-T_1 (since all singletons are δ-closed sets) but it is not δ-T_2 (since the points a and b are not separated by δ- neighborhoods).

3.34 Theorem: If (X,τ,Ω) is a δ-T_2 B.S and Y is a subset of X ,then (Y,τ_Y,Ω_Y) is δ-T_2 too.

Proof : It is obvious.

3.35 Definition : A B.S (X,τ,Ω) is called δ-$T_{2\frac{1}{2}}$,or δ-Urysohn , if any two distinct points in X are separated by δ-closed neighborhoods . Not that a δ-$T_{2\frac{1}{2}}$ B.S must be δ-T_2.

3.36 Definition : A B.S (X,τ,Ω) is called δ-regular if given any point x and δ-closed set F in X such that $x\notin F$, then they are separated by δ- neighborhoods.

3.37 Remark : If (X,τ,Ω) is a B.S ,then the cases that (X,τ) is regular and (X,τ,Ω) is δ-regular are independent, see the following two examples.

3.38 Example :

X=$\{a,b,c,d\}$, $\tau=\{X,\phi,\{a\},\{c\},\{a,c\},\{b,c\},\{c,d\},\{a,c,d\},\{a,b,c\},\{b,c,d\}\}$, $\Omega=\{X,\phi,\{a\}\}$ then δ-o(X)= $\tau\cup\{\{a,b\},\{a,d\},\{a,b,d\}\}$ and (X,τ,Ω) is δ-regular but (X,τ) is not regular since the point b and the closed set $\{a,d\}$ are not separated by neighborhoods.

3.39 Example : X=$\{a,b,c,d\}$, $\tau=\{X,\phi,\{a\},\{d\},\{a,d\},\{b,c\},\{a,b,c\},\{b,c,d\}\}$, $\Omega=\{X,\phi,\{a\}\}$,then δ-o(X)= $\tau\cup\{\{a,b\},\{a,c\},\{a,b,d\},\{a,c,d\}\}$ and it is clear that (X,τ) is regular but (X,τ,Ω) is not δ-regular ,since the point b and the δ-closed set $\{c,d\}$ are not separated by δ- neighborhoods.

3.40 Remark : The notions of δ-T_1 and δ-regular are independent ,and the notions of δ-T_2 and δ-regular are independent too. In example 3.39 (X,τ,Ω) is δ-T_1 and δ-T_2 but not δ-regular, for the other part see the following example .

3.41 Example : X=$\{a,b,c\}$, $\tau=\{X,\phi,\{a\},\{b,c\}\}$, $\Omega=\{X,\phi,\{a\},\{c\},\{a,c\}\}$,then δ-o(X)= τ and (X,τ,Ω) is δ-regular but it is neither δ-T_1 nor δ-T_2.

3.42 Theorem: A B.S (X,τ,Ω) is δ-regular if and only if for each δ-open set U and $x\in U$ there exists a δ-open set V such that $x\in V$ and δ-cl$V\subseteq U$.

proof: Suppose that (X,τ,Ω) is a δ-regular B.S ,and let $x\in U$ where U is a δ-open set, take H =X-U ,then H is δ- closed and $x\notin H$,so there exist two δ-open sets V and W such that $x\in V$,$H\subseteq W$ and $V\cap W=\phi$ i.e$V\subseteq X-W$, which implies δ-cl$V\subseteq\delta$-cl(X-W)=X-W.On the other hand $H\subseteq W$ implies $X-W\subseteq X-H=U$,therefore δ-cl$V\subseteq U$.

Conversely ,if H is a δ- closed set and $x\notin H$, then $x\in X-H$ (=U),U is a δ-open set, and by the condition of the theorem there exists a δ-open set V such that $x\in V$ and δ-cl$V\subseteq U$ therefore $H\subseteq X-\delta$-clV, $x\in V$ and $V\cap(X-(\delta$-clV$))=\phi$.

Hence (X,τ,Ω) is δ-regular.

3.43 Definition : A B.S (X,τ,Ω) is said to be δ-T_3, if it is δ-T_1 and δ-regular.

δ-T_1 and

3.44 Definition : A B.S X, τ, Ω is said to be δ-normal if any two disjoint δ-closed sets in X are separated by δ- neighborhoods.

3.45 Remark : The notions of δ-T_1 and δ-normal and are independent , and the notions of δ-T_2 and δ-normal are independent too. See examples 3.39 and 3.41.

3.46 Remark : If X, τ, Ω is a B.S ,then the cases that X, τ is normal and X, τ, Ω is δ-normal are independent. In example 3.38 X, τ, Ω is δ-normal but X, τ is not normal since the closed sets $\{b\}$ and $\{a,d\}$ are not separated by neighborhoods.

On the other hand ,in example 3.39 X, τ is a normal space where the B.S X, τ, Ω is not δ-normal ,since the closed sets $\{b\}$ and $\{c,d\}$ are not separated by δ-neighborhoods.

3.47 Definition : A B.S X, τ, Ω is said to be δ-T_4 if it is δ-T_1 and δ-normal.

3.48 Theorem: If Y is a δ-closed subset of a δ-T_4 B.S X, τ, Ω, then the B.S Y, τ_Y, Ω_Y is δ-T_4 too.

Proof: Since X, τ, Ω is δ-T_1, Y, τ_Y, Ω_Y is δ-T_1 (3.24).Since Y is δ-closed ,a subset F of Y is δ-closed in Y if and only if F is δ-closed in X .Therefore if F and H are disjoint δ-closed subsets of Y , they are also disjoint δ-closed subsets of X . there are thus δ-open sets U and V such that $F \subseteq U$,$H \subseteq V$ and $U \cap V = \phi$.But then $F \subseteq Y \cap U$, $H \subseteq Y \cap V$ where $Y \cap U$ and $Y \cap V$ are disjoint subsets of Y which are δ-open in Y .Therefore Y is δ-normal ,hence Y is δ-T_4

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