On Feebly –closed mappings in bitopological spaces

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Abstract

This search discusses the α-set and feebly –closed sets in bitopological space, and these concepts define the feebly-closed function , semi-closed function and pre-closed function also we defines an α-closed function and study the relation between these concepts.

Introduction

let S be a subset of a bitopological spaces (X,t_1,t_2), we denote the closure of S and the interior of S with respect to t_1, t_2 by $cl_{t1}(S)$, $int_{t1}(S)$ and $cl_{t2}(S)$, $int_{t2}(S)$ respectively.

[O.Njastad,1965] introduced the concept of α-set in a topological space (X,t) . A subset S of (X, t) is called an α-set if S⊆int(cl(int(S))) . the notion of semi open set , per -open set were introduced by[N.Levine, 1963].a subset A is said to be Feebly-open set [S.N.Maheshwari and U.D.Tapi,1978] in (X,t) if there exist an open set U such that U⊆A⊆scl(U) , the complement of Feebly- open set is called Feebly closed set.

In this search , we shall define the α-set and Feebly -open set in bitopological space (X,t_1,t_2) . A subset S of a bitopological $(X,t_1,t_2)$is said to be α-set if S isα-set with respect to $t_1$or t_2 , that is if S $\subseteq in_{ti}(cl_{ti}(in_{ti}(S)))$, i=1or2, we shall define a Feebly open set in a bitopological space(X,t_1,t_2) if there exist a open set U with respect to $t_1$or t_2 such that U⊆A⊆$scl_{ti}(U)$, i=1or2, the complement of Feebly- open set is called Feebly- closed set .

Feebly closed and α-closed mapping

The concept of α-closed and Feebly- closed mapping have been introduced by[Mashhour A.S. , etral,1983] and [Maheshwari S.N. and Tapi U.D.,1978] respectively.

Definintion (2-1) [Greenwood S.G. and Reilly I.L. , 1986]

Let (X, t) and (Y,σ) are two topological spaces , a function f: (X, t)→(Y,σ) is said to be:

1- Feebly –closed if the image of each closed set in X is Feebly- closed in Y.

2- α-closed if the image of each closed set in X is α-closed set in Y.

Lemma: (2-2)[Greenwood S.G. and Reilly I.L. , 1986]

Let A be a subset of (X, t) then sint(cl(A))=cl(int(cl(A)).

Proposition(2-3)[Greenwood S.G. and Reilly I.L., 1986]

Let (X, t)be a topological spaces , a subset A of (X,t) is Feebly closedif and only if A is α-closed set.

Definition (2-4)[Greenwood S.G. and Reilly I.L. , 1986]

Let (X, t) and (Y,σ) are two topological spaces , a function f: (X,t)→(Y,σ) is said to be:

1- semi-closed if the image of each closed set in X is semi -closed set in Y.

2- pre-closed if the image of each closed set in X is pre- closed in Y.

Proposition (2-5)[Greenwood S.G. and Reilly I.L. , 1986]

Let (X,t) and (Y,σ) are two topological spaces , a function f: (X,t)→(Y,σ) is α-closed if and only if it is semi- closed and pre-closed .

Feebly-closed and α-closed mapping in bitopological space.

Definition(3-1)

Let (X,t_1,t_2) and (Y,σ_1,σ_2) are two bitopological spaces , a function

$f: (X,t_1,t_2) \rightarrow (Y,\sigma_1,\sigma_2)$said to be:

1- Feebly- closed if the induced maps $f: (X,t_1)\rightarrow(Y,\sigma_1)$ or $f: (X,t_2)\rightarrow(Y,\sigma_2)$ is Feebly-closed.

2- α -closed the induced maps $f: (X,t_1)\rightarrow(Y,\sigma_1)$ or $f: (X,t_2)\rightarrow(Y,\sigma_2)$ is α- closed.

Proposition (3-2):

Let (X,t_1,t_2) be a bitopological space and A be a subset of X then $sint_{ti}(A)=cl_{ti}(int_{ti}(A))$, i=1or2.

Proof: since we have $cl_{ti}(int_{ti}(cl_{ti}(A))$ is semi open set with respect to ti and $cl_{ti}(int_{ti}(cl_{ti}(A))= cl_{ti}(int_{ti}(cl_{ti}(int_{ti}(A)))$ and $cl_{ti}(int_{ti}(cl_{ti}(A))\subseteq cl_{ti}(A)$ then $cl_{ti}(int_{ti}(cl_{ti}(A))\subseteq sint_{ti}(cl_{ti}(A))$ ….(1).

Now if V is any ti-semi-open set contained in $cl_{ti}(A)$ then $U \subseteq cl_{ti}(int_{ti}(V)) \subseteq cl_{ti}(int_{ti}(cl_{ti}(A)))$ and hence $sint_{ti}(cl_{ti}(A))\subseteq cl_{ti}(int_{ti}(cl_{ti}(A))$ ….(2). By (1) and (2) we get the result.

Proposition(3-3)

Let (X,t_1,t_2) be a bitopological space a subset A of X is Feebly- closed set in X if and only if A is α-closed.

Proof :

It follows from the definition of an α-set and α-closed set in bitopological space . That a subset A of (X,t_1,t_2) is α-closed set if and only if $cl_{ti}(int_{ti}(cl_{ti}(A))\subseteq A$, i=1or2 , since $cl_{ti}(int_{ti}(cl_{ti}(A))\subseteq A$ if and only if $sint_{ti}(cl_{ti}(A))\subseteq A$ by lemma (2-2) in bitopological space the result exist.

Definition(3-4)

Let (X,t_1,t_2) and (Y,σ_1,σ_2) are two bitopological spaces , a function

$f: (X,t_1,t_2) \rightarrow (Y,\sigma_1,\sigma_2)$said to be:

1- semi-closed if the induced maps $f: (X,t_1)\rightarrow(Y,\sigma_1)$ or $f: (X,t_2)\rightarrow(Y,\sigma_2)$ is semi-closed.

2- pre -closed the induced maps $f: (X,t_1)\rightarrow(Y,\sigma_1)$ or $f: (X,t_2)\rightarrow(Y,\sigma_2)$ is pre- closed.

Proposition (3-5):

Let (X,t_1,t_2) and (Y,σ_1,σ_2) are two bitopological spaces , a function

$f: (X,t_1,t_2) \rightarrow (Y,\sigma_1,\sigma_2)$is α-closed if and only if it is semi-closed and pre-closed.

Proof: since $f: (X,t_1)\rightarrow(Y,\sigma_1)$ is α-closed if and only if it is semi closed and pre-closed [theorem (3), I.l.Reilly and M.R.VamanaMurthy] , similarly $f: (X,t_2)\rightarrow(Y,\sigma_2)$
Is α-closed if and only if it is semi-closed and pre-closed [theorem (3), I.l.Reilly and M.R.VamanaMurthy] and hence the result.

This example show that if $f: (X,t_1,t_2) \rightarrow (X,\sigma_1,\sigma_2)$ is pre closed then f not to be α-closed.

Example (3-6): let X={1,2,3} and defined t_1 to be the discrete topology and $t_2=\{X,\varnothing,\{1\},\{3\},\{1,3\}\}$, $\sigma_1=\{X,\varnothing,\{1\},\{3\},\{1,3\}\}$ and σ_2 be the discrete topology. Define $f: (X,t_1)\rightarrow(Y,\sigma_1)$ by f(1)=f(2)=f(3)=1 then f is pre closed but not α-closed since {1} is pre closed in (Y,σ_1) but not α-closed in (X,t_1) and define $f: (X,t_2)\rightarrow(Y,\sigma_2)$ by f(1)=f(2)=f(3)=1 then f is pre closed but not α-closed since {1} is pre closed in (Y,σ_2) but not α-closed in (X,t_2).

This example show that if f: $(X,t_1,t_2) \rightarrow (X,\sigma_1,\sigma_2)$ is semi- closed then f not to be α-closed.

Example(3-7)

let X={1,2,3} and defined $t_1$and t_2 to be the discrete topologies and$\sigma_1$and σ_2 be the indiscrete topology. Define f: $(X,t_1)\rightarrow(Y,\sigma_1)$ by f(1)=f(2)=f(3)=1 then f is semi closed but not α-closed and f: $(X,t_2)\rightarrow(Y,\sigma_2)$ defined by f(1)=f(2)=f(3)=1 then f is semi closed but not α-closed since {1} is

Conclusion

From this paper we can conclude that the type of a bitopological space (X,t_1,t_2) is depend on the type of a topological space (X,t_1) and(X,t_2) since the subset of these topologies will induce the subset of (X,t_1,t_2) and because these subset is the same in (X,t_1,t_2), hence any definition and proposition which is true in (X,t_1) and(X,t_2) will be true in (X,t_1,t_2) in this search.

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