ϑ_{MK}	10	3	0.01225	0.0564	0.1145	0.0157	0.0578	0.1174	0.01292	0.05739	0.1119
		5	0.0147	0.05718	0.1183	0.0187	0.0591	0.1218	0.01915	0.05983	0.1158
	40	8	0.01201	0.05617	0.1127	0.0147	0.0568	0.1153	0.01224	0.05651	0.1080
		10	0.01442	0.05695	0.1164	0.0175	0.0581	0.1195	0.01818	0.05887	0.1117
	100	10	0.01133	0.05296	0.1063	0.0124	0.0564	0.1115	0.01145	0.05562	0.1062
		20	0.0136	0.0537	0.1098	0.0148	0.0577	0.1157	0.017	0.05795	0.1096
ϑ^{*}	10	3	0.01081	0.0509	0.1088	0.0139	0.0522	0.1116	0.0114	0.0518	0.1063
		5	0.01297	0.05161	0.1124	0.0165	0.0534	0.1157	0.0169	0.054	0.1100
	40	8	0.0106	0.0507	0.1071	0.013	0.0513	0.1096	0.0108	0.051	0.1026
		10	0.01272	0.0514	0.1106	0.0155	0.0525	0.1136	0.01604	0.05313	0.1061
	100	10	0.010	0.0502	0.101	0.011	0.0509	0.106	0.0101	0.0502	0.1009
		20	0.012	0.0509	0.1043	0.0131	0.0521	0.1099	0.015	0.0523	0.1042

Table (2) : Empirical powers of
ϑ^{*} , ϑ_{ML} , ϑ_{LB} , ϑ_{MK} , ϑ_{BP} , *and* ϑ_{M} test statistics

إحصاءة الاختبار	T	m	$\lambda = 0.5$			$\lambda = 1$			$\lambda = 1.1$		
			$\alpha = 0.01$	$\alpha = 0.05$	$\alpha = 0.1$	$\alpha = 0.01$	$\alpha = 0.05$	$\alpha = 0.1$	$\alpha = 0.01$	$\alpha = 0.05$	$\alpha = 0.1$
ϑ_{BP}	10	3	0.7121	0.7303	0.7552	0.6644	0.6814	0.7356	0.6887	0.7062	0.730
		5	0.698	0.7171	0.753	0.6513	0.6690	0.7023	0.675	0.6935	0.728
	40	8	0.758	0.7684	0.801	0.707	0.717	0.7472	0.7328	0.7431	0.774
		10	0.7345	0.766	0.7726	0.6853	0.7147	0.7139	0.7102	0.7407	0.7471
	100	10	0.7776	0.8016	0.829	0.7255	0.7479	0.7735	0.7519	0.7752	0.8016
		20	0.7677	0.7740	0.7801	0.7163	0.7221	0.7278	0.7423	0.7485	0.7544
ϑ_{LB}	10	3	0.804	0.8246	0.8527	0.7501	0.7694	0.8305	0.7775	0.7974	0.8246
		5	0.7881	0.8096	0.850	0.7353	0.7553	0.7930	0.7621	0.783	0.822

	40	8	0.8555	0.8677	0.9042	0.7982	0.8095	0.8436	0.8274	0.839	0.874
		10	0.8293	0.8649	0.872	0.7737	0.8069	0.8061	0.802	0.8363	0.8435
	100	10	0.878	0.9051	0.936	0.8192	0.8444	0.8733	0.849	0.8753	0.9051
		20	0.8667	0.874	0.881	0.8087	0.8153	0.8217	0.8381	0.8451	0.852
ϑ_{ML}	10	3	0.811	0.8317	0.860	0.7565	0.7760	0.8376	0.7842	0.8042	0.8317
		5	0.795	0.8166	0.8572	0.7416	0.762	0.8000	0.7686	0.7900	0.829
	40	8	0.8628	0.8751	0.9119	0.805	0.8165	0.8510	0.8345	0.8462	0.8817
		10	0.8364	0.8723	0.880	0.7803	0.8138	0.813	0.809	0.8435	0.8507
	100	10	0.8855	0.9128	0.944	0.8262	0.8517	0.8808	0.8562	0.8827	0.9128
		20	0.8741	0.8814	0.8883	0.8156	0.8223	0.8287	0.8453	0.8523	0.859
ϑ_{M}	10	3	0.5695	0.584	0.604	0.5313	0.545	0.5883	0.5508	0.5648	0.5841
		5	0.5582	0.5735	0.602	0.5208	0.535	0.5617	0.5332	0.5546	0.5821
	40	8	0.606	0.6146	0.6405	0.5654	0.5734	0.5976	0.5861	0.5943	0.6192
		10	0.5874	0.6126	0.618	0.5480	0.5716	0.5710	0.5681	0.5924	0.5975
	100	10	0.622	0.6411	0.663	0.5803	0.5982	0.6186	0.6013	0.6200	0.6411
		20	0.614	0.619	0.624	0.573	0.5775	0.582	0.5937	0.5986	0.6033
ϑ_{MK}	10	3	0.774	0.7938	0.8208	0.7221	0.7406	0.80	0.7485	0.7676	0.7938
		5	0.759	0.7794	0.8181	0.7078	0.7271	0.7633	0.7336	0.7537	0.7911
	40	8	0.8235	0.8352	0.8704	0.7684	0.7793	0.8121	0.7965	0.8077	0.8415
		10	0.7983	0.833	0.84	0.7448	0.7768	0.7759	0.7720	0.8050	0.812
	100	10	0.8451	0.8713	0.901	0.7886	0.8129	0.8406	0.8172	0.8425	0.8713
		20	0.8343	0.8413	0.848	0.7785	0.7849	0.7910	0.8068	0.8135	0.82
ϑ^{*}	10	3	0.859	0.881	0.911	0.8014	0.8220	0.8873	0.8307	0.8519	0.881
		5	0.842	0.865	0.908	0.7856	0.807	0.8472	0.8142	0.8365	0.878
	40	8	0.914	0.927	0.966	0.8528	0.8649	0.9013	0.8840	0.8964	0.934
		10	0.886	0.924	0.932	0.8266	0.8621	0.8612	0.8568	0.8935	0.9012
	100	10	0.938	0.967	0.978	0.8752	0.9022	0.9330	0.9070	0.9351	0.967
		20	0.926	0.9337	0.941	0.8640	0.8711	0.8779	0.8954	0.9029	0.9100

Table (1) : Empirical significant levels of ϑ^{*} , ϑ_{MH} , ϑ_{LR} , ϑ_{MK} , ϑ_{BP} , *and* ϑ_{M} statistics

إحصاءة الاختبار	T	m	$\lambda = 0.5$			$\lambda = 1$			$\lambda = 1.1$		
			$\alpha = 0.01$	$\alpha = 0.05$	$\alpha = 0.1$	$\alpha = 0.01$	$\alpha = 0.05$	$\alpha = 0.1$	$\alpha = 0.01$	$\alpha = 0.05$	$\alpha = 0.1$
ϑ_{BP}	10	3	0.0068	0.0289	0.057	0.0055	0.0287	0.0601	0.00763	0.0285	0.0571
		5	0.0066	0.0284	0.0552	0.0051	0.0294	0.058	0.00513	0.0297	0.0551
	40	8	0.00689	0.0299	0.0604	0.0085	0.0282	0.0621	0.00815	0.0281	0.058
		10	0.00647	0.0283	0.0585	0.0078	0.0289	0.0599	0.0055	0.02923	0.0561
	100	10	0.00668	0.0296	0.0614	0.0068	0.028	0.0632	0.0086	0.0298	0.06
		20	0.0061	0.028	0.059	0.0066	0.0286	0.061	0.0058	0.02877	0.0581
ϑ_{LB}	10	3	0.01234	0.058	0.1093	0.0158	0.0594	0.1121	0.01302	0.06211	01068
		5	0.01481	0.05881	0.1129	0.0188	0.0640	0.1162	0.0193	0.06474	0.1105
	40	8	0.0121	0.05777	0.1076	0.0148	0.0615	0.1101	0.01233	0.06115	0.1030
		10	0.01452	0.05857	0.1111	0.0177	0.0629	0.1141	0.01831	0.0637	0.1066
	100	10	0.01142	0.0572	0.1014	0.0125	0.0610	0.1065	0.01153	0.06019	0.1013
		20	0.0137	0.058	0.1048	0.0149	0.0624	0.1104	0.01713	0.06271	0.1047
ϑ_{ML}	10	3	0.0119	0.0523	0.1091	0.0152	0.0536	0.1119	0.01254	0.05322	0.1066
		5	0.01427	0.05303	0.1127	0.0182	0.0548	0.1160	0.01859	0.05549	0.1103
	40	8	0.01166	0.05209	0.1074	0.0143	0.0527	0.1099	0.01188	0.0524	0.1029
		10	0.014	0.05281	0.1109	0.0170	0.0539	0.1139	0.01764	0.0546	0.1064
	100	10	0.011	0.05158	0.1012	0.0121	0.0523	0.1063	0.01111	0.05158	0.1011
		20	0.0132	0.0523	0.1046	0.0144	0.0535	0.1102	0.0165	0.05374	0.1045
ϑ_{M}	10	3	0.0054	0.021	0.051	0.0058	0.0215	0.0537	0.00675	0.0216	0.0510
		5	0.0045	0.02071	0.0494	0.0049	0.021	0.0518	0.00455	0.02071	0.0493
	40	8	0.00472	0.0212	0.0541	0.0069	0.0217	0.0555	0.00722	0.02192	0.0519
		10	0.00477	0.02092	0.0524	0.0058	0.0212	0.0536	0.00486	0.02104	0.0501
	100	10	0.00584	0.0213	0.0549	0.0074	0.022	0.0566	0.00761	0.0223	0.0538
		20	0.00486	0.021	0.0532	0.0062	0.0215	0.0546	0.00513	0.02137	0.052

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3) The values of T are taken to be 10 (small sample size) , 40 (moderate sample size) and 100 (large sample size) .
4) All simulation experiments were based 0n R=1000 replications .
5) The a_t's generated as zero mean , unit variance white noise .
6) The values of significant level α are taken to be 0.1 , 0.05 and 0.01 .
7) The values of m are taken to be m= 3,5 for (T=10) , m=8,10 for (T=40) and m=10,20 for (T=100) .

In order to obtain some information concerning the validity of the asymptotic performance of ϑ_{BP} , ϑ_{LB} , ϑ_{ML} , ϑ_{M} , ϑ_{MK} *and* ϑ^{*} test statistics , a Visual Basic program was written by the author to calculate empirical significance levels and empirical powers of the test statistics under consideration .

Our results are reported in table (1) and table (2) respectively .

Our main conclusions are :

1) As is it to be expected , as sample size increases , the performance of the different test statistics improve dramatically .

2) If the series obeys to the random walk process $(\lambda = 1)$, the performance of the different test statistics will be less than their performance for stationary and non stationary models .

3) The performance of the different test statistics for stationary models better than their performance for non stationary models .

4) The goodness of performance of the test statistics under consideration , are as follows , respectively , ϑ^{*}, ϑ_{ML} , ϑ_{LB} , ϑ_{MK} , ϑ_{BP} , *and finally* ϑ_{M} .

where $\hat{F}_a(w) = \frac{1}{\eta}(number\ of\ w_i\ less\ than\ or\ equal\ to\ w)$ and $\eta = 2\pi / \varepsilon$ represent the number of points which will be taken , to calculate $\hat{F}_a(w)$, $i.e.\ i = 1,2,...,\eta\ and\ \varepsilon$ an arbitrary number , like 0.01 .

Now, to test the hypothesis in (11) , we can use the kolmogorov - smirnov one sample test , with the following test statistic ,

$$D = Max \left| \hat{F}_a(w_i) - F_a(w_i) \right| \qquad , i = 1,2,..., \eta \qquad ----(12)$$

where the sampling distribution of D under H_o is known [10] , and a lot of references give critical values from that sampling distribution .

Intuitively , There are some advantages for the above procedure :

1) It deal with the frequency domain , which provides an alternative way of viewing the process . For some applications , the frequency domain analysis may be more illuminating than the time domain analysis [3] .

2) Since the test based on non parametric statistic , then the advantages of non Parametric statistical tests will be risen .

III- An Empirical study

A monte Carlo study was conducted to generate sets of observations from the Markov model $y_t = \lambda y_{t-1} + a_t$, under the following assumptions:

1) The initial value y_o equal to zero .

2) The values of Markov model parameter λ are taken to be 0.5 (to generate stationary process) , 1 (to generate random walk process , i.e. a process has a boundary value problem) and 1.1 (to generate non stationary process).

Nasir conclused that almost all time , the null hypothesis is accepted and that the statistics $\vartheta_{BP}, \vartheta_{LB}, \vartheta_{M}$ and ϑ_{MK} have nearly the same performances at large sample sizes .

II- Another point of view

Since a white noise process has a flat spectral density function $f_a(w)$ as in (7) , we can simply represent the problem under consideration by the hypothesis ,

$$\left.\begin{aligned} H_o &: \hat{f}_a(w) = f_a(w) \\ &vs \\ H_1 &: \hat{f}_a(w) \neq f_a(w) \end{aligned}\right\} \qquad ----(9)$$

where

$$\hat{f}_a(w) = \frac{1}{2\pi}\sum_{v=-M}^{M} \hat{r}_v\, k_M(v)\cos(vw) \quad , \quad -\pi \le w \le \pi \qquad ----(10)$$

is a consistent estimator of $f_a(w)$, i.e. the frequencies obtained from the actual performance of an experiments .

where M is the truncation point parameter , $k_M(v)$ is the lag window and $\hat{r}_v$ is the sample autocorrelation function , defined as in (2) .

Since , the hypothesis tested is how good the observed frequencies $\hat{f}_a(w_i)$ fit a given pattern $f_a(w_i) = 1/2\pi$, we can rewrite the hypothesis in (9) in terms of cumulative spectrum as follows :

$$\left.\begin{aligned} H_o &: \hat{F}_a(w) = F_a(w) \\ &vs \\ H_1 &: \hat{F}_a(w) \neq F_a(w) \end{aligned}\right\} \qquad ----(11)$$

are replaced by the sample autocorrelations of the squared data , $\tilde{r}_k$, giving ,

$$\vartheta_{ML} = T(T+2)\sum_{k=1}^{m}(T-k)^{-1}\tilde{r}_k^{\,2} \qquad ----(5)$$

The hypothesis of *iid* data is then rejected at level α if the observed value of ϑ_{ML} is larger than the $1-\alpha$ quantile of the χ^2_{m-p-q} distribution .

Monti 1994 [7] proposed another portmanteau test , based on the partial autocorrelation function of residuals ,

$$\vartheta_{M} = T(T+2)\sum_{k=1}^{m}(T-k)^{-1}\hat{\phi}_{kk} \qquad ----(6)$$

The ϑ_M statistic is asymptotically distributed as χ^2_{m-p-q} also .

Since each frequency in the spectrum of white noise process contributes equally to the variance of the process , then the white noise process has a flat spectral density function

$$f_a(w) = 1/2\pi \quad , \quad -\pi \le w \le \pi \qquad ----(7)$$

Mokkadem 1994 [9] derived another portmanteau test statistic based on the hypothesis $H_o : f_a(w) = c \quad vs \quad H_1 : f_a(w) \ne c$, where c is any constant . The formula of Mokkadem statistic is :

$$\vartheta_{MK} = Ln\left(\frac{\hat{R}_o}{2\pi}\right) - \frac{1}{2\pi}\int_{-\pi}^{\pi} Ln\left|\hat{f}_a(w)\right| dw$$

$$\cong \sum_{k=1}^{m}\hat{r}_k^{\,2} \qquad ----(8)$$

Al-Nasir 2000 [1] , generated sets of observations from the markov process for comparison among test statistics ϑ_{BP} , ϑ_{LB} , ϑ_M *and* ϑ_{MK} . Al-

$$\hat{r}_v = \frac{\sum_{t=v+1}^{T} \hat{a}_t \hat{a}_{t-v}}{\sum_{t=1}^{T} \hat{a}_t^2} \quad , \ k = 1,2,... \qquad ----(2)$$

Box and Pierce 1970 [2] show that , under the correct model specification
(null hypothesis) provided that m is moderately large , the statistic

$$\vartheta_{BP} = T \sum_{k=1}^{m} \hat{r}_k^2 \qquad ----(3)$$

is asymptotically distributed as χ^2 with $(m-q-p)$ degrees of freedom . Tests of model adequacy based on this statistic are generally called portmanteau tests .

It has been shown by David , Triggs and Newbold 1977 [5] that , for sample sizes commonly found in practice , the actual significance levels of ϑ_{BP} can be considered lower than those predicted by asymptotic theory . However , a simple modification , studied by Ljung and Box 1978 [6]

$$\vartheta_{LB} = T(T+2) \sum_{k=1}^{m} (T-k)^{-1} \hat{r}_k^2 \qquad ----(4)$$

appears to have a distribution very much closer to the asymptotic χ^2 with $(m-p-q)$ degrees of freedom .

Davis and Newbold 1979 [4] concentrated on the behavior of the modified statistic ϑ_{LB} , and in particular investigated the frequency with which it detects misspecification , relating this to the increase in forecast error variance resulting from use of the incorrect model .

Another portmanteau , formulated by Mcleod and Li 1983 [8] , can be used as a farther test for the *iid* hypothesis , since if the data are *iid* , then the squared data are also *iid* . It is based on the same statistic used for the Ljung and Box test , except that the sample autocorrelations of the data

On The Lack Of Fit In Time Series Models

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Summary

In this paper, we present another test for lack of fit in time series models . A simulation study was conducted to compare among model adequacy tests . Our conclusion is that the proposed statistic achieves a high level of success to detect time series model misspecification, at all situations considered .

I- Introduction

Consider a discrete time series $\{y_t\}$ generated by a stationary autoregressive-moving average process

$$y_t = \lambda_1 y_{t-1} + \dots + \lambda_p y_{t-p} + a_t - \beta_1 a_{t-1} - \dots - \beta_q a_{t-q} \qquad ------(1)$$

and $\{a_t\}$ is a sequence of zero mean , finite variance , independent and identically distributed random deviates .

The y_t's can in general represent the d-th difference or some other suitable transformation of a non stationary series $\{z_t\}$.

After a model of this form has been fitted to a series $y_1, y_2, \dots, y_t$, it is useful to study the adequacy of the fit by examining the residuals $\hat{a}_1, \hat{a}_2, \dots, \hat{a}_t$ and in particular , their autocorrelations

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