# Re-sampling of Digital Images by Spatial Domain Method

**Wasna Jafar Hamad**
*College of Science/ University of Babylon*

## Abstract

The present research will be devoted to investigate re-sampling method that is performed in the spatial domain. Wavelet trasnsform has been adopted and used as an image re-sampling technique. Re-sampling process is required in many digital image processing applications and is particularly important in remote sensing to magnify or reduce image sizes, correct radiometric distortions, correct geometric distortions, and so on. Special transformation procedures using the Haar wavelet transform are adopted to produce wavelet bands having (1/4) and (1/8) size of the original image size. Moreover, the present research involves proposal to improve the quality of the reconstructed image and preserve sharp features of the image, especially, if less bands are adopted. Eventually, it should be noted that this method may be considered a fast algorithm.

## الخلاصة

إن البحث الحالي تضمن طريقة جديدة لإعادة تقسيم الصور منفذة في المجال الفضائي (الحيزي). التقنية المقترحة لإعادة تقسيم الصور تعتمد التحويل المويجي أساساً لها. إعادة تشكيل الصور بإستخدام عملية إعادة التقسيم ذات أهمية كبيرة في الكثير من تطبيقات معالجة الصور الرقمية، على سبيل المثال لا الحصر، في تطبيقات التحسس النائي تعد عمليات تكبير أو تصغير أحجام الصور وتصحيح التشوهات فيها من العمليات المهمة والمفيدة في الكثير من تطبيقاتها. في العمل الحالي بإستخدام تحويل مويجة هار تم تطوير هذه الطريقة الحديثة لتوليد حزم تحويل مويجي بحجم (4/1) و (8/1) من حجم الصورة الأصلية أستغلت في عمليات تصغير أحجام الصور. إن البحث الحالي تضمن مقترح لتحسين جودة الصورة المسترجعة والمحافظة على التفاصيل والملامح الدقيقة للصورة خاصة إذا ما تم تبني عدد أقل من حزم التحويل المويجي. إن هذه الطريقة تعتبر من الطرق السريعة والقابلة للتحسين إذا ما تم تبني معاملاتها بدقة.

## Introduction

Analog (continuous) images must first be sampled before being manipulated by digital computer. Sampling is the process of extracting a discrete set of points from the continuous image, it is carried out by some sampling devices, e.g. Page-Scanners, TV-Cameras, Micro-densitometers, …etc. (Gomes and Velho, 1997; Niblack, 1986). The aim of sampling operation is to represent an original continuous image by a finite string or array of numbers called *samples*. The only constraint on these numbers is that it should be possible to reconstruct the original image from them (Rosenfeld and Kak, 1982).

To reconstruct the complete image from the sampled values, a form of interpolation operation has to be used. Interpolation is the process of estimating the intermediate values of a continuous event from discrete samples. This process is, usually, referred to as *re-sampling operation* (Rosenfeld and Kak, 1982). Re-sampling process is used in digital image processing applications and is particularly important in remote sensing for several different purposes: magnifying or reducing image sizes, correction of image radiometric distortions, correction of image geometric distortions, and so on (Richards and Jia, 1999). Digital re-sampling methods could be categorized according to their implementation. Some of them are implemented in the spatial domain, whereas others are performed in the frequency domain. The present work will be implemented in spatial domain. Spatial domain method is directly performed on image pixels in the (x, y) plane. The most common techniques that are used to perform image re-sampling in the spatial domain are nearest neighbor, and bilinear interpolation (Parker, Kenyon and Troxel, 1983; Maeland, 1983). In this work, we have been adopted and used wavelet transform as an image re-sampling technique.

## 1 Wavelet Transform

Wavelets are powerful mechanisms for analyzing and processing digital signals. The concept of wavelets is rooted in many disciplines, including mathematics, physics, engineering, and computer science (Asuncion, 2003). A wavelet, which literally means small wave, is a waveform function of limited duration that has an average value of zero. Unlike sinusoids (cosine and sine signals) that theoretically extend from minus to plus infinity, wavelets have a beginning and an end. Fig. (1) shows a representation of a continuous sinusoid (a cosine wave is shown here) and a so-called “continuous” wavelet (a Daubechies wavelet is depicted here). Wavelets are irregular, of compact support, and often non-symmetrical (Fugal, 2006). A wavelet function, known as a *mother wavelet*, gives rise to a family of wavelets that are translated (shifted) and dilated (stretched or compressed) versions of the original mother wavelet (Asuncion, 2003). There are an infinite number of possible mother wavelets. Two of the more common are the Haar wavelet and the Daubechies wavelet (Torrence and Compo, 1998).

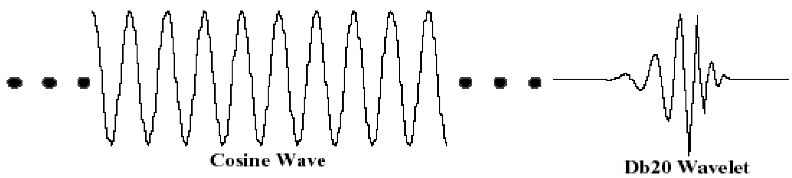
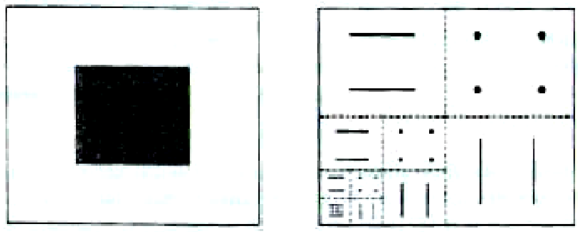



**Figure (1): A portion of an infinitely long sinusoid and a finite length wavelet, (Fugal, 2006).**

Wavelet transform is computationally efficient, and the reason for this is that the wavelet function used is often of compact support. Additionally, the wavelet transform requires only “N” computational operations (i.e. multiplication and addition) for N-valued input signals, while the number of computational operations is “$N \log_2 N$” for the other transforms. The wavelet transform performs a decomposition of the original data, allowing operations to be performed on the wavelet coefficients and then the data reconstructed, (Starck, *et al.*,1998; Newland, 1993).

Another advantage gained from the wavelet transform is that it can be used for signal as well as image filteration. The filtering process can be performed by means of application of low pass and high pass filters. The collection of filters is termed an *analysis bank* or a *filter bank*. These filters have decomposed an input signal into several sub-bands. First, the input data are partitioned into four sub-bands labeled as lowpass-lowpass (LL), lowpass-highpass (LH), highpass-lowpass (HL), and highpass-highpass (HH). The LL component is further partitioned into the two-level of four sub-bands, since most of the energy in an image is contained in the low frequency components. Decomposition process can be continued to as many levels as needed. Since each sub-band can be further partitioned, the filter bank implementation of the wavelet transform can be used for multi-resolution analysis (Shi and Sun, 1999; Starck, Murtagh, and Bijaoui, 1998). The following simple example will demonstrate the procedures involved in the wavelet transformation, see Figs. (2 & 3).



(a) (b)

(a)

**Figure (2): (a) A simple "centered-black-block" image and, (b) A wavelet transform of image in (a), (Parker, 1997).**

The forward wavelet transform is performed by the following steps:

(1) Perform averaging operation on the rows of the original image.
(2) Perform averaging operation on the columns of the results of (1) to produce the LL sub-image, (lower-left quarter of Fig. (2-b)).
(3) Perform subtraction operation on the columns of the results of (1) to produce the LH sub-image, (lower-right quarter of Fig. (2-b)).
(4) Perform subtraction operation on the rows of the original image.
(5) Perform averaging operation on the columns of the results of (4) to produce the HL sub-image, (upper-left quarter of Fig. (2-b)).
(6) Perform subtraction operation on the columns of the results of (4) to produce the HH sub-image, (upper-right quarter of Fig. (2-b)).

According to the above procedures, the original image will be transformed into four sub-images, illustrated in Fig. (2). Practically, the wavelet transformation may be carried out several times, each time a filter bank is used to produce a low-pass and high-pass filtered images which then transformed into four half-size sampled images. Fig. (3), shows an example of two-level discrete wavelet transform.

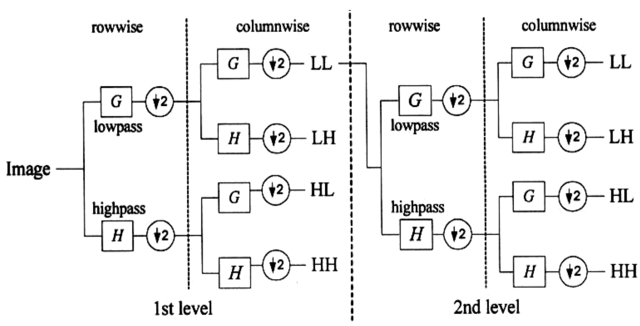



**Figure (3): Two-level wavelet transform, (Mallat, 1989).**

The inverse wavelet transform is used to reconstruct the original image from its wavelet decomposition. Hence, the generation of the transformed sub-images in a recursive process is considered a *decomposition process* of the processed image, whereas the reverse operation, i.e. reproducing the original image, is referred to as a *reconstruction process*. Fig. (4) demonstrates the decomposition and reconstruction operations in the wavelet transform (Shi and Sun, 1999; Parker, 1997; Mallat, 1989).

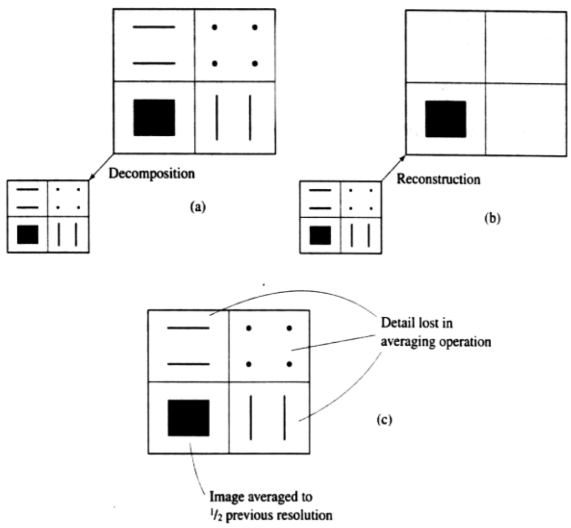



**Figure (4): Wavelet decomposition and reconstruction processes, (Parker, 1997).**

In this work, we have used the Haar wavelet and adopted other discrete form of wavelet transform by which the number of pixels in the reproduced filtered images is (1/4) and (1/8) of the total number of pixels in the processed image. This is because the multi-resolution analysis is not convenient for our re-sampling method.

## 2 The Haar Wavelet Transform

The Haar wavelet is probably the simplest of all wavelets, and is also the earliest. The basic Haar wavelet is a step function taking the values +1 and –1 that is defined as follows, see Fig. (5) (Asuncion, 2003; Parker, 1997):

$$\Psi(r) = \begin{cases} 1 & 0 \le r \le 0.5 \\ -1 & 0.5 \le r \le 1 \\ 0 & otherwise \end{cases} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

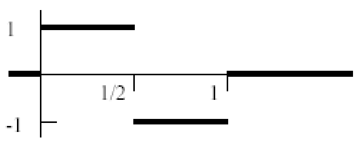



**Figure (5): The Haar wavelet, (Asuncion, 2003).**

For the Haar transform, the forward wavelet filters can be expressed as

Low- pass filter: $\frac{1}{\sqrt{2}}[1,\ 1]$

High- pass filter: $\frac{1}{\sqrt{2}}[1,\ -1]$

The inverse wavelet transform is performed by reducing the wavelet transform data into its original size. The inverse Haar transform filters like the forward Haar transform filters (Sandberg, 2000).

## 3 The Experimental Procedures

In what follows, we shall summarize the steps used for evaluation results;

- Select a test image f (x, y),
- Generate a sampled image $f_s$ (x, y) of f (x, y) using scanner device,
- Perform sampling-rate reduction of $f_s$ (x, y) by extracting every intermediate original sample value by utilizing scanner device itself,
- Select a scale-change factor defined as (1/i), where “i” is an integer smaller than 1,
- Use the adopted technique to reconstruct all intermediate pixels omitted in the above step to reproduce an approximation $f_r$ (x, y) of f (x, y),
- Compare $f_r$ (x, y) and f (x, y) to determine how well the re-sampling algorithm is performed.

We shall illustrate the results quantitatively, using set of the quantitative measures and subjectively by demonstrating, visually, the performance of technique. In fact, it is well known that a useful quantitative measure should be coincident well with subjective test. Two types of test images have been utilized to demonstrate the performance of adopted technique; i.e. low contrast astronomical image (e.g. representing the Spiral Galaxy image), and high contrast real scene image (e.g. representing Flower picture).

**Table (1):** The quantitative measures (root-mean-square error, peak signal-to-noise ratio, mean, variance, and standard deviation) of wavelet transform with scale-change factor (2) for the Galaxy image.

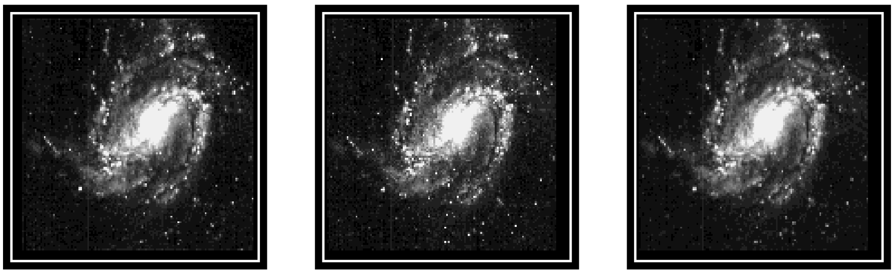
| Wavelet transform. | rms. | psnr. | mean | var. | std. |
|---|---|---|---|---|---|
| | | | 47.661 | 2979.741 | 54.587 |
| Haar wavelet transform with four bands. | 15.831 | 64.088 | 48.382 | 2978.722 | 54.578 |
| Haar wavelet transform with two bands. | 7.134 | 95.474 | 48.317 | 2790.148 | 52.822 |

**Table (2):** The quantitative measures (root-mean-square error, peak signal-to-noise ratio, mean, variance, and standard deviation) of wavelet transform with scale-change factor (4) for the Galaxy image.

| Wavelet transform. | rms. | psnr. | mean | var. | std. |
|---|---|---|---|---|---|
| | | | 51.060 | 2984.698 | 54.632 |
| Haar wavelet transform with four bands. | 19.164 | 58.251 | 47.422 | 2941.303 | 54.234 |
| Haar wavelet transform with two bands. | 9.449 | 82.955 | 47.663 | 2738.822 | 52.334 |

**Table (3):** The quantitative measures (root-mean-square error, peak signal-to-noise ratio, mean, variance, and standard deviation) of wavelet transform with scale-change factor (2) for the Flower image.

| Wavelet transform. | rms. | psnr. | mean | var. | std. |
|---|---|---|---|---|---|
| | | | 148.421 | 7842.743 | 88.559 |
| Haar wavelet transform with two bands. | 18.284 | 59.635 | 148.486 | 7516.24 | 86.696 |



(a) (b) (c)

**Figure (1):** The performance of Haar wavelet transform with scale-change factor (2). (a) Test image, a Spiral Galaxy, 128×128 pixels. Images ((b) and (c)) are 128×128 pixels images produced from a 256×256 test image with four and two bands.

(a) (b) (c)

**Figure (2):** The performance of Haar wavelet transform with scale-change factor (4). (a) Test image, a Spiral Galaxy, 64×64 pixels. Images ((b) and (c)) are 64×64 pixels images produced from a 256×256 test image with four and two bands.

(a) (b) (c)

**Figure (3):** The performance of Haar wavelet transform with scale-change factor (2). (a) Test image, a Flower, 128×128 pixels. Images ((b) and (c)) are 128×128 pixels images produced from a 256×256 test image with four and two bands.

## 4 Discussion and Conclusions

In general, wavelet transform produces quite good quantitative and subjective results. Therefore, it can be said that this technique has a successful coincidence between quantitative and subjective tests. In particular, the quantitative results are substantially better when the test image involves less details and lower contrast level; i.e. a Galaxy image yielded higher PSNR values than the Flower image, see Tables ((1) to (3)). Also, these results have proved better especially, when less bands are adopted and when scale-change factors is 2. Subjectively, this method can preserve fine details and the contrast of the image especially, if less bands are adopted. The Haar wavelet algorithm has the advantage of being simple to compute and easier to understand.

## References

Asuncion, A. , "Signal processing applications of wavelets", 2003, University of California.

Fugal, L., "Conceptual wavelets in digital signal processing", 2006, Space & Signals technologies LLc, U.S.A..

Gomes, J. and Velho, L., "Image processing for computer graphics", 1997, Springer-Verlag New York, U. S. A..

Maeland, E., "On the comparison of interpolation methods", 1988, IEEE transactions on medical imaging, vol. (7), no. (3).

Mallat, S., "A theory of multiresolution signal decomposition: the wavelet representation", IEEE Trans. pattern analysis and machine inteelligence, 1989, vol. (11).

Newland, D. E., "An introduction to random vibrations, spectral and wavelet analysis", 1993, Longman scientific & Technical, Singapore .

Niblack, W., "An introduction to digital image processing", 1986, Prentic-Hall international (UK) Ltd.

Parker, J. A., Kenyon, R. V. and Troxel, D. E., "Comparison of interpolation methods for image resampling", 1983, IEEE transactions on medical imaging, vol. (MI-2), no. (1).

Parker, J. R., "Algorithms for image processing and computer vision", 1997, John Wiley & Sons, U.S.A..

Richards, J. A. and Jia, X., "An Introduction to Remote Sensing Digital Image Analysis ", 1999, Springer-Verlag Berlin Heidelberg, Germany.

Rosenfeld, A. and Kak, A. C., "Digital picture processing", 1982, Academic press, U. S. A..

Sandberg, K. , "The Haar wavelet transform", 2000, University of Colorado at Boulder.

Shi, Y. Q. and Sun, H., "Image and video compression for multimedia engineering", 1999, CRC press, U.S.A..

Starck, J. L., Murtagh, F. and Bijaoui, A., "Image processing and data analysis", 1998, Cambridge university press, United Kingdom.

Torrence, C. and Compo, G. P. , "A practical guide to wavelet analysis", 1998, American meteorological society, vol. (79), no. (1).