Various Resolvable Space in Ideal Topological Spaces

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Abstract

In this research,we introduce a new definitions of resolvable spaces of the depended on the idea:Weakly-$\mathfrak{I}$-dense ,$\mathfrak{I}$-dense,$\mathfrak{T}$*-dense and dense.The definitions are: $\mathfrak{W}\mathfrak{I}$-resolvable space, e-resolvable space, $\mathfrak{W}\mathfrak{I}$-$\mathfrak{I}$-resolvable space, $\mathfrak{W}\mathfrak{I}$-$\mathfrak{T}$*-resolvable space, $\mathfrak{W}\mathfrak{I}$-$\mathfrak{T}$-resolvable space, $\mathfrak{I}$-$\mathfrak{T}$*-resolvable space, $\mathfrak{I}$-$\mathfrak{T}$-resolvable space and $\mathfrak{T}$*-$\mathfrak{T}$-resolvable space. We prove various results in the field resolvable space.

Keywords: dense,resolvability,Hausdorff and Ψ -operator.

الخلاصة

في هذا البحث ,نقدم تعاريف جديدة من الفضاءات القابلة للحل والمعتمدة على فكرة كثافة–*T–كثافة I, –كثافة,ضعيفة –I– كثيفة وهذه التعريفات هي : WI–الفضاء القابل للحل, E–الفضاء القابل للحل , WI–I –الفضاء القابل للحل, *WI–T– الفضاء القابل للحل,WI–T –الفضاء القابل للحل,*I–T– الفضاء القابل للحل,I–T–الفضاء القابل للحل,T*–T– الفضاء القابل للحل.وحصلنا على النتائج المختلفة في حقل الفضاء القابل للحل.

الكلمات المفتاحية: الكثافة ,الفضاء القابل للحل ,هاوسدورف, Ψ – العاملة .

1. Introduction and Preliminaries

In (Hewitt, 1943) introduced the result: If there exists two disjoint union dense subsets, then this means that $\mathfrak{Z}$ is resolvable space. Chattopadhyay(1992) have been studied the resolvability, irresolvability space and properties of maximal spaces. Furthermore, the prove of density topology is resolvable such as: Dontchev *et al.*, (1999). In 1966,Kuratowski define an ideal I on topological space ($\mathfrak{Z}$,$\mathfrak{T}$) is a nonempty collection of subsets of $\mathfrak{Z}$ which satisfies:

1. If $D \in \mathfrak{I}$ and $G \subseteq D$ implies $G \in \mathfrak{I}$.
2. If $D \in \mathfrak{I}$ and $G \in \mathfrak{I}$ implies $D \bigcup G \in \mathfrak{I}$.

Moreover, a σ-ideal on a topological space($\mathfrak{Z}$,$\mathfrak{T}$) is an ideal which satisfies (1),(2) the following condition:

3.If $\{D_i: i=1,2,3,...\} \subseteq \mathfrak{I}$, then $\bigcup \{D_i=1,2,3,...\} \in \mathfrak{I}$ (countable additively), for further information see(Kuratowski,1966).

For a space ($\mathfrak{Z}$,$\mathfrak{T}$,$\mathfrak{I}$) and a subset $D \subseteq \mathfrak{Z}$,$D^*(\mathfrak{I})=\{z \in \mathfrak{Z}: W \bigcap D \notin \mathfrak{I}$ for every $W \in \mathfrak{T}(z)\}$ is called the local function of D with respect to $\mathfrak{I}$ and $\mathfrak{T}$ (Kuratowski,1933). A Kuratowski closure operator cl*(.) for a topology $\mathfrak{T}^*(\mathfrak{I},\mathfrak{T})$ called the $*$-topology, finer than $\mathfrak{T}$,is defined by $cl^*(D)=D \bigcup D^*$.$D \subseteq \mathfrak{Z}$ is called $*$-closed (Jankovic and Hamlett,1990) if cl*(D)=D, and D is called $*$-open (i.e.,$D \in \mathfrak{T}^*$) if $\mathfrak{Z}$-D is $*$-closed. Obviously, D is $*$-open if and only if int*(D)=D. Let ($\mathfrak{Z}$,$\mathfrak{I}$,$\mathfrak{T}$) be an ideal space and let $\mathfrak{P} \subseteq \mathfrak{Z}$. Then $(\mathfrak{P},\mathfrak{T}_{\mathfrak{P}},\mathfrak{I}_{\mathfrak{P}})$ is an ideal space, where $\mathfrak{T}_{\mathfrak{P}}=\{Q \bigcap \mathfrak{P}: Q \in \mathfrak{T}\}$ and $I_{\mathfrak{P}}=\{\mathfrak{I} \bigcap \mathfrak{P}: I \in \mathfrak{I}\}=\{I \in \mathfrak{I}: I \subseteq \mathfrak{P}\}$. A subset D of an ideal space ($\mathfrak{Z}$,$\mathfrak{T}$,$\mathfrak{I}$) is called dense ($*$-dense ($\mathfrak{I}$-

dense (Dontchev, ,1999))),if cl(D)=$\mathfrak{Z}$,(resp.cl*(D)=$\mathfrak{Z}$,(D*=$\mathfrak{Z}$)). For an ideal space ($\mathfrak{Z}$,$\mathfrak{T}$,$\mathfrak{I}$), $\mathfrak{I}$ is codense (Devi, Sivaraj and Chelvam,2005) if $\mathfrak{T}\cap\mathfrak{I}=\Phi$. A subset D of an ideal space ($\mathfrak{Z}$,$\mathfrak{T}$,$\mathfrak{I}$) is said to be $\mathfrak{I}$-open(Jankovic and Hamlett,1992)(pre-$\mathfrak{I}$-open(Dontchev,1996), if $D\subseteq$int(D*),(resp. $D\subseteq$intcl*(D)). A subset D of a space ($\mathfrak{Z}$,$\mathfrak{T}$) is said to be preopen (Mashhour, Abd El-Monsef and El-Deeb,1982) if $D\subseteq$ intcl(D).A set $D\subseteq\mathfrak{Z}$ is said to be scattered(Jankovic and Hamlett,1990) if D contains no nonempty dense-in-itself subset. An ideal space ($\mathfrak{Z}$,$\mathfrak{T}$,$\mathfrak{I}$) is called Hausdorff (Hausdorff,1957),if for each two points $z\neq p$, there exist open sets U and V containing z and p respectively, such that $U\cap V=\Phi$.T. Natkaniec (Natkaniec,1986) used the idea of ideals to define another operator known as Ψ-operator. The definition of $\Psi_{\mathfrak{T}}$(D) for a subset D of $\mathfrak{Z}$ is as follows: $\Psi_{\mathfrak{T}}$(D)=$\mathfrak{Z}$-($\mathfrak{Z}$-D)*.Equivalently $\Psi_{\mathfrak{T}}(D)=\bigcup\{M\in\mathfrak{T}:M-D\in\mathfrak{I}\}$.It is obvious that $\Psi_{\mathfrak{T}}$(D) for any D is a member of $\mathfrak{T}$.(Hatir, Keskin and Noiri,2005 for an ideal space ($\mathfrak{Z}$,$\mathfrak{T}$,$\mathfrak{I}$) and let $D\subseteq\mathfrak{P}\subseteq\mathfrak{Z}$. Then $cl_{\mathfrak{P}}$*(D)=cl*(D)$\cap\mathfrak{P}$. (Devi, Sivaraj and Chelvam,2005) for an ideal space ($\mathfrak{Z}$,$\mathfrak{T}$,$\mathfrak{I}$) and $D\subset\mathfrak{Z}$.If $D\subset D^*$,then D*=cl(D*)=cl(D)=cl*(D).In this paper, we define and formulate a new definitions: $\mathfrak{W}\mathfrak{I}$-resolvable space and its generalizations, we investigate various results in the filed of resolvable space .

2- Weakly-$\mathfrak{I}$-Dense Sets and Generalizations.

In this section, we are using formula M** to define a new type of closure operators and denoted to be $cl^e(M)=M\cup M^{**}$,for any $M\subseteq\mathfrak{Z}$. This enable us to define a new types of dense.

Definition 2.1: Let ($\mathfrak{Z}$,$\mathfrak{T}$,$\mathfrak{I}$) be an ideal space and $M\subseteq\mathfrak{Z}$. Then M is called.

1. Weakly-$\mathfrak{I}$-dense ,if (M*)*=$\mathfrak{Z}$.
2. Emaciated-dense, if $cl^e(M)=\mathfrak{Z}$.
3. $\mathfrak{I}$-codense, if $\mathfrak{Z}$-M is $\mathfrak{I}$-dense.
4. $\mathfrak{W}\mathfrak{I}$-codense, if $\mathfrak{Z}$-M is weakly-$\mathfrak{I}$-dense.

Definition 2.2 : An ideal topological space ($\mathfrak{Z}$,$\mathfrak{T}$,$\mathfrak{I}$) is called thick if $M\subset M^*$ for every $M\subseteq\mathfrak{Z}$.

Remark 2.3: Every weakly-$\mathfrak{I}$-dense is $\mathfrak{I}$-dense and $\mathfrak{T}$*-dense and hence dense.

This tells us that: If M is weakly-$\mathfrak{I}$-dense ,then M is emaciated-dense which leads to:

Lemma 2.4: Let($\mathfrak{Z}$,$\mathfrak{T}$,$\mathfrak{I}$) be an ideal space and $M\subset\mathfrak{Z}$, then $M^{**}=cl^e(M)$ if $M\subset M^{**}$.

Proof: Since $M^{**}\subset cl^e(M)$ and $M\subset M^{**}$,then $M^{**}=cl^e(M)$.

Remark 2.5:Let ($\mathfrak{Z}$,$\mathfrak{T}$,$\mathfrak{I}$) be an ideal topological space.Then the following statement is hold: If M is emaciated-dense, then M is weakly-$\mathfrak{I}$-dense.

Proof: Get it from Lemma 2.4.

Remark 2.6:Let ($\mathfrak{Z}$,$\mathfrak{T}$,$\mathfrak{I}$) be an ideal topolgical space and $\mathfrak{I}$ is codense.Then the following statement is hold: If M is $\mathfrak{I}$-dense, then M is weakly-$\mathfrak{I}$-dense.

Proof: Let $z \in \mathfrak{Z}$ and if possible $z \notin M^{**}$,then there exists $U_z \in \mathfrak{T}(z)$ such that $U_z \cap M^* \in \mathfrak{I}$.Since $M^*=\mathfrak{Z}$,then $U_z \cap M^*=U_z \cap \mathfrak{Z}=U_z \in \mathfrak{I}$ which contradiction.Hence $z \in M^{**}$.Therefore M is weakly-$\mathfrak{I}$-dense in $\mathfrak{Z}$.

This an immediate consequence of Definition 2.2.

Theorem 2.7: Let $(\mathfrak{Z},\mathfrak{T},\mathfrak{I})$ be a thick and an ideal topological space.Then the following statements are holds.

1. If M is $\mathfrak{T}^*$-dense ,then M is $\mathfrak{I}$-dense.
2. If M is dense, then M is $\mathfrak{T}^*$-dense.

Example 2.8 :(1) Let $\mathfrak{Z}=\{a,b,c,d\}$, $\mathfrak{T}=\{\Phi,\{a,c\},\{b,d\},Z\}$, $\mathfrak{I}=\{\Phi,\{a\},\{c\},\{a,c\}\}$.Let $M=\{b,c\}$

Then $M^*=\{b,d\}$, $cl^*(M) \neq \mathfrak{Z}$ and $cl(M)=\mathfrak{Z}$.Thus M is dense but not $\mathfrak{T}^*$-dense.

(2) Let $\mathfrak{Z}=\{a,b,c\}$, $\mathfrak{T}=\{\Phi,\{a\},\{a,b\},Z\}$, $\mathfrak{I}=\{\Phi,\{a\}\}$.Let $M=\{a,b\}$.

Then $M^*=\{b,c\}$ and $cl^*(M)=\mathfrak{Z}$.Thus M is $\mathfrak{T}^*$-dense but not $\mathfrak{I}$-dense.

(3) Let $\mathfrak{Z}=\{a,b,c\}$, $\mathfrak{T}=\{\Phi,\{a\},\{a,b\},Z\}$, $\mathfrak{I}=\{\Phi,\{a,b\}\}$.Let $M=\{a,c\}$.

Then $M^*=\mathfrak{Z}$, $M^{**}=\{a,c\} \neq \mathfrak{Z}$. Thus M is $\mathfrak{I}$-dense but not weakly-$\mathfrak{I}$-dense.

Note that 1. M is $\mathfrak{I}$-codense, if and only if $\Psi(M)=\Phi$.

2. M is $\mathfrak{W}\mathfrak{I}$-codense, if and only if $\Psi(\Psi(M))=\Phi$.

We focus on the properties of the local function regarding to the formula M^{**} in the following theorem.

Theorem 2.9: Let $(\mathfrak{Z},\mathfrak{T})$ be a topological space with $\mathfrak{I}_1$ and $\mathfrak{I}_2$ are ideals on $\mathfrak{Z}$ and let M subset of $\mathfrak{Z}$. Then

1. If $\mathfrak{I}_1 \subseteq \mathfrak{I}_2$,then $M^{**}(\mathfrak{I}_2) \subseteq M^{**}(\mathfrak{I}_1)$.
2. For every $I \in \mathfrak{I}$,then $(M \cup I)^{**}=M^{**}=(M-I)^{**}$.
3. If $\mathfrak{T} \subset \sigma$,then $M^{**}(\mathfrak{I},\sigma) \subset M^{**}(\mathfrak{I},\mathfrak{T})$.

Proof: (1) Clear .

Proof: (2) Clearly from [(Jankovic and Hamlett,1990), Theorem 2.3(h)].

Corollary 2.10:

1. Let $\mathfrak{I}_1$ and $\mathfrak{I}_2$ being ideals on $\mathfrak{Z}$ such that $\mathfrak{I}_1 \subseteq \mathfrak{I}_2$.If M is weakly-$\mathfrak{I}_2$-dense,then M is weakly-$\mathfrak{I}_1$-dense.
2. Let $\mathfrak{I}_1$ and $\mathfrak{I}_2$ being ideals on $\mathfrak{Z}$ such that $\mathfrak{I}_1 \subseteq \mathfrak{I}_2$.If M is emaciated-dense with respect to $\mathfrak{I}_2$,then M is emaciated-dense with respect to $\mathfrak{I}_1$.

Proof: Get it from Theorem 2.9 .

Corollary 2.11:

1. Let $(\mathfrak{Z},\mathfrak{T},\mathfrak{I})$ and $(\mathfrak{Z},\sigma,\mathfrak{I})$ be two topological spaces with $\mathfrak{T}\subset\sigma$.If M is σ-weakly-$\mathfrak{I}$-dense, then M is $\mathfrak{T}$-weakly-$\mathfrak{I}$-dense.
2. Let $(\mathfrak{Z},\mathfrak{T},\mathfrak{I})$ and $(\mathfrak{Z},\sigma,\mathfrak{I})$ be two topological spaces with $\mathfrak{T}\subset\sigma$.If M is emaciated-dense with respect to σ,then M is emaciated-dense with respect to $\mathfrak{T}$.

Proof: Get it from Theorem 2.9.

Lemma 2.12:

1. Let $(\mathfrak{Z},\mathfrak{T},\mathfrak{I})$ be an ideal topological space and $M\subset\mathfrak{Y}\subset\mathfrak{Z}$,then $(M_{\mathfrak{y}}^*)_{\mathfrak{y}}^*=(M_{\mathfrak{z}}^*)_{\mathfrak{z}}^*\cap\mathfrak{Y}$ and $\mathfrak{Y}\in\mathfrak{T}$.
2. If M be weakly-$\mathfrak{I}$-dense subsets of $\mathfrak{Z}$ and U is open subset of $\mathfrak{Z}$,then $U\subseteq(U\cap M)^{**}$.

Proof.1. Let $(M_{\mathfrak{y}}^*)_{\mathfrak{y}}^*=(M_{\mathfrak{z}}^*\cap\mathfrak{Y})_{\mathfrak{y}}^*=(M_{\mathfrak{z}}^*\cap\mathfrak{Y})_{\mathfrak{z}}^*\cap\mathfrak{Y}\subset(M_{\mathfrak{z}}^*)_{\mathfrak{z}}^*\cap\mathfrak{Y}$.Hence $(M_{\mathfrak{y}}^*)_{\mathfrak{y}}^*\subset(M_{\mathfrak{z}}^*)_{\mathfrak{z}}^*\cap\mathfrak{Y}$ ......(1).

Now let $a\in(M_{\mathfrak{z}}^*)_{\mathfrak{z}}^*\cap\mathfrak{Y}$,then $a\in(M_{\mathfrak{z}}^*)_{\mathfrak{z}}^*$ and $a\in\mathfrak{Y}$,then $U\cap M_{\mathfrak{z}}^*\notin\mathfrak{I}$ for every $U\in\mathfrak{T}(a)$ and $a\in\mathfrak{Y}$.Since $\mathfrak{I}_{\mathfrak{y}}\subset\mathfrak{I}$,then $U\cap M_{\mathfrak{z}}^*\notin\mathfrak{I}_{\mathfrak{y}}$,for every $U\in\mathfrak{T}(a)$ and $a\in\mathfrak{Y}\in\mathfrak{T}$,then $a\in U\cap\mathfrak{Y}\in\mathfrak{T}(a)$,we have $U\cap\mathfrak{Y}\cap M_{\mathfrak{z}}^*\notin\mathfrak{I}_{\mathfrak{y}}$,for every $U\cap\mathfrak{Y}\in\mathfrak{T}_{\mathfrak{y}}(a)$.Hence $a\in(M_{\mathfrak{y}}^*)_{\mathfrak{y}}^*$.Therefore $(M_{\mathfrak{z}}^*)_{\mathfrak{z}}^*\cap\mathfrak{Y}\subset(M_{\mathfrak{y}}^*)_{\mathfrak{y}}^*\rightarrow(2)$.From (1) and (2),it follows that$(M_{\mathfrak{y}}^*)_{\mathfrak{y}}^*=(M_{\mathfrak{z}}^*)_{\mathfrak{z}}^*\cap\mathfrak{Y}$.

2. Let $z\in U$ and if possible $z\notin(U\cap M)^{**}$,then there exists $V\in\mathfrak{T}(z)$ such that $V\cap(U\cap M)^*\in\mathfrak{I}$.Since $U\cap M^*\subset(U\cap M)^*$,then $V\cap U\cap M^*\subset V\cap(U\cap M)^*$,then $V\cap U\cap M^*\in\mathfrak{I}$ which contradiction with $z\in M^{**}$.Therefore $U\subseteq(U\cap M)^{**}$.

Proposition 2.13: Let $(\mathfrak{Z},\mathfrak{T},\mathfrak{I})$ be an ideal topological space. If $\mathfrak{Y}\subset\mathfrak{Z}$ and $(M_{\mathfrak{z}}^*)_{\mathfrak{z}}^*=\mathfrak{Z}$, then $(M_{1\mathfrak{y}}^*)_{\mathfrak{y}}^*=\mathfrak{Y}$ where $M_1=M\cap\mathfrak{Y}$.

Proof: The proof is clear using Lemma 2.12.

Lemma 2.14:

1. If M be emaciated-dense subsets of $\mathfrak{Z}$ and U is open subset of $\mathfrak{Z}$, then $U\subseteq cl^e(U\cap M)$.
2. let $(\mathfrak{Z},\mathfrak{T},\mathfrak{I})$ be an ideal topological space and $M\subseteq\mathfrak{Y}\subseteq\mathfrak{Z}$, then $cl^e_{\mathfrak{y}}(M)=cl^e_{\mathfrak{z}}(M)\cap\mathfrak{Y}$ and $\mathfrak{Y}\in\mathfrak{T}$.

Proof: Clear.

Proposition 2.15: Let $(\mathfrak{Z},\mathfrak{T},\mathfrak{I})$ be an ideal topological space. If $\mathfrak{Y}$ is open subsets of $\mathfrak{Z}$ and $cl^e_{\mathfrak{z}}(M)=\mathfrak{Z}$, then $cl^e_{\mathfrak{y}}(M_1)=\mathfrak{Y}$ where $M_1=M\cap\mathfrak{Y}$.

Proof: The proof is clear using Lemma 2.14.

Lemma 2.16:

1. If M be $\mathfrak{T}$*-dense subsets of $\mathfrak{Z}$ and U is $\mathfrak{T}$*-open subset of $\mathfrak{Z}$, then U$\subseteq$cl*(U$\cap$M).
2. If M be $\mathfrak{T}$-dense subsets of $\mathfrak{Z}$ and U is open subset of $\mathfrak{Z}$, then U$\subseteq$(U$\cap$M)*.

Proof: Clear.

Proposition 2.17:

1. Let ($\mathfrak{Z}$,$\mathfrak{T}$,$\mathfrak{I}$) be an ideal topological space. If $\mathfrak{Y}$ is open subset of $\mathfrak{Z}$ and $M_{\mathfrak{z}}$*=$\mathfrak{Z}$, then $M_{1\mathfrak{y}}$*=$\mathfrak{Y}$ where M_1=M$\cap\mathfrak{Y}$.
2. Let ($\mathfrak{Z}$,$\mathfrak{T}$,$\mathfrak{I}$) be an ideal topological space. If $\mathfrak{Y}$ is $\mathfrak{T}$*-open subset of $\mathfrak{Z}$ and $cl_{\mathfrak{z}}$*(M)=$\mathfrak{Z}$, then $cl_{\mathfrak{y}}$*(M_1)=$\mathfrak{Y}$ where M_1=M$\cap\mathfrak{Y}$.
3. Let ($\mathfrak{Z}$,$\mathfrak{T}$,$\mathfrak{I}$) be an ideal topological space. If $\mathfrak{Y}$ is open subset of $\mathfrak{Z}$ and $cl_{\mathfrak{z}}$(M)=$\mathfrak{Z}$, then
   $Cl_{\mathfrak{y}}$(M_1)=$\mathfrak{Y}$, where M_1=M$\cap\mathfrak{Y}$.

   Proof.Get it from Lemma 2.16.

Definition 2.18: A function $\mathfrak{F}$:($\mathfrak{Z}$,$\mathfrak{T}$,$\mathfrak{I}$)→($\mathfrak{Y}$,σ,$\mathfrak{J}$) is called:

1. $\mathfrak{W}$-$\mathfrak{I}$-map,if $\mathfrak{F}$(M**)$\subseteq$($\mathfrak{F}$(M))**.
2. $\mathfrak{E}$-$\mathfrak{I}$-map,if $\mathfrak{F}$(cl^e(M))$\subseteq$$cl^e$($\mathfrak{F}$(M)).
3. $\mathfrak{W}_0$-$\mathfrak{I}$-map,if $\mathfrak{F}$(M*)$\subseteq$($\mathfrak{F}$(M))*.

This tells us that: $\mathfrak{W}_\circ$-$\mathfrak{I}$-map → $\mathfrak{W}$-$\mathfrak{I}$-map and $\mathfrak{W}$-$\mathfrak{I}$-map → $\mathfrak{E}$-$\mathfrak{I}$-map.

Theorem 2.19: Let $\mathfrak{F}$:($\mathfrak{Z}$,$\mathfrak{T}$,$\mathfrak{I}$)→($\mathfrak{Y}$,σ,$\mathfrak{J}$) be a $\mathfrak{W}$-$\mathfrak{I}$-map and bijection. If M is weakly-$\mathfrak{I}$-dense in $\mathfrak{Z}$, then $\mathfrak{F}$(M) is weakly-$\mathfrak{J}$-dense in $\mathfrak{Y}$.

Proof: Suppose that M is weakly-$\mathfrak{I}$-dense in $\mathfrak{Z}$, then M**=$\mathfrak{Z}$, implies that $\mathfrak{Y}$=$\mathfrak{F}$($\mathfrak{Z}$)=$\mathfrak{F}$(M**)$\subseteq$($\mathfrak{F}$(M))** because $\mathfrak{F}$ is $\mathfrak{W}$-$\mathfrak{I}$-map and bijection. Thus $\mathfrak{Y}$=($\mathfrak{F}$(M))**. Hence $\mathfrak{F}$(M) is weakly-$\mathfrak{J}$-dense in $\mathfrak{Y}$.

Corollary 2.20: Let $\mathfrak{F}$:($\mathfrak{Z}$,$\mathfrak{T}$,$\mathfrak{I}$)→($\mathfrak{Y}$,σ,$\mathfrak{J}$) be a $\mathfrak{W}_\circ$-$\mathfrak{I}$-map and bijection. If M is weakly-$\mathfrak{I}$-dense in $\mathfrak{Z}$, then $\mathfrak{F}$(M) is weakly-$\mathfrak{J}$-dense in $\mathfrak{Y}$.

Proof: Clear.

Corollary 2.21: Let $\mathfrak{F}$:($\mathfrak{Z}$,$\mathfrak{T}$,$\mathfrak{I}$)→($\mathfrak{Y}$,σ,$\mathfrak{J}$) be a $\mathfrak{W}$-$\mathfrak{I}$-map and bijection. If M is weakly-$\mathfrak{I}$-dense in $\mathfrak{Z}$,then $\mathfrak{F}$(M) is emaciated-dense in $\mathfrak{Y}$.

Proof: Get it from Theorem 2.19.

Theorem 2.22: Let $\mathfrak{F}$:($\mathfrak{Z}$,$\mathfrak{T}$,$\mathfrak{I}$)→($\mathfrak{Y}$,σ) be a $\mathfrak{W}_\circ$-$\mathfrak{I}$-map and bijection. If M is $\mathfrak{I}$-dense in $\mathfrak{Z}$, then $\mathfrak{F}$(M) is $\mathfrak{J}$-dense in $\mathfrak{Y}$.

Proof: Proof resemble proof Theorem 2.19.

Theorem 2.23: Let $\mathfrak{F}$:($\mathfrak{Z}$,$\mathfrak{T}$,$\mathfrak{I}$)→($\mathfrak{Y}$,σ,$\mathfrak{J}$) be a $\mathfrak{E}$-$\mathfrak{I}$-map and bijection. If M is emaciated-dense in $\mathfrak{Z}$, then $\mathfrak{F}$(M) is emaciated-dense in $\mathfrak{Y}$.

Proof: Suppose that M is emaciated-dense in $\mathfrak{Z}$, then $cl^e(M)=\mathfrak{Z}$ implies that $\mathfrak{Y}=\mathfrak{F}(\mathfrak{Z})=\mathfrak{F}(cl^e(M))\subseteq cl^e(\mathfrak{F}(M))$ because $\mathfrak{F}$ is $\mathfrak{E}$-$\mathfrak{I}$-map and bijection.Thus $\mathfrak{Y}=cl^e(\mathfrak{F}(M))$. Hence $\mathfrak{F}(M)$ is emaciated-dense in $\mathfrak{Y}$.

Corollary 2.24: Let $\mathfrak{F}:(\mathfrak{Z},\mathfrak{T},\mathfrak{I})\rightarrow(\mathfrak{Y},\sigma,\mathfrak{J})$ be a $\mathfrak{W}$-$\mathfrak{I}$-map and bijection. If M is emaciated-dense in $\mathfrak{Z}$, then $\mathfrak{F}(M)$ is emaciated-dense in $\mathfrak{Y}$.

3- $\mathfrak{WI}$-Resolvability and Generalizations.

In this section, we offer a new definitions of resolvable spaces in terms of formula M** and indicate its properties and relationships.

Definition 3.1: An ideal topological space $(\mathfrak{Z},\mathfrak{T},\mathfrak{I})$ is called

1. $\mathfrak{WI}$-resolvable, if $\mathfrak{Z}$ is the disjoint union of two weakly-$\mathfrak{I}$-dense subsets.
2. e-resolvable, if $\mathfrak{Z}$ is the disjoint union of two emaciated-dense subsets.
3. $\mathfrak{WI}$-$\mathfrak{I}$-resolvable, if $\mathfrak{Z}$ is the disjoint union of two weakly-$\mathfrak{I}$-dense and $\mathfrak{I}$-dense subsets.
4. $\mathfrak{WI}$-$\mathfrak{T}$*-resolvable, if $\mathfrak{Z}$ is the disjoint union of two weakly-$\mathfrak{I}$-dense and $\mathfrak{T}$*-dense subsets.
5. $\mathfrak{WI}$-$\mathfrak{T}$-resolvable, if $\mathfrak{Z}$ is the disjoint union of two weakly-$\mathfrak{I}$-dense and $\mathfrak{T}$-dense subsets.
6. $\mathfrak{I}$-$\mathfrak{T}$*-resolvable, if $\mathfrak{Z}$ is the disjoint union of two $\mathfrak{I}$-dense and $\mathfrak{T}$*-dense subsets.
7. $\mathfrak{I}$-$\mathfrak{T}$-resolvable, if $\mathfrak{Z}$ is the disjoint of two $\mathfrak{I}$-dense and $\mathfrak{T}$-dense subsets.
8. $\mathfrak{T}$*-$\mathfrak{T}$-resolvable, if $\mathfrak{Z}$ is the disjoint union of two $\mathfrak{T}$*-dense and $\mathfrak{T}$-dense subsets.

Theorem 3.2: Let $(\mathfrak{Z},\mathfrak{T},\mathfrak{I})$ be an ideal topological space. The following statements are holds.

1. Every $\mathfrak{WI}$-resolvable space is $\mathfrak{WI}$-$\mathfrak{I}$-resolvable space.
2. Every $\mathfrak{WI}$-$\mathfrak{I}$-resolvable space is $\mathfrak{WI}$-$\mathfrak{T}$*-resolvable space.
3. Every $\mathfrak{WI}$-$\mathfrak{T}$*-resolvable space is $\mathfrak{WI}$-$\mathfrak{T}$-resolvable space.
4. Every $\mathfrak{WI}$-$\mathfrak{T}$-resolvable space is $\mathfrak{I}$-$\mathfrak{T}$-resolvable space.
5. Every $\mathfrak{I}$-$\mathfrak{T}$*-resolvable space is $\mathfrak{I}$-$\mathfrak{T}$-resolvable space
6. Every $\mathfrak{I}$-$\mathfrak{T}$-resolvable space is $\mathfrak{T}$*-$\mathfrak{T}$-resolvable space.
7. Every $\mathfrak{WI}$-resolvable space is e-resolvable space.

Proof.Get it from Remark 2.3.

This direct consequence of Remark 2.6.

Proposition 3.3: Let $(\mathfrak{Z},\mathfrak{T},\mathfrak{I})$ be an ideal topological space and $\mathfrak{I}$ is codense.

If Z is $\mathfrak{WI}$-$\mathfrak{I}$-resolvable space,then $\mathfrak{Z}$ is $\mathfrak{WI}$-resolvable space.

This direct consequence of Remark 2.6 and Theorem 2.7.

Proposition 3.4: Let (Ƶ,$\mathfrak{T}$,$\mathfrak{I}$) be a thick and an ideal topological space and $\mathfrak{I}$ is codense.Then the following statements are holds.

(1) If Ƶ is $\mathfrak{I}$-$\mathfrak{T}$*-resolvable space,then Ƶ is $\mathfrak{WI}$-$\mathfrak{I}$-resolvable space.

(2) If Ƶ is $\mathfrak{I}$-$\mathfrak{T}$-resolvable space,then Ƶ is $\mathfrak{WI}$-$\mathfrak{T}$*-resolvable space.

(3) If Ƶ is $\mathfrak{T}$*-$\mathfrak{T}$-resolvable space,then Ƶ is $\mathfrak{WI}$-$\mathfrak{T}$-resolvable space.

This an immediate consequence of Theorem 2.7.

Proposition 3.5: Let (Ƶ,$\mathfrak{T}$,$\mathfrak{I}$) be a thick and an ideal topological space.Then the following statements are hold.

(1) If Ƶ is $\mathfrak{WI}$-$\mathfrak{T}$-resolvable space ,then Ƶ is $\mathfrak{WI}$-$\mathfrak{T}$*-resolvable space.

(2) If Ƶ is $\mathfrak{WI}$-$\mathfrak{T}$* -resolvable space ,then Ƶ is $\mathfrak{WI}$-$\mathfrak{I}$-resolvable space.

(3) If Ƶ is $\mathfrak{T}$*-$\mathfrak{T}$-resolvable space ,then Ƶ is $\mathfrak{I}$ -$\mathfrak{T}$-resolvable space.

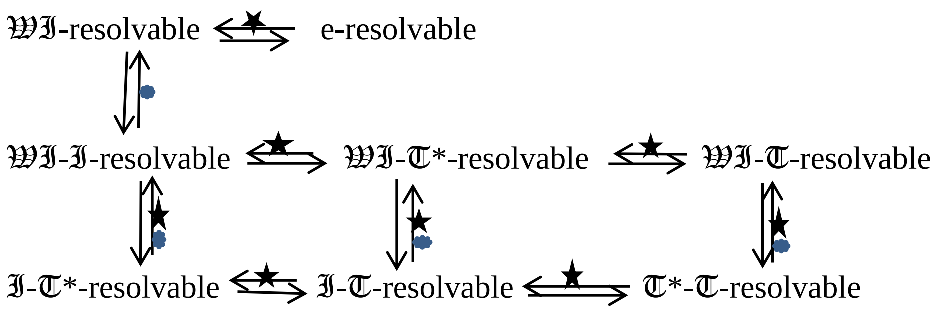
(4) If Ƶ is $\mathfrak{I}$-$\mathfrak{T}$-resolvable space ,then Ƶ is $\mathfrak{I}$-$\mathfrak{T}$* -resolvable space.

Example 3.6: Let Ƶ={a,b,c,d} , $\mathfrak{T}$={ Φ ,{a,c},{a,c,d},Z} , $\mathfrak{I}$={ Φ ,{c},{a,c}}.Let M={a,b} and G={c,d}.Then cl(G)=Ƶ and M*=Ƶ.Now G$\cap${a,c}={c}$\in\mathfrak{I}$,then a,c$\notin$ G*, G$\cap${Z}={c,d}$\in\mathfrak{I}$,then b$\notin$G* and G$\cap${a,c,d}={c,d}$\notin\mathfrak{I}$,then d$\in$G*.Thus G*={b,d}$\neq$Ƶ and cl*(G)$\neq$Ƶ.Therefore Ƶ is $\mathfrak{I}$-$\mathfrak{T}$-resolvable but not $\mathfrak{I}$-$\mathfrak{T}$*-resolvable.

Question: (1) We claim $\mathfrak{WI}$-$\mathfrak{I}$-resolvability does not imply $\mathfrak{WI}$-resolvability,but we could we find an example.

(2) We claim e-resolvability does not imply $\mathfrak{WI}$-resolvability,but we could we find an example.

Remark 3.7: By the above definitions, we obtain the following applications.



: $\mathfrak{I}$ is codense.

★: Z is thick

Remark 3.8:

1. If $\mathfrak{I}_1$ and $\mathfrak{I}_2$ are ideals with $\mathfrak{I}_1\subseteq\mathfrak{I}_2$,Ƶ is $\mathfrak{WI}_2$-resolvable implies that Ƶ is $\mathfrak{WI}_1$-resolvable.

2. If $\mathfrak{T}$ and σ are topological spaces with $\mathfrak{T}\subseteq\sigma$,$\mathfrak{Z}$ is σ-$\mathfrak{WI}$-resolvable implies that $\mathfrak{Z}$ is $\mathfrak{T}$-$\mathfrak{WI}$-resolvable.

Theorem 3.9: Let ($\mathfrak{Z}$,$\mathfrak{T}$,$\mathfrak{I}$) be an ideal topological space .Then the following statements are hold:If $\mathfrak{Z}$ be a $\mathfrak{WI}$-resolvable and $\mathfrak{Y}$ is open subset of $\mathfrak{Z}$. Then $\mathfrak{Y}$ is $\mathfrak{WI}$-resolvable subspace of $\mathfrak{Z}$.

Proof: Assume that $\mathfrak{Z}$ is $\mathfrak{WI}$-resolvable, then $M\cup G=\mathfrak{Z}$ and $M\cap G=\Phi$,where $M^{**}=\mathfrak{Z}$ and $G^{**}=\mathfrak{Z}$. Note that $\mathfrak{Y}=(\mathfrak{Y}\cap M)\cup(\mathfrak{Y}\cap G)$ and $(\mathfrak{Y}\cap M)\cap(\mathfrak{Y}\cap G)=\Phi$.Put $M_1=M\cap\mathfrak{Y}$ and $G_1=G\cap\mathfrak{Y}$,then $M_1\cup G_1=\mathfrak{Y}$ and $M_1\cap G_1=\Phi$.To prove($M_1{}_{\mathfrak{y}}{}^*)_{\mathfrak{y}}{}^*=\mathfrak{Y}$ and $(G_1{}_{\mathfrak{y}}{}^*)_{\mathfrak{y}}{}^*=\mathfrak{Y}$. So by Proposition 2.13,it follows that $(M_1{}_{\mathfrak{y}}{}^*)_{\mathfrak{y}}{}^*=\mathfrak{Y}$ and $(G_1{}_{\mathfrak{y}}{}^*)_{\mathfrak{y}}{}^*=\mathfrak{Y}$. Hence ($\mathfrak{Y}$,$\mathfrak{T}_{\mathfrak{y}}$,$\mathfrak{I}_{\mathfrak{y}}$) is $\mathfrak{WI}$-resolvable.

Theorem 3.10: Let ($\mathfrak{Z}$,$\mathfrak{T}$,$\mathfrak{I}$) be an ideal topological space .Then the following statements are hold:

1. If $\mathfrak{Z}$ be a $\mathfrak{WI}$-$\mathfrak{I}$-resolvable and $\mathfrak{Y}$ is open subset of $\mathfrak{Z}$. Then $\mathfrak{Y}$ is $\mathfrak{WI}$-$\mathfrak{I}$-resolvable subspace of $\mathfrak{Z}$.
2. If $\mathfrak{Z}$ be a $\mathfrak{WI}$-$\mathfrak{T}^*$-resolvable and $\mathfrak{Y}$ is open subset of $\mathfrak{Z}$. Then $\mathfrak{Y}$ is $\mathfrak{WI}$-$\mathfrak{T}^*$-resolvable subspace of $\mathfrak{Z}$.
3. If $\mathfrak{Z}$ be a $\mathfrak{WI}$-$\mathfrak{T}$-resolvable and $\mathfrak{Y}$ is open subset of $\mathfrak{Z}$. Then $\mathfrak{Y}$ is $\mathfrak{WI}$-$\mathfrak{T}$-resolvable subspace of $\mathfrak{Z}$.
4. If $\mathfrak{Z}$ be a $\mathfrak{I}$-$\mathfrak{T}^*$-resolvable and $\mathfrak{Y}$ is open subset of $\mathfrak{Z}$. Then $\mathfrak{Y}$ is $\mathfrak{I}$-$\mathfrak{T}^*$-resolvable subspace of $\mathfrak{Z}$.
5. If $\mathfrak{Z}$ be a $\mathfrak{I}$-$\mathfrak{T}$-resolvable and $\mathfrak{Y}$ is open subset of $\mathfrak{Z}$. Then $\mathfrak{Y}$ is $\mathfrak{I}$-$\mathfrak{T}$-resolvable subspace of $\mathfrak{Z}$.
6. If $\mathfrak{Z}$ be a $\mathfrak{T}^*$-$\mathfrak{T}$-resolvable and $\mathfrak{Y}$ is open subset of $\mathfrak{Z}$. Then $\mathfrak{Y}$ is $\mathfrak{T}^*$-$\mathfrak{T}$-resolvable subspace of $\mathfrak{Z}$.
7. If $\mathfrak{Z}$ be a e-resolvable and $\mathfrak{Y}$ is open subset of $\mathfrak{Z}$.Then $\mathfrak{Y}$ is e-resolvable subspace of $\mathfrak{Z}$.

Proof: The same prove Theorem 3.9.

Definition 3.11: Let ($\mathfrak{Z}$,$\mathfrak{T}$,$\mathfrak{I}$) be an ideal topological space and a subset M of $\mathfrak{Z}$ is called $\mathfrak{WI}$-open, if $M\subseteq int(M^{**})$.

Definition 3.12: An ideal topological space ($\mathfrak{Z}$,$\mathfrak{T}$,$\mathfrak{I}$) is called

1. $\mathfrak{WI}$-Hausdorff, if for every $p\neq q$, $\exists$ $\mathfrak{WI}$-open sets K and L containing p and q singly, where $K\cap L=\Phi$.
2. $\mathfrak{WI}$-$\mathfrak{I}$-Hausdorff,if for every $p\neq q$, $\exists$ $\mathfrak{WI}$-open and $\mathfrak{I}$-open sets K and L containing p and q singly, where $K\cap L=\Phi$.
3. $\mathfrak{WI}$-$\mathfrak{P}^*$-Hausdorff,if for every $p\neq q$, $\exists$ $\mathfrak{WI}$-open and pre-$\mathfrak{I}$-open sets K and L containing p and q singly,where $K\cap L=\Phi$.
4. $\mathfrak{WI}$-$\mathfrak{P}$-Hausdorff,if for every $p\neq q$, $\exists$ $\mathfrak{WI}$-open and pre-open sets K and L containing p and q singly, where $K\cap L=\Phi$.
5. $\mathfrak{I}$-$\mathfrak{P}^*$-Hausdorff,if for every $p\neq q$, $\exists$ $\mathfrak{I}$-open and pre-$\mathfrak{I}$-open sets K and L containing p and q singly, everywhere $K\cap L=\Phi$.

6. $\mathfrak{I}$-$\mathfrak{P}$-Hausdorff,if for every $p \neq q$, $\exists$ $\mathfrak{I}$-open and pre-open sets K and L containing p and q individually, everywhere $K \cap L = \Phi$.

Theorem 3.13: Let ($\mathfrak{Z}$,$\mathfrak{T}$,$\mathfrak{I}$) be an ideal topological space and the scattered sets of ($\mathfrak{Z}$,$\mathfrak{T}$*) are in $\mathfrak{I}$.The following statement are holds: Every $\mathfrak{WI}$-resolvable space is $\mathfrak{WI}$-Hausdorff space.

Proof: Suppose that $\mathfrak{Z}$ is a $\mathfrak{WI}$-resolvable, then there exists M,G be disjoint weakly-$\mathfrak{I}$-dense subsets of $\mathfrak{Z}$ such that $M \cup G = \mathfrak{Z}$.We get that M and B are $\mathfrak{WI}$-open. Let $z,p \in \mathfrak{Z}$.We have to show that ($\mathfrak{Z}$,$\mathfrak{T}$,$\mathfrak{I}$) is $\mathfrak{WI}$-Hausdorff.So by Theorem 2.9,it follows that $(M-\{p\})^{**}=M^{**}=(G \cup \{p\})^{**}$,then $(M-\{p\})^{**}=\mathfrak{Z}=(G \cup \{p\})^{**}$.Thus $K=M-\{p\}$ and $L=G \cup \{p\}$ are disjoint $\mathfrak{WI}$-open sets having z and p individually.

Theorem 3.14: Let ($\mathfrak{Z}$,$\mathfrak{T}$,$\mathfrak{I}$) be an ideal topological space and the scattered sets of ($\mathfrak{Z}$,$\mathfrak{T}$*) are in $\mathfrak{I}$.The following statements are holds**:**

1. Every $\mathfrak{WI}$-$\mathfrak{I}$-resolvable space is $\mathfrak{WI}$-$\mathfrak{I}$-Hausdorff space.
2. Every $\mathfrak{WI}$-$\mathfrak{T}$*-resolvable space is $\mathfrak{WI}$-$\mathfrak{P}$*-Hausdorff space.
3. Every $\mathfrak{WI}$-$\mathfrak{T}$-resolvable space is $\mathfrak{WI}$-$\mathfrak{P}$-Hausdorff space.
4. Every $\mathfrak{I}$-$\mathfrak{T}$*-resolvable space is $\mathfrak{I}$-$\mathfrak{P}$*-Hausdorff space.
5. Every $\mathfrak{I}$-$\mathfrak{T}$-resolvable space is $\mathfrak{I}$-$\mathfrak{P}$-Hausdorff space.

Proof: The same prove Theorem 3.13.

Theorem 3.15: Let $\mathfrak{F}$:($\mathfrak{Z}$,$\mathfrak{T}$,$\mathfrak{I}$)→($\mathfrak{Y}$,σ) be $\mathfrak{W}$-$\mathfrak{I}$-map and bijection. If $\mathfrak{Z}$ is $\mathfrak{WI}$-resolvable, then $\mathfrak{Y}$ is $\mathfrak{WJ}$-resolvable.

Proof: Suppose that $\mathfrak{Z}$ is $\mathfrak{WI}$-resolvable, then there exists M and G subsets of $\mathfrak{Z}$ such that $M \cup G = \mathfrak{Z}$ and $M \cap G = \Phi$,where $M^{**}=\mathfrak{Z}$ and $G^{**}=\mathfrak{Z}$. So by Theorem 2.19,we have f(M) and f(G) are weakly-$\mathfrak{J}$-dense in $\mathfrak{Y}$. Hence $\mathfrak{Y}$ is $\mathfrak{J}$-resolvable.

Theorem 3.16: Let $\mathfrak{F}$:($\mathfrak{Z}$,$\mathfrak{T}$,$\mathfrak{I}$)→($\mathfrak{Y}$,σ,$\mathfrak{J}$) be a $\mathfrak{W}$-$\mathfrak{I}$-map and bijection. If $\mathfrak{Z}$ is $\mathfrak{WI}$-resolvable, then $\mathfrak{Y}$ is $\mathfrak{J}$-resolvable.

Proof: The same prove Theorem 3.15.

Corollary 3.17: Let $\mathfrak{F}$:($\mathfrak{Z}$,$\mathfrak{T}$,$\mathfrak{I}$)→($\mathfrak{Y}$,σ,$\mathfrak{J}$) be a $\mathfrak{W}$-$\mathfrak{I}$-map and bijection.If $\mathfrak{Z}$ is $\mathfrak{WI}$-resolvable,then $\mathfrak{Y}$ is e-resolvable.

Proof: Get it from Theorem 3.15.

Theorem 3.18: Let $\mathfrak{F}$:($\mathfrak{Z}$,$\mathfrak{T}$,$\mathfrak{I}$)→($\mathfrak{Y}$,σ) be a $\mathfrak{E}$-$\mathfrak{I}$-map and bijection. If $\mathfrak{Z}$ is e-resolvable, then $\mathfrak{Y}$ is e-resolvable.

Proof: Get it from Theorem 2.23.

Here, we will characterize wI-resolvable spaces by means of Ψ -operator.

Theorem 3.19: Let $(\mathfrak{Z},\mathfrak{T},\mathfrak{I})$ be $\mathfrak{I}$-resolvable if and only if there exists M subset of $\mathfrak{Z}$ such that $\Psi(M)=\Psi(M^c)=\Phi$.

Proof: Necessity. Let $(\mathfrak{Z},\mathfrak{T},\mathfrak{I})$ be $\mathfrak{I}$-resolvable, then there exists M and G subsets of $\mathfrak{Z}$ such that $M\bigcup G=\mathfrak{Z}$ and $M\bigcap G=\Phi$ where $M^*=G^*=\mathfrak{Z}$, $M^c=G$ and $G^c=M$. Since $(M^*)^c=\Phi$,then $(((M^c)^c)^*)^c=\Phi$,this implies $\Psi(M^c)=\Phi$ and since $(G^*)^c=\Phi$,then $((M^c)^*)^c=\Phi$,this implies $\Psi(M)=\Phi$.

Sufficiency. Let $M\subseteq\mathfrak{Z}$ such that $\Psi(M)=\Psi(M^c)=\Phi$.Since $((M^c)^*)^c=\Phi$,then $G^*=\mathfrak{Z}$. Since $(((M^c)^c)^*)^c=\Phi$,then $M^*=\mathfrak{Z}$,$M\bigcup M^c=\mathfrak{Z}$ and $M\bigcap M^c=\Phi$.Hence $\mathfrak{Z}$ is $\mathfrak{I}$-resolvable.

Theorem 3.20: Let $(\mathfrak{Z},\mathfrak{T},\mathfrak{I})$ be $\mathfrak{W}\mathfrak{I}$-resolvable if and only if there exists M subset of $\mathfrak{Z}$ such that $\Psi(\Psi(M))=\Psi(\Psi(M^c)=\Phi$.

Proof: The same prove Theorem 3.19.

Proposition 3.21: Let $(\mathfrak{Z},\mathfrak{T},\mathfrak{I})$ be a $\mathfrak{W}\mathfrak{I}$-resolvable if and only if there exist weakly-$\mathfrak{I}$-dense subset M of $\mathfrak{Z}$ with $\Psi(\Psi(M))=\Phi$.

Proof: Necessity.By Theorem 3.20,then there exist $M\subseteq\mathfrak{Z}$ such that $\Psi(\Psi(M))=\Psi(\Psi(M^c))=\Phi$,then $((((M^c)^c)^*)^*)^c=\Phi$,this implies $M^{**}=\mathfrak{Z}$. Hence M is weakly-$\mathfrak{I}$-dense.

Sufficiency. Let M is weakly-$\mathfrak{I}$-dense with $\Psi(\Psi(M))=\Phi$. Since $M^{**}=\mathfrak{Z}$, then $((((M^c)^c)^*)^*)^c=\Phi$, which implies that $\Psi(\Psi(M^c))=\Phi$.So by theorem 3.20,it follows that $\mathfrak{Z}$ is $\mathfrak{W}\mathfrak{I}$-resolvable.

Lemma 3.22: Let $(\mathfrak{Z},\mathfrak{T},\mathfrak{I})$ is $\mathfrak{I}$-resolvable if and only if there exists $\mathfrak{I}$-dense subset M of $\mathfrak{Z}$ with $\Psi(M)=\Phi$.

Proof: Clear.

Theorem 3.23: Let $(\mathfrak{Z},\mathfrak{T},\mathfrak{I})$ be an ideal space and $M\subseteq\mathfrak{I}$, then M^c is weakly-$\mathfrak{I}$-dense if and only if $\Psi(\Psi M))=\Phi$.

Proof: Let M^c is weakly-$\mathfrak{I}$-dense in $\mathfrak{Z}$ iff $((M^c)^*)^*=\mathfrak{Z}$ iff $(((M^c)^*)^*)^c=\Phi$ iff $\Psi(\Psi(M))=\Phi$.

Theorem 3.24: Let $(\mathfrak{Z},\mathfrak{T},\mathfrak{I})$ is $\mathfrak{I}$-$\mathfrak{T}^*$-resolvable, if and only if there exists a subset M of $\mathfrak{Z}$ such that $\Psi(M^c)=int^*(M)=\Phi$.

Proof: Necessity. Let $\mathfrak{Z}$ be a $\mathfrak{I}$-$\mathfrak{T}^*$-resolvable, then there exist M and G subsets of $\mathfrak{Z}$ such that $M\bigcup G=\mathfrak{Z}$,$M\bigcap G=\Phi$ where $M^*=\mathfrak{Z}=cl^*(G)$, $M=G^c$ and $G=M^c$. Since $(cl^*(G))^c=\Phi$,implies that $int^*(M)=\Phi$ and since $M^*=\mathfrak{Z}$, then $\Psi(M^c)=\Phi$.

Sufficiency. Clear.

Definition 3.25:Let $(\mathfrak{Z},\mathfrak{T},\mathfrak{J})$ be an ideal topological space,let M be any subset of $\mathfrak{Z}$.The e-interior of M is denoted by $int^e(M)$ and is of form $int^e(M)=M\cap\Psi(\Psi(M))$.

Proposition 3.26: Let $(\mathfrak{Z},\mathfrak{T},\mathfrak{J})$ be a $\mathfrak{WJ}$-resolvable, then there exists weakly-$\mathfrak{J}$-dense subset M of $\mathfrak{Z}$ such that $int^e(M)=\Phi$.

Proof: By Proposition 3.21 ,we have $int^e(M)=M\cap\Psi(\Psi(M))=\Phi$.

Theorem 3.27: Let $(\mathfrak{Z},\mathfrak{T},\mathfrak{J})$ be an ideal topological space. The following statements are equivalent.

1. $\mathfrak{Z}$ is $\mathfrak{J}$-resolvable.
2. There exists $\mathfrak{J}$-dense subset M and G of $\mathfrak{Z}$ such that $\Psi(M)=\Phi$.
3. There exists disjoint union subsets M and G of $\mathfrak{Z}$ such that $\Psi(M)=\Psi(G)=\Phi$.
4. There exists disjoint union $\mathfrak{J}$-codense subsets M and G of $\mathfrak{Z}$.

Proof:

(1)↔(2). Get it from Lemma 3.22.

(2)→(3).By (2) there exist $M\subset\mathfrak{Z}$ such that $M^*=\mathfrak{Z}$ and $\Psi(M)=\Phi$,then $(((M^c)^c)^*)^c=\Phi=\Psi(M^c)=\Phi$,put $M^c=G$.thus, we get $\Psi(G)=\Phi$.

(3)→(4).Let M and G subsets of $\mathfrak{Z}$ such that $M\cup G=\mathfrak{Z}$ and $M\cap G=\Phi$.By(3) $\Psi(M)=\Psi(G)=\Phi$.So $((M^c)^*)^c=\Phi$,then $(M^c)^*=\mathfrak{Z}$. Thus M^c is $\mathfrak{J}$-dense. Hence M is $\mathfrak{J}$-codense. Similarity ,we get that G is $\mathfrak{J}$-codense.

(4)→(1). Since $M\cup G=\mathfrak{Z}$ and $M\cap G=\Phi$, $M=G^c$ and $G=M^c$. By(4),we have $(M^c)^*=\mathfrak{Z}$. Therefore $M^*=\mathfrak{Z}$ and $G^*=\mathfrak{Z}$ which means that $\mathfrak{Z}$ is $\mathfrak{J}$-resolvable.

Theorem 3.28: Let $(\mathfrak{Z},\mathfrak{T},\mathfrak{J})$ be an ideal topological space. The following statements are equivalent.

1. $\mathfrak{Z}$ is $\mathfrak{WJ}$-resolvable.
2. There exists weakly-$\mathfrak{J}$-dense subsets M and G of $\mathfrak{Z}$ such that $\Psi(\Psi(M))=\Phi$.
3. There exists disjoint union subsets M and G of $\mathfrak{Z}$ such that $\Psi(\Psi(M))=\Psi(\Psi(G))=\Phi$.
4. There exists disjoint union $\mathfrak{WJ}$-codense subsets M and G of $\mathfrak{Z}$.

Proof:

(1)↔(2). Get it from Proposition 3.21.

(2)→(3). By (2) there exist $M\subset\mathfrak{Z}$ such that $(M^*)^*=\mathfrak{Z}$ and $\Psi(\Psi(M))=\Psi(\Psi(M^c))=\Phi$,put $M^c=G$.thus we get $\Psi(\Psi(G))=\Phi$.

(3)→(4). Let M and G subsets of $\mathfrak{Z}$ such that $M\cup G=\mathfrak{Z}$ and $M\cap G=\Phi$.By(3) $\Psi(\Psi(M))=\Psi(\Psi(G))=\Phi$,so $((M^c)^*)^c=\Phi$,then $(M^c)^*=\mathfrak{Z}$. Thus M^c is weakly-$\mathfrak{J}$-dense. Hence M is $\mathfrak{WJ}$-codense.Similarit we get that G is $\mathfrak{WJ}$-codense.

(4)→(1). Since $M \bigcup G = \mathcal{Z}$ and $M \bigcap G = \Phi$, $M=G^c$ and $G=M^c$. Since M is $\mathfrak{WI}$-codense, then $(M^c)^{**}=\mathcal{Z}$. Therefore $M^{**}=\mathcal{Z}$ and $G^{**}=\mathcal{Z}$ which means that $\mathcal{Z}$ is $\mathfrak{WI}$-resolvable.

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