Preliminary test estimation of a location parameter in a population of normal distribution

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المستخلص

يهدف هذا البحث ايجاد مقدر الاختبار الأولي لمعلمة الموقع في مجتمع يتوزع توزيعا طبيعيا عندما كون الفرضية البديلة للمقدار الأولى من جانب واحد ($H_1 : \mu \geq \mu_0$) حيث μ هو متوسط المجتمع ، μ_0 هو أي قيمة ابتدائية. بعد ذلك تم اشتقاق وايجاد دالة الخطر ومن ثم حساب دالة الخطر عند احجام العينات الصغيرة لمعرفة خصائص هذا المقدر عند العينات الصغيرة. كما بينا أن القيمة الحرجة المثلى لمقدر الاختبار الأولى هو $\sqrt{\frac{(n-1)}{(n+1)}}$.

ABSTRACT:

In this paper. we find pretest estimator of the variance in a normal population when the a alternative hypothesis in the preest is one sided ($H_1 : \mu \geq \mu_0$) , Where μ is a population mean & μ_0 is a initial value, then drive the risk function & examine it for the small sample size properties of the pretest stimator. It is shown that the optimal critical value for pretest estimator is $\sqrt{\frac{(n-1)}{(n+1)}}$

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1. Introduction:- let $(X_1, X_2, ..., X_n)$ be a random samples from a population of normal distribution with unknown mean μ and variance σ^2, where μ is shape parameter $-\infty < \mu < \infty$ and σ^2 is location parameter $\sigma^2 > 0$.

The unbiased estimator for σ^2 is:-

$$S^2 = \frac{\sum_{i=1}^{n} (Xi - \overline{X})^2}{(n-1)} \quad \text{.......................... (1)}$$

Where $\overline{X}$ is the sample mean. Thus, we know that the unbiased estimator is dominated by the estimator in terms of the mean squared error.

$$S_1^{2*} = \frac{\sum_{i=1}^{n} (Xi - \overline{X})^2}{(n+1)} \quad \text{..............(2)}$$

The estimator S_1^{2*} is called the usual estimator, the researcher stein in 1964 observed that the usual estimator is dominated by the estimator

$$\hat{\sigma}^2 = \min\left[S_1^{2*}, S_2^{2*}\right] \quad \text{.............(3)}$$

where:

$$S_2^{2*} = \frac{\sum_{i=1}^{n} (Xi - \mu_\circ)^2}{(n+2)} \quad \text{............(4)}$$

where $\mu_\circ$ is any constant. Many researchers are made studies and extensions on improved estimators of σ^2, for example, the researcher Brown in 1968,the researchers Klotz, Milton and zacks in 1969, the researcher strawderman in 1989.

2. Preliminary test estimators:- If we take the usual estimator S_1^{2*}, then we modified it by multiply and divide σ^2, then:

$$S_1^{2*} = \frac{\dfrac{\sigma^2 \sum (Xi - \overline{X})^2}{\sigma^2}}{n+1} = \frac{\sigma^2 W}{n+1} \qquad (5)$$

where W is a random variable and defined as:

$$W = \frac{\sum (Xi - \overline{X})^2}{\sigma^2} \sim \chi^2_{(n-1)} \qquad (6)$$

If we take the estimator S_2^{2*} and modified it as follows:

$$S_2^{2*} = \frac{\sum Xi^2 - 2\mu_0 \sum Xi + n\mu_\circ^2}{n+2} \qquad (7)$$

$$S_2^{2*} = \frac{\sum Xi^2 - 2\mu_\circ \sum Xi + n\mu_\circ^2 + n\overline{X}^2 + n\overline{X}^2 - 2n\overline{X}^2}{n+2} \qquad (8)$$

$$S_2^{2*} = \frac{\sum Xi^2 - 2n\overline{X}\dfrac{\sum Xi}{n} + n\overline{X}^2 + n\overline{X}^2 - 2\mu_\circ \sum Xi + n\mu_\circ^2}{n+2} \qquad (9)$$

$$S_2^{2*} = \frac{\frac{\sigma^2 \sum (Xi - X)^2}{\sigma^2} + \frac{\sigma^2 (\overline{X} - \mu_\circ)^2}{\sigma^2 / n}}{n+2} = \frac{\sigma^2 \left(w + z^2\right)}{n+2} \qquad (10)$$

where z is a random variable and defined as:

$$Z = \frac{\overline{X} - \mu_\circ}{\sigma / \sqrt{n}} \sim N(\lambda, 1) \qquad (11)$$

Where $\lambda = \frac{(\mu - \mu_\circ)}{\sigma / \sqrt{n}}$

We can write the t test as:

$$t = \frac{\overline{X} - \mu_\circ}{S / \sqrt{n}} = \frac{\frac{\overline{X} - \mu_\circ}{1 / \sqrt{n}}}{\sqrt{\frac{\sum (Xi - \overline{X})^2}{n-1}}} \qquad (12)$$

$$t = \frac{\frac{\overline{X} - \mu_\circ}{\sigma / \sqrt{n}}}{\sqrt{\frac{\sum (Xi - \overline{X})^2}{\sigma^2 (n-1)}}} = \frac{Z}{\sqrt{\frac{w}{n-1}}} \qquad (13)$$

If we have the prior information $\mu \geq \mu_\circ$, where $\mu_\circ$ is initial value, then

the pretest estimator is:

$$\tilde{\sigma}^2 = \begin{cases} S_1^{2*} & if\,|t| \geq c \\ S_2^{2*} & if\,|t| < c \end{cases} \qquad (14)$$

Where c is positive critical value, Gelfand and Dey in 1988a show that the stein estimator is pretest estimator with $C = \sqrt{\frac{n-1}{n+1}}$. The preliminary test

estimator is written as:

$$\tilde{\sigma}^2 = \begin{cases} \dfrac{\sigma^2}{n+1} w & if \quad \dfrac{z}{\sqrt{w}} \geq c' \\ \dfrac{\sigma^2}{n+2}(w+z^2) & if \quad \dfrac{z}{\sqrt{w}} < c' \end{cases} \qquad (15)$$

Where $c' = \dfrac{c}{\sqrt{n-1}} = \dfrac{1}{\sqrt{n+1}}$

The aim of this research is to find preliminary test estimator as shown is equation (15) to estimate the location parameter in normal distribution with critical value c when the prior information is one-sided $\mu \geq \mu_\circ$ and true, then we drive the risk function and calculate the risk function for the small sample properties of the preliminary test estimator in (15).

Note:- The pretest estimator in this research is differ from stein estimator, the alternative hypothesis in a stein estimator is two-sided $H_1 : \mu \neq \mu_\circ$, but the hypothesis in this estimator is one- sided $H_1 : \mu \geq \mu_\circ$, the risk function and its numerical results in this research are differ for stein

3- Risk function:- To find & drive the risk function of the preliminary test estimator for the variance of normal distribution, we assume that the loss fnction is $L(\dot{\sigma}^2) = \dfrac{\left[\dot{\sigma}^2 - \sigma^2\right]}{\sigma^2}$, then:

$$R(\tilde{\sigma}^{2}) = E\left[L\tilde{\sigma}^{2}\right] \quad \text{.........................(16)}$$

$$= \int \frac{(\tilde{\sigma}^{2}-\sigma^{2})^{2}}{\sigma^{4}} f(\tilde{\sigma}^{2})\, d\tilde{\sigma}^{2} \quad \text{....................(17)}$$

$$= \int_{z=-\infty}^{\infty} \int_{S=0}^{\infty} I(C',\infty)\frac{Z}{\sqrt{W}} \cdot \frac{\left(\frac{\sigma^{2}}{n+1}w-\sigma^{2}\right)^{2}}{\sigma^{4}} f(z) f(w)\, dz dw +$$

$$= \int_{z=-\infty}^{\infty} \int_{S=0}^{\infty} I(-\infty,C')\frac{Z}{\sqrt{W}} \cdot \frac{\left(\frac{\sigma^{2}}{n+1}w-\sigma^{2}\right)^{2}}{\sigma^{4}} f(z) f(w)\, dz dw \quad \text{................(18)}$$

$$R(\tilde{\sigma}^{2}) = \int_{z=-\infty}^{\infty} \int_{S=0}^{\infty} I(C',\infty)\frac{Z}{\sqrt{W}}\left(\frac{w}{n+1}-1\right)^{2} f(Z) f(w)\, dz dw +$$

$$= \int_{z=-\infty}^{\infty} \int_{S=0}^{\infty} I(-\infty,C')\frac{Z}{\sqrt{W}}\left(\frac{Z^{2}+w}{n+2}-1\right)^{2} f(z) f(w)\, dz\, dw$$

$$R(\tilde{\sigma}^{2}) = \int_{z=-\infty}^{\infty} \int_{S=0}^{\infty} I(C',\infty)\frac{Z}{\sqrt{W}} f(Z) f(w)\, dz dw + \int_{z=-\infty}^{\infty} \int_{S=0}^{\infty} I(C',\infty)\frac{Z}{\sqrt{W}}\left[\frac{w^{2}}{(n+1)^{2}}-\frac{2w}{n+1}\right]$$

$$f(z) f(w)\, dz dw + \int_{z=-\infty}^{\infty} \int_{S=0}^{\infty} I(-\infty,C')\frac{Z}{\sqrt{W}} f(z) f(w)\, dz dw + \int_{z=-\infty}^{\infty} \int_{S=0}^{\infty} (-\infty,C')\frac{Z}{\sqrt{W}}$$

$$\left[\frac{(Z^{2}+W)^{2}}{(n\div 2)^{2}}-\frac{2(Z^{2}+w)}{n+2}\right] f(z) f(w) dz dw \quad \text{..................(19)}$$

Where I(a,b) (X) is an indicator function:

$$I(a,b)(X)=\begin{cases}1 & if\ a\le X\le b\\ 0 & if\ X<a\ or\ X>b\end{cases}$$

We notice that the first state in equation (15) satisfy the following:

$$I(C'',\infty)\frac{Z}{\sqrt{W}}=1 \quad where\ C''\le\frac{Z}{\sqrt{W}}\le\infty$$

but the second state in the same equation satisfy the following:

$$I(-\infty,c')\frac{Z}{\sqrt{W}}=0 \qquad Where\ \frac{Z}{\sqrt{W}}<-\infty\ \ or\ \ \frac{Z}{\sqrt{W}}>c'$$

$$R(\tilde{\sigma}^2)=1+\int_{-\infty}^{\infty}\int_{0}^{\infty}I(c',\infty)\frac{Z}{\sqrt{W}}\left[\frac{w^2}{(n+1)^2}-\frac{2w}{n+1}\right]f(z)f(w)\,dz\,dw+$$

$$\int_{-\infty}^{\infty}\int_{0}^{\infty}I(-\infty,c')\frac{Z}{\sqrt{W}}\left[\frac{(z^2+w)^2}{(n+2)^2}-\frac{2(z^2+w)}{n+2}\right]f(z)f(w)\,dz\,dw \qquad (20)$$

$$R(\tilde{\sigma}^2)=1+\int_{-\infty}^{\infty}\int_{0}^{\infty}I(c',\infty)\frac{Z}{\sqrt{W}}\left[\frac{w^2}{(n+1)^2}-\frac{2w}{n+1}\right]\frac{1}{\sqrt{2\pi}}e^{\frac{-(z-\lambda)^2}{2}}\ \frac{1}{\Gamma\frac{r}{2}\ 2^{r/2}}W^{\frac{r}{2}-1}e^{-\frac{w}{2}}\,dz\,dw$$

$$+\int_{-\infty}^{\infty}\int_{0}^{\infty}I(-\infty,c')\frac{Z}{\sqrt{W}}\left[\frac{(z^2+w)^2}{(n+2)^2}-\frac{2(z^2+w)}{n+2}\right]\frac{1}{\sqrt{2\pi}}e^{\frac{-(z-\lambda)^2}{2}}\ \frac{1}{\Gamma\frac{r}{2}\ 2^{r/2}}W^{\frac{r}{2}-1}e^{-\frac{w}{2}}\,dz\,dw$$

(21)

Recall that:-

$$e^{\frac{-(z-\lambda)^2}{2}}=e^{\frac{-z^2}{2}+z\lambda\frac{-\lambda^2}{2}}=e^{\frac{-(z^2+\lambda^2)}{2}}\ e^{z\lambda}$$

$$=e^{-\frac{(z^2+\lambda^2)}{2}}\sum_{i=0}^{\infty}\frac{z^i\lambda^i}{i!}$$

$$R(\tilde{\sigma}^2)=1+\int_{-\infty}^{\infty}\int_{0}^{\infty}\sum_{i=0}^{\infty}Ai\,\frac{Z}{\sqrt{W}}\left[\frac{w^2}{(n+1)^2}-\frac{2w}{n+1}\right]e^{-z^2/2}Z^iW^{r/2-1}e^{-w/2}\,dzdw+$$

$$\int_{-\infty}^{\infty}\int_{0}^{\infty}\sum_{i=0}^{\infty}Ai\frac{Z}{\sqrt{W}}\left[\frac{(z^2+w)^2}{(n+2)^2}-\frac{2(z^2+w)}{n+2}\right]e^{-z^2/2}Z^iW^{r/2-1}e^{-w/2}\,dwdz \qquad (22)$$

where:

$$\sum_{i=0}^{\infty}Ai=\sum_{i=0}^{\infty}\frac{1}{\sqrt{2\pi}\;\Gamma\frac{r}{2}\;2^{r/2}}\;\frac{\lambda^i}{i!}\;e^{-\frac{\lambda^2}{2}}$$

Applying the following transformation of variables:

$$t=\frac{z}{w} \qquad and \qquad u=w$$

$$dz=\sqrt{u}\;dt \qquad and \qquad du=dw$$

$$R(\tilde{\sigma}^2)=1+\sum_{i=0}^{\infty}Ai\int_{-\infty}^{\infty}\int_{0}^{\infty}\left[\frac{u^2}{(n+1)^2}-\frac{2u}{n+1}\right]t^{i+1}\,u^{\frac{1}{2}(i+1)}\,e^{-\frac{u(t^2+1)}{2}}\,u^{\frac{r}{2}-1}\,du\,dt+$$

$$\sum_{i=0}^{\infty}Ai\int_{-\infty}^{\infty}\int_{0}^{\infty}\left[\frac{u^2(t^2+1)^2}{(n+2)^2}-\frac{2u(t^2+1)}{n+2}\right]t^{i+1}u^{\frac{1}{2}(i+1)}\,e^{\frac{-u(t^2+1)}{2}}\,u^{\frac{r}{2}-1}du\,dt \qquad (23)$$

Again applying the following transformation of variables:

$$y=\frac{u(t^2+1)}{2} \qquad and \qquad du=\frac{2}{t^2+1}dy$$

$$R(\tilde{\sigma}^2)=1+\sum_{i=0}^{\infty}Ai\int_{-\infty}^{\infty}\int_{0}^{\infty}\left[\frac{2^2\;\;y^2}{(n+1)^2\;\;(t^2+1)^2}-\frac{2^2\;\;y}{(n+1)\,(t^2+1)}\right]t^{i+1}\;\;e^{-y}$$

$$\frac{2^{\frac{1}{2}(i+1)} \; y^{\frac{1}{2}(i+1)} \; 2^{\frac{r}{2}-1} \; y^{\frac{r}{2}-1} \; 2}{(t^2+1)^{\frac{1}{2}(i+1)} \; (t^2+1)^{\frac{r}{2}-1} \; (t^2+1)} dy\,dt + \sum_{i=0}^{\infty} Ai \int_{c'}^{\infty}\int_{0}^{\infty} \left[\frac{2^2 y^2}{(n+2)^2} - \frac{2^2 y}{(n+2)}\right]$$

$$t^{i+1} e^{-y} \; \frac{2^{\frac{1}{2}(i+1)} \; y^{\frac{1}{2}(i+1)} \; 2^{\frac{r}{2}-1} \; y^{\frac{r}{2}-1} \; 2}{(t^2+1)^{\frac{1}{2}(i+1)} (t^2+1)^{\frac{r}{2}-1} (t^2+1)} dy\,dt \qquad (24)$$

$$R(\widetilde{\sigma}^2) = 1 + \sum_{i=0}^{\infty} Ai \int_{c'}^{\infty}\int_{0}^{\infty} \frac{2^{\frac{1}{2}(i+r+5)} \quad y^{\frac{1}{2}(i+r+3)}}{(n+1)^2 \; (t^2+1)^{\frac{1}{2}(i+r+5)}} \; t^{i+1} \; e^{-y} \; dy\,dt -$$

$$\sum_{i=0}^{\infty} Ai \int_{c'}^{\infty}\int_{0}^{\infty} \frac{2^{\frac{1}{2}(i+r+5)} \quad y^{\frac{1}{2}(i+r+1)}}{(n+1) \; (t^2+1)^{\frac{1}{2}(i+r+3)}} \; t^{i+1} \; e^{-y} \; dy\,dt \; +$$

$$\sum_{i=0}^{\infty} Ai \int_{c'}^{\infty}\int_{0}^{\infty} \frac{2^{\frac{1}{2}(i+r+5)} \quad y^{\frac{1}{2}(i+r+3)}}{(n+2)^2 \; (t^2+)^{\frac{1}{2}(i+r+1)}} \; t^{i+1} \quad e^{-y} \quad dy\,dt \quad -$$

$$\sum_{i=0}^{\infty} Ai \int_{c'}^{\infty}\int_{0}^{\infty} \frac{2^{\frac{1}{2}(i+r+5)} \quad y^{\frac{1}{2}(i+r+1)}}{(n+2) \; (t^2+1)^{\frac{1}{2}(i+r+1)}} t^{i+1} e^{-y} \; dy\,dt \qquad (25)$$

The integrating is similar to gamma function, where :
$\alpha = \frac{1}{2}(i+r+5)$, $\frac{1}{2}(i+r+3)$ **respectively and** $\beta = 1$**, then:**

$$R(\widetilde{\sigma}^2) = 1 + \sum_{i=0}^{\infty} Ai \; 2^{\frac{1}{2}(i+r+5)} \; \Gamma\tfrac{1}{2}(i+r+5) \int_{c'}^{\infty} \frac{t^{i+1}}{(n+1)^2} (1+t^2)^{\frac{-1}{2}(i+r+5)} dt -$$

$$\sum_{i=0}^{\infty} Ai \quad 2^{\frac{1}{2}(i+r+5)}\ \Gamma\tfrac{1}{2}(i+r+5) \quad \int_{c'}^{\infty}\frac{t^{i+1}}{(n+1)}\ (1+t^2)^{-\frac{1}{2}(i+r+3)}\ dt\ +$$

$$\sum_{i=0}^{\infty} Ai\ 2^{\frac{1}{2}(i+r+5)}\Gamma\tfrac{1}{2}(i+r+5) \quad \int_{c'}^{\infty}\frac{t^{i+1}}{(n+2)^2}\ (1+t^2)^{-\frac{1}{2}(i+r+1)}\ dt -$$

$$\sum_{i=0}^{\infty} Ai\ 2^{\frac{1}{2}(i+r+5)}\Gamma\tfrac{1}{2}(i+r+3)\int_{c'}^{\infty}\frac{t^{i+1}}{(n+2)}\ (1+t^2)^{-\frac{1}{2}(i+r+1)}dt \qquad (26)\mathbf{AA}$$

pplying the following transformation of variable :

$$y = t^2 \qquad and \qquad dt = \frac{1}{2\sqrt{y}}dy$$

$$R(\tilde{\sigma}^2) = 1 + \sum_{i=0}^{\infty} Ai\ 2^{\frac{1}{2}(i+r+5)}\ \Gamma\tfrac{1}{2}(i+r+5)\ \int_{c'^2}^{\infty}\frac{y^{\frac{1}{2}i}}{2(n+1)^2}\ (1+y)^{-\frac{1}{2}(i+r+5)}\ dy -$$

$$\sum_{i=0}^{\infty} Ai\, 2^{\frac{1}{2}(i+r+3)}\ \Gamma\tfrac{1}{2}(i+r+3)\ \int_{c'^2}^{\infty}\frac{y^{\frac{1}{2}i}}{2(n+1)}(1+y)^{-\frac{1}{2}(i+r+3)}\ dy\ +$$

$$\sum_{i=o}^{\infty} Ai 2^{\frac{1}{2}(i+r+5)}\ \Gamma\tfrac{1}{2}(i+r+5)\ \int_{c'^2}^{\infty}\frac{y^{\frac{1}{2}i}}{2(n+2)^2}(1+y)^{-\frac{1}{2}(i+r+1)}\ dy -$$

$$\sum_{i=0}^{\infty} Ai2^{\frac{1}{2}(i+r+5)}\Gamma\tfrac{1}{2}(i+r+3)\int_{c'^2}^{\infty}\frac{y^{\frac{1}{2}i}}{2(n+2)^2}(1+y)^{-\frac{1}{2}(i+r+1)}dy \qquad (27)$$

Applying the transformation of variable, then:

$$t=\frac{y}{1+y} \qquad and \qquad dy=\frac{1}{(1-t)^2}dt$$

Where:

$$\beta\left[\tfrac{1}{2}i+1,\tfrac{1}{2}(r+3)\right]\int_{k}^{1}\frac{t^{\frac{1}{2}i}\ (1-t)^{\frac{1}{2}(r+1)}}{\beta\left[\frac{1}{2}i+1,\frac{1}{2}(r+3)\right]}dt$$

Hower, the integral is incomplete beta function, where:

$$K=\frac{c'^2}{(1+c'^2)}, \quad and \quad a=\tfrac{1}{2}i+1, \quad b=\tfrac{1}{2}(r+3),$$ **then:**

$$R(\tilde{\sigma}^2)=1+\sum_{i=0}^{\infty}Ai\,2^{\frac{1}{2}i-2}\ \Gamma\tfrac{1}{2}(i+r+5)\ \frac{\beta\left[\frac{1}{2}i+1,\frac{1}{2}(r+7)\right]}{2(n+1)^2}\left[I_K\left(\tfrac{1}{2}i+1,\tfrac{1}{2}(r+7)\right)\right]$$

$$-\sum_{i=0}^{\infty}Ai2^{\frac{1}{2}i-2}\Gamma\tfrac{1}{2}(i+r+3)\ \frac{\beta\left[\frac{1}{2}i+1,\frac{1}{2}(r+5)\right]}{2(n+1)}\left[I_K\left(\tfrac{1}{2}i+1,\tfrac{1}{2}(r+5)\right)\right]$$

$$+\sum_{i=0}^{\infty}Ai\ 2^{\frac{1}{2}i-2}\Gamma\tfrac{1}{2}(i+r+5)\frac{\beta\left[\frac{1}{2}i+1,\frac{1}{2}(r+3)\right]}{2(n+2)^2}\left[I_K\left(\tfrac{1}{2}i+1,\tfrac{1}{2}(r+3)\right)\right]$$

$$-\sum_{i=0}^{\infty} Ai\,2^{\frac{1}{2}i+2}\,\Gamma\tfrac{1}{2}(i+r+3)\frac{\beta\left[\frac{1}{2}i+1,\frac{1}{2}(r+3)\right]}{2(n+2)}\left[I_K\left(\tfrac{1}{2}i+1,\tfrac{1}{2}(r+3)\right)\right] \qquad (28)$$

Where:

$$\int_{k}^{1} \frac{t^{\frac{1}{2}i}\,(1-t)^{\frac{1}{2}(r+1)}}{\beta\left[\frac{1}{2}i+1,\frac{1}{2}(r+3)\right]} = \left[I_K\left(\tfrac{1}{2}i+1,\tfrac{1}{2}(r+3)\right)\right]$$

and:

$$\sum_{i=0}^{\infty} Ai = \frac{1}{\sqrt{\pi}\ \Gamma\frac{1}{2}} \sum_{i=0}^{\infty} e^{-\frac{\lambda^2}{2}}\ \frac{\lambda^i}{i!}$$

4. Numerical Results:-

After drive the risk function for $\tilde{\sigma}^2$, then numerical calculations of this risk function assuming that : n = 3, 5, 9, 15, 21, where r = n-1 and various values of $\lambda = 0, 0.5, 1, 1.5, 2, 2.5, 3$, $\pi = 3.14$, The incomplete beta function find and compute from the reference (Abramowitz and stegun 1972). Thus, the numerical results shown in the following table

The table represent the risk function in pretest estimator

n	λ	I_K	C	Approximate (α)	$R(\tilde{\sigma}^2$
3	0	0.345	0.707	0.267	0.254
	0.5				0.294
	1				0.321
	1.5				0.385
	2				0.455
	2.5				0.496
	3				0.512
5	0	0.401	0.816	0.227	0.305
	0.5				0.366
	1				0.395
	1.5				0.427
	2				0.496
	2.5				0.553
	3				0.607
9	0	0.439	0.894	0.203	0.351
	0.5				0.386
	1				0.442
	1.5				0.576
	2				0.611
	2.5				0.692
	3				0.732
15	0	0.464	0.935	0.184	0.413
	0.5				0.479
	1				0.542
	1.5				0.599
	2				0.633
	2.5				0.682
	3				0.795
21	0				0.509

	0.5				0.572
	1				0.641
	1.5	0.518	0.953	0.174	0.721
	2				0.774
	2.5				0.810
	3				0.866

5. Recommendations and Conclusions:-

1- We see that the risk functions for pretest estimators when $\mu = \mu_\circ$ has minimum risks among all sample size.

2- We notice that the risk functions for pretest estimators are increasing when $\mu > \mu$ among all sample size.

3- We observe that the risk functions for pretest estimators are increasing when the approximate level of significants are decreasing among all values of λ.

4- We see that the risk functions for pretest estimators are increasing when the sample size are increasing for all values of λ.

5- We notice that the optimal critical region value is $C = \sqrt{\frac{(n-1)}{(n+1)}}$.

6-We recommend to find the risk functions for S_1^{*2} and S_2^{*2} and compare it with risk function for pretest estimator.

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