On Partional Valued Characters of The Group Q2n Where n=2p and p is any prime number greater than 2

Nesir Rasool Mahamoed
Dept. of Math. College of Al - Kied
University of Kufa

Abstract

In this paper, We have proved that each rational valued character of the quaternion group Q_{2n} where n=2p and p is any prime number greater than 2, can be written as a Z-linear combination of induced characters $1_{c_i}^{Q_{2n}}$, C_1 runs over all cyclic subgroup of Q_{2n}.

الخلاصة

في هذا البحث برهنا على انه يمكن التعبير عن كل شاخص من شواخص الزمرة Q_{2n} عندما يكون n=2p حيث ان p أي عدد اولي اكبر من 2 بصيغة تركيب خطي للشواخص المحتثة $1_{c_i}^{Q_{2n}}$ وبمعاملات اعداد صحيحة حيث ان C_i زمرة جزئية دائرية من الزمرة Q_{2n}.

Introduction

Assuming Q_{2n} a quaternion group, Q a rational field and x, y$\in$ Q_{2n} then x is Q-conjugate to y if the cyclic group generated by x is conjugate in Q_{2n} the cyclic group generate by y. Denote this relation by x ~ y, clearly ~ is equivalence relation on Q_{2n} their equivalence classes are called the F-classes of Q_{2n}.

Let these be 1 = Γ1, Γ2, … Γm

Let $x_i \in$ ΓI be the representative of the class Γi and $C_i = <x_i>$.

Due to Artin theorem asserts if G be a finite group and θ be a rational valued character of G;

Then $\theta = \sum_{i=1}^{m} a_i \ 1_{c_i}^{G}, ..., a_i \in Q$ …(1)

So according to this theorem each valued character *x* of Q_{2n} can be written as:

$$x = \sum_{i=1}^{m} a_i \ 1_{c_i}^{G}, ..., a_i \in Q \qquad \text{…(2)}$$

In the present research, we have proved that each rational valued character *x* of Q_{2n}, which is expressible as in (1) with $a_i \in Z$, where n=2p such that p is any prime number greater than 2.

$$x = \sum_{i=1}^{m} a_i \ 1_{c_i}^{Q_{2n}}, ..., a_i \in Z \qquad \text{…(3)}$$

There is no doubt that the problem of constructing the rational valued characters table of Q_{2n}, where n=2p and p is any prime number greater than 2, would be rather simplified if we knew that each rational valued characters of Q_{2n} can be written as a Z-linear combination of the induced characters $1_{c_i}^{Q_{2n}}$.

On the group $K(Q_{2n})$

Definition (1) : For each positive integer n the general quaternion group of order 4n can be defined as follows:

$Q_{2n} = \{<x, y> : X^{2n} = y^4 = 1 , y\times y^{-1} = x^{-1}\}$

For more information see (Board *et al.*, 1973; Curtise, 1988)

Definition (2): Let cf (G, Z) be the set of all Z-Valued class functions of G which are constant on Q-Classes. Let R(Q, Z), be the intersection of cf (G, Z) with R(G) the group of generalized characters of G. R(Q, G) is a finitely generated Z-module with bases Q-Characteristics of G.

$$x_i = \sum_{QE\ Gal\ ((Q(X_I)/Q)}^{m} x_I^Q \qquad , x_i \in I_{rr}(c)$$

Theorem (1): For a finite group, we have;

$$|K(G)| = \left(\frac{\prod_{I=1}^{m} n_i |N(C_I)|}{\phi(|C_I|)} \right)^{1/2}$$

For proof see (Kirdar, 1988)

Theorem (2): Let G be a finite group $\{C_i = <x_i> , 1 \leq m\}$ be a full set of non-conjugate cyclic subgroups of G, and n_i be the number of conjugate cyclic subgroup of G contained in the Q-Class Γi, then each rational valued character of G can be written as a Z-linear combination of $1_{C_i}^{C_n}$ if and only if:

$$\left(\prod_{i=1}^{m} \frac{n_i}{\phi(|C_i|)} \right)^{1/2} = \prod_{i=1}^{m} \frac{(|N(C_i)|)^{1/2}}{|C_i|}$$

for proof see (Kirdar, 1988)

Corollary 1: The rational valued character of a cyclic group C_n of order n can be written as a Z-linear combination of characters $1_{C_i}^{Q_{2n}}$, where C_i runs over the set of subgroup of C_n.

For proof see (Kirdar, 1988)

Lemma 1: Let m be the number of Q-classes Γi of Q_{2n}, where n is an even number, then $m=m_1+2$, where m_1 is the number of Q-classes Γ1 of the cyclic subgroup C2n=<x>.

For proof see (Nesir, 1995)

Corollary 2: We have,

$K(Q_{2n}) = k(Q_{4p}) = k(C_{2n}) \oplus C_8 \oplus C_2 \oplus C_2/C_4$

Where n=2p and p is any prime number greater than 2.

For proof see (Nesir, 1995)

Conclusion

Assuming n=2p where p is any prime number greater than 2, by theorem (1) we get;

$$|k(Q_{2n})| = |k(Q_{4p})| = \left(\prod_{i=1}^{m} \frac{n_i(|N(C_i)|)}{\phi(|C_i|)}\right)^{1/2} \qquad \ldots(1)$$

By corollary (2) we get;

$$|k(Q_{2n})| = |k(Q_{4p})| = 8|k(C_{2n})| \qquad \ldots(2)$$

Further more by equation [1] lemma (1) and corollary (2), we get;

$$|k(Q_{2n})| = |k(Q_{4p})|$$

$$= \frac{\left(\prod_{i=1}^{m} \frac{n_i(|N(C_i)|)}{\phi(|C_i|)}\right)^{1/2} \cdot \left(\frac{n_r|N(C_8)|^{1/2}}{\phi|(C_8)|}\right) \cdot \left(\frac{n_t|N(C_2)|}{\phi|(C_2)|}\right)^{1}}{\left(\frac{n_s|N(C_4)|^{1/2}}{\phi|(C_4)|}\right)}$$

but;

$$\phi(|C_8|) = \phi(8) = 4$$

$$\phi(|C_4|) = \phi(4) = 2$$

$$\phi(|C_2|) = \phi(2) = 1$$

and $n_r = 4$, $n_s = 2$, $n_t = 1$

$$\therefore |k(Q_{2n})| = \left(\prod_{i=1}^{m_1} \frac{n_i(|N(C_i)|)}{\phi(|C_i|)}\right)^{1/2} \cdot \frac{(|N(C_8)|)^{1/2}\,(|N(C_2)|)}{(|N(C_4)|)^{1/2}} \qquad \ldots(3)$$

Also, by equation [2] and equation [3], we get;

$$8|k(Q_{2n})| = \left(\prod_{i=1}^{m_1} \frac{n_i(|N(C_i)|)}{\phi(|C_i|)}\right)^{1/2} \cdot \frac{(|N(C_8)|)^{1/2}\,(|N(C_2)|)}{(|N(C_4)|)^{1/2}}$$

Since

$$|k(C_{2n})| = \left(\prod_{i=1}^{m_1} \frac{n_i(|N(C_i)|)}{\phi(|C_i|)}\right)^{1/2}$$

$$\therefore\ 8 = \frac{(|N(C_8)|)^{1/2}\,(|N(C_2)|)}{(|N(C_4)|)^{1/2}} \qquad \ldots(4)$$

Hence

$$\prod_{i=1}^{m_1} \frac{(|N(C_i)|)^{1/2}}{(|C_i|)} = \prod_{i=1}^{m_1} \frac{(|N(C_i)|)^{1/2}}{(|C_i|)} \cdot \frac{(|N(C_8)|)^{1/2} \cdot (|N(C_2)|)}{8(|N(C_4)|)^{1/2}}$$

but from equation [4] we get;

$$\frac{(|N(C_8)|)^{1/2}\,(|N(C_2)|)}{8(|N(C_4)|)^{1/2}} = 1$$

$$\therefore \prod_{i=1}^{m} \frac{(|N(C_i)|)^{1/2}}{(|C_i|)} = \prod_{i=1}^{m_1} \frac{(|N(C_i)|)^{1/2}}{(|C_i|)} \qquad \ldots(5)$$

on the other hand;

$$\left(\prod_{i=1}^{m} \frac{n_i}{\phi(|C_i|)}\right)^{1/2} = \left(\prod_{i=1}^{m_1} \frac{n_i}{\phi(|C_i|)}\right)^{1/2} \cdot \left(\frac{n_r}{\phi(|C_8|)}\right)^{1/2} \cdot \left(\frac{n_t}{\phi(|C_2|)}\right)^{1/2} \cdot \left(\frac{\phi(|C_4|)}{n_s}\right)^{1/2}$$

and as above $n_r = n_s = 2$, $n_t = 1$

and $\phi(|C_8|) = 4$, $\phi(|C_4|) = 2$, $\phi(|C_2|) = 1$,

Therefore;

$$\left(\prod_{i=1}^{m} \frac{n_i}{\phi(|C_i|)}\right)^{1/2} = \left(\prod_{i=1}^{m_1} \frac{n_i}{\phi(|C_i|)}\right)^{1/2} \qquad \ldots[6]$$

by comparison [5] and [6] , we get;

$$\left(\prod_{i=1}^{m} \frac{n_i}{\phi(|C_i|)}\right)^{1/2} = \prod_{i=1}^{m} \frac{(N(|C_i|)^{1/2}}{(|C_i|)} \qquad \ldots[7]$$

This main result is the following; the rational valued characters of equation group Q_{2n}, where n=2p and p is any prime number greater than 2, can be written as Z-linear combination of the induced characters $1_{c_i}^{Q_{2n}}$, where C_i runs over the set of all subgroup of Q_{2n}.

References

Board, A.D., Ocnner, D.E., and Young; 1973. Symmetry and Its Application in Science: Mologaw Hill Book Company Limited (UK).

Curtise, C.R., Reiner, 1988. Representation of Finite Groups and Associative Algebras; Wiley Interscience .

Isacc, I.M.; 1976. Character Theory of Finite Groups. Academic Press, New York.

Kirdar, M.S. 1988. On Artins Theorem. Arab Journal of Mathematics Vol.(9), No.182.

Nesir R.; 1995. The Cyclic Decomposition of the Factor Group of (Q_{2m}, Z)/ $\overline{R}C(Q_{2m})$, M.Sc., thesis University of Technology.

Serre, J.P.; 1977. Linear Representations of Finite group Springier-Verlag (42), New York,