A New Design for Linear Phase IIR Digital Filter with Efficient Realization

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Abstract

Linear phase requirement can be achieved by direct design with an FIR filter at the expense of high order and high computational burden for the given specification. Alternatively, IIR digital filters can be designed with much smaller order than their FIR counterparts, but at the expense of the non-linear phase. In this paper, a new design of linear phase IIR digital filter is presented by composing a zero phase efficiently designed FIR digital filter with maximally flat group delay. The new IIR digital filter is designed with a lower order satisfying the same roll-off magnitude response of the corresponding FIR filter and preserving the approximate pass-band characteristics.

الخلاصة:

أن متطلبات تصميم مرشحات رقمية ذات طور خطي يمكن الحصول عليها بتصميم مرشحات من نوع الاستجابة المحدودة الدفقة (FIR) وعلى حساب الزيادة في مرتبة المرشح و ما يتبعها من عمليات حسابية معقدة للحصول على مواصفات الاستجابة المطلوبة. وعلى العكس من ذلك ، فأن المرشحات الرقمية ذات الاستجابة اللامحدودة الدفقة (IIR) يمكن أن تعطي هذه المواصفات عند مرتبة للمرشح أقل بكثير على حساب غياب الطور الخطي .

في هذا البحث تم تصميم مرشح رقمي من نوع (IIR) ذي طور خطي باستخدام مرشح رقمي نوع (FIR) كبسط مع دالة قطبية خطية الطور (maximally flat group delay all-pole function) كمقام للمرشح نوع (IIR) الجديد. يمتاز هذا المرشح أيضاً بتحقيق كفوء بالإضافة إلى تقليل المرتبة مقارنة مع المرشح الرقمي نوع (FIR) مع ضمان نفس طبيعة خصائص التفريق للاستجابة المقدارية مقارنة مع نوع (FIR).

1- Introduction

Many authors have studied the synthesis of IIR digital filters with linear phase[1]-[11]. The problem to be solved is the simultaneous approximation of both magnitude and phase characteristics. The preponderant technique employs all-pass equalizers to obtain linear phase of the IIR digital filter, which was initially designed from magnitude characteristics. Hence the approximations of magnitude and phase characteristics are handled separately. Although the stability of the filter is guaranteed using all-pass equalizer, it is difficult to achieve a good

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approximation of the linear phase characteristic, especially the constant group delay in general [1]. There are some design philosophies in [2] analogous to the equalizer method, in which, the magnitude and phase approximations are handled separately.

Many design methods have been successfully applied to the design of linear phase IIR digital filters. Linear programming is one of these methods, and is used for the design of IIR digital filters with approximated linear phase that unifies the approximation of both magnitude and phase characteristics [3].

A new methodology was presented in [4] to design linear phase IIR digital filters. Such design is based on a non-causal FIR model that represents the ideal filter response with linear phase. A linear phase IIR filter is then synthesized as an approach of the ideal FIR filter. The stability of such IIR filter is guaranteed since these IIR filters are obtained leaning on FIR pattern.

Other techniques for designing linear phase IIR filter is a two-path polyphase structure [5]-[7], Frequency Masking (FM) technique [8]-[10], and the new design method of Multirate approximately linear phase IIR filter structure for arbitrary bandwidth that is presented in [11].

The specification of linear phase is normally given in terms of the desired constant group delay t for the IIR digital filter. The desired linear phase specification can be written as

$$f(w) = -t \cdot w \qquad \ldots(1)$$

where w is the normalized frequency variable. Varying t value, one can control the linear phase specification.

Precise linear phase transfer function can be easily realized as an FIR structure. Unfortunately, this structure requires many times more multiplies per output sample to satisfy a good roll-off magnitude response specification such as that of IIR filters. On the other hand, the IIR filter can not be designed to exhibit exact linear phase response [12]. An approximate linear phase response is acceptable for IIR filter. Such approximation may be carried out for the pass-band only. The stop-band phase approximation is ignored since it is of no significance as the magnitude values appear with very low levels in this region [3]. Our analysis in this paper depends on such type approximate linear phase design of IIR filters.

A new design method for approximate linear phase IIR digital filters is presented in this paper. A mirror image polynomial of the same type designed in [13] for FIR filter is used as an IIR filter's numerator to design the magnitude response. Linear phase all-pole function is cascaded with this numerator, this cascaded structure produces an approximate linear phase IIR filter. Such a design is achieved with optimum frequency response, and then with reduced complexity. Finally a merged design is obtained with both optimal response and efficient realization properties. Section 2 includes the design of the all-pole function that is used in the design of the linear phase IIR digital filter in Section 3. Illustrative example is shown in Section 4. In Section 5 some conclusions are given.

2- Linear Phase All-Pole Function Design

An all-pole digital function is used here to represent the denominator. It is of the type with maximally flat group delay, and can be expressed as [14]

$$\frac{1}{D(z^{-1})} = \frac{(2N)!}{N!} \cdot \frac{1}{\prod_{i=N+1}^{2N}(2t+i)}$$

$$\frac{1}{\sum_{k=0}^{N}\left[(-1)^k \binom{N}{k} \prod_{i=0}^{N} \frac{2t+i}{2t+i+k} z^{-k}\right]}$$

…(2)

$$\frac{1}{D(z^{-1})} = \frac{1}{G} \cdot \frac{1}{\sum_{i=0}^{N} b_i z^{-i}}$$

…(3)

where N is the all-pole function order,

$\frac{1}{G} = \frac{2N!}{N!} \cdot \frac{1}{\prod_{i=N+1}^{2N}(2t+i)}$ is the all-pole function constant,

t is the desired group delay (sec.), and

b_i's are the all-pole function coefficients.

Thus, the general form of the IIR digital filter can be written as

$$H(z) = \frac{1}{G} \cdot \frac{\sum_{m=0}^{M} a_m z^{-m}}{\left(1 + \sum_{n=1}^{N} b_n z^{-n}\right)}$$

…(4)

where a_m is the IIR digital filter numerator coefficient,

The above form of $H(z)$ is identical to that in [15]. To reduce the number of the required multipliers, we can divide the numerator polynomial by G. This will reduce the number of multipliers by one.

It should be noted that the function in Equation (2) is stable for all positive values of group delay, t. The cutoff frequency (w_c) of the filter is determined by the desired group delay value, and the filter order *N*. High group delay value gives lower cutoff frequency, while higher all-pole filter function order leads to higher cutoff frequency [16].

This maximally flat group delay all-pole filter function has a magnitude value equal to unity at $w = 0$, and starts decreasing with very small values up to cutoff frequency. Magnitude decreasing amount depends on the group delay value. That gives almost constant pass-band magnitude at low frequencies, this property is used, here, to design the magnitude response of low-pass filters. It can be modified to design band-pass filter when small group delay values are chosen. In addition, the all-pole filter function has linear phase at low frequencies. Figs. 1 and 2 show the magnitude and phase shapes of the all- pole function with 11th order and group delay values of 0.1, 0.5, 1, 1.5, 2 sec.s respectively.

3- Linear Phase IIR Digital Filter Design Procedure

As mentioned before, the numerator of the proposed linear phase IIR digital filter is a mirror image polynomial, that gives zero phase. This numerator is designed in [13] as an FIR filter. This non-recursive filter has a zero group delay. Normally, this type of FIR filter function is used to approximate the overall filter magnitude after being multiplied by $z^{-N/2}$ for casualty purposes.

Cascading the linear phase all-pole filter function in Section 2, with the FIR filter gives a new linear phase IIR digital filter. The resulting IIR digital filter will be stable for all finite positive values of group delay (t)[16].

In the next sub-sections, different algorithms will be used to design this linear phase IIR digital filter. First algorithm gives some reduction in complexity of IIR filter realization. The second one presents an optimum linear phase IIR filter magnitude response in stop-band peak ripple sense. Finally, a linear phase IIR filter which is optimally designed with efficient realization is achieved.

i- Efficient Realization of Linear Phase IIR Filter

The linear phase IIR filter designed here is composed of FIR filter that is previously designed in [13] as a numerator, and an all-pole filter function of the type used in Section 2 as a denominator. The FIR filter is designed by a new procedure, this procedure results in an FIR filter with some zero coefficient values. The number of zero coefficients may be increased to reach $N/2$. These zero coefficients lead to a reduction in the total required number of multipliers in the filter realization.

Cascading the FIR filter function with the all-pole function gives a new linear phase IIR digital filter with some zero coefficients in the numerator. This type of IIR filters gives more simple realization with a good response, compared to the corresponding IIR filter of the same order.

ii- Optimal Design of Linear Phase IIR filter

The FIR filter that is designed in Section 2 of [13], could be used as a numerator of the linear phase IIR filter. This filter function numerator is designed with optimal single or double transition-band samples. These optimum values are obtained by employing the Golden Section Search optimization method in Single Transition- Band Sample, and with the aid of Steepest Descent algorithm in Double Transition- Band Samples. The main advantages of these optimization procedures are reduced in Pass-Band Average Deviation (PBAD), and minimum Peak Stop-Band Ripple (PSBR) levels. Optimal linear phase IIR digital filter response is obtained by composing this FIR filter function with the all-pole filter function to produce a new optimal response IIR digital filter. The resulting IIR filter exhibits the phase linearity property, in addition to more improvement in stop-band response. In other words, if it is required to obtain the same FIR filter response, an IIR filter with lower order can be used.

iii- A Novel Linear Phase IIR Filter Design and Realization

In the Sub-section 3.*i*, an approximate linear phase IIR digital filter with some zero coefficient values is designed. This IIR filter gives some reduction in required number of multipliers in IIR filter realization. In Sub-section 3.*ii*, an optimal design for IIR filter is achieved in the sense of stop-band ripple levels.

In this sub-section, the linear phase IIR filter is designed utilizing the above two approaches (i.e., efficient realization, and optimal response). The resulting IIR filter represents a modernistic design and realization.

4- Design Example

It is required to design a low pass IIR digital filter meeting the following specification: 11th order, $w_c = 5p/11$ rad. , and group delay values (t) of 0.1, 0.5, and 1 sec.s respectively.

In *Case* **1**, the designed IIR digital filter is as in Section 3.*i*. The values of transition-band samples of the filter numerator are chosen arbitrarily. The numerator and denominator coefficients of the designed IIR filter function are listed in Table 1. Fig. 3 shows the magnitude response of the overall filter. The PSBR levels are -23dB at t = 0.1 sec., -39dB at t = 0.5 sec., -47dB at t =1 sec.. For the same above sequence of group delay values, the corresponding PBAD are 1.05%, 6.51%, and 13.20%.

In *Case* **2**, the designed IIR digital filter is as in Section 3.*ii*. The double transition-band sample values of the filter numerator are optimized using Golden Section search method with the aid of Steepest Descent algorithm (a_{t1} = 0.58073, a_{t2} = 0.0983101) to give minimum peak ripple in the stop-band. The IIR filter coefficients are listed in Table 2. The magnitude response of the linear phase IIR filter is shown in Fig. 4. The PSBR levels are reduced for t = 0.1, 0.5, and 1 sec., to -79dB, -91dB, -103dB, respectively. The corresponding PBAD are 0.70%, 6.17%, and 12.87%.

In *Case* **3**, the IIR digital filter is designed by using the same procedure illustrated in Section 3.*iii*. The double transition-band sample values of the filter numerator are optimized to give minimum peak ripple in the stop-band besides having some of zero coefficients values. The sum of these transition-band samples is still equal to unity. The IIR filter coefficients (with optimized transition-band sample a_{t1}=0.58073, a_{t2}=0.0983101) are listed in Table 3. The magnitude response of the linear phase IIR filter is shown in Fig. 5. The PSBR levels are reduced for t = 0.1, 0.5, and 1 sec., to -55dB, -61dB, -69dB, respectively. The corresponding PBAD are 0.70%, 6.17%, and 12.87%. The final IIR filter has five zero coefficients values.

In all the above three cases, the average Pass-Band Phase Error (PBPE) for t = 0.1, 0.5, and 1 sec. takes the values 3.87×10^{-9}, 1.653×10^{-7}, and 2.78×10^{-6} rad. respectively (see Fig. 6). This means that a good approximation for pass-band phase linearity is achieved in the design of this IIR digital filter.

5- Conclusions

In this paper, the design of linear phase IIR digital filter has been resulted from cascading separated zero phase numerator and approximated linear phase denominator. A mirror image polynomial (FIR filter function) has been utilized here to design the numerator of the linear phase IIR filter function. A linear phase all-pole function has been designed to simulate the denominator of this new IIR filter. It has been noticed that the magnitude and the phase shapes of the designed linear phase all-pole function are highly dependent of the desired filter order (N), and the assigned group delay value (τ).

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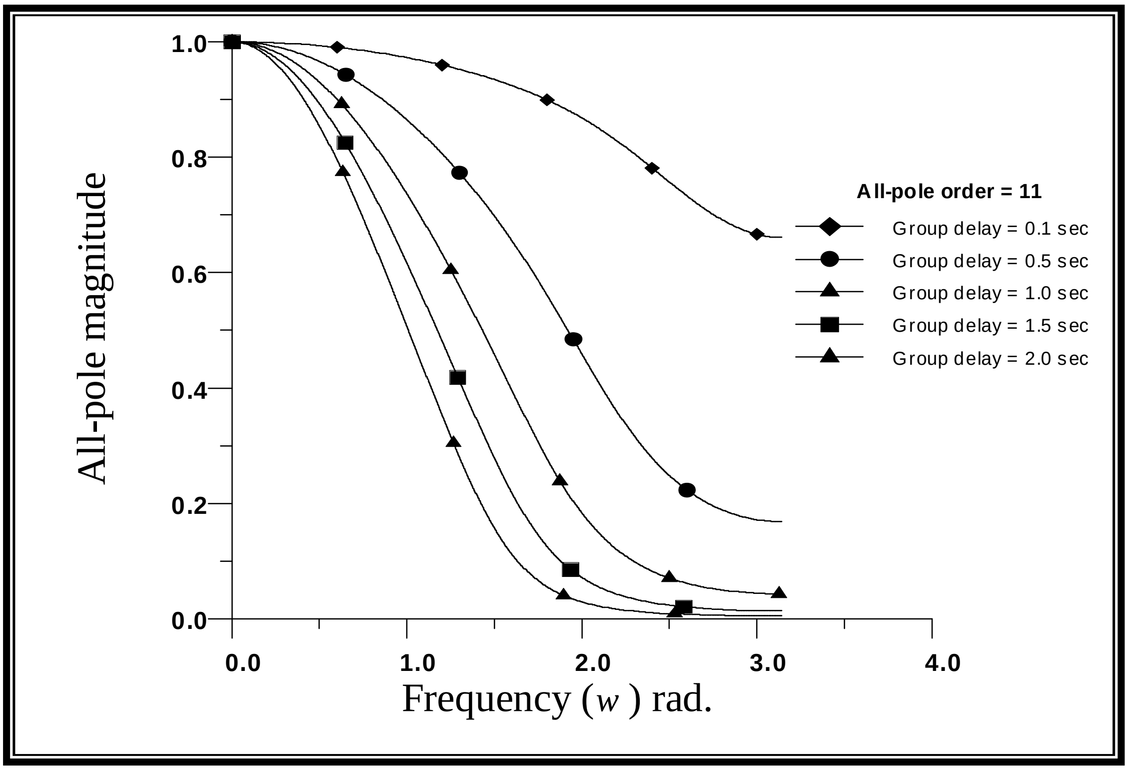

Fig. 1 Magnitude response for an 11^th order all-pole function with group delay values τ = 0.1,0.5,1.0,1.5 and 2 secs.

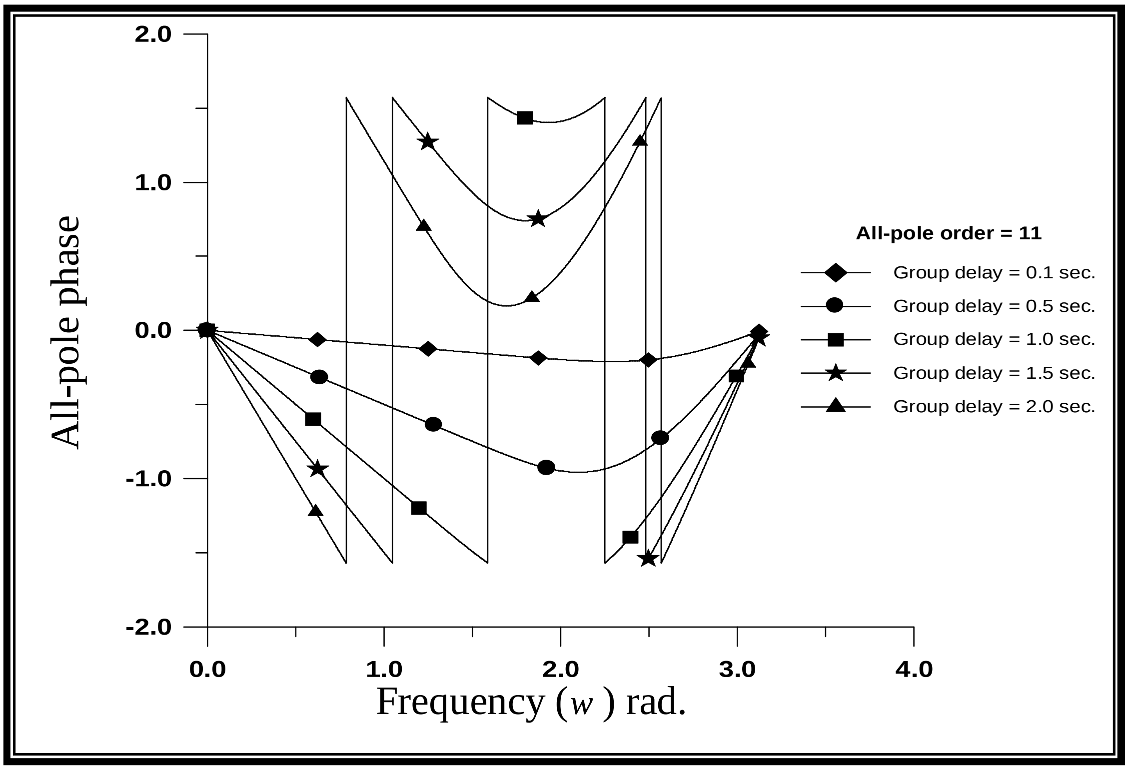

Fig. 2 Phase response for an 11^th order all-pole function with group delay values τ = 0.1,0.5,1.0,1.5 and 2 secs.

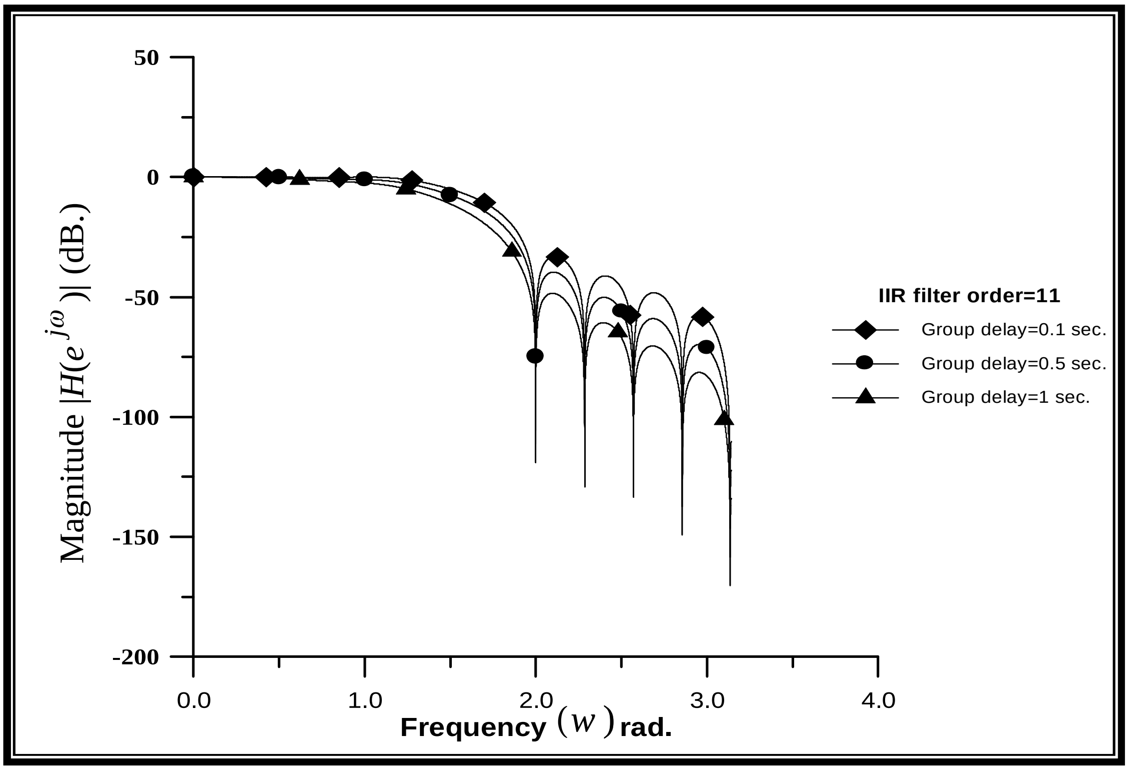

Fig. 3 Magnitude response for an 11th order LP IIR filter with a_{t1} =0.7,a_{t2} =0.3, τ = 0.1,0.5, and 1 secs.

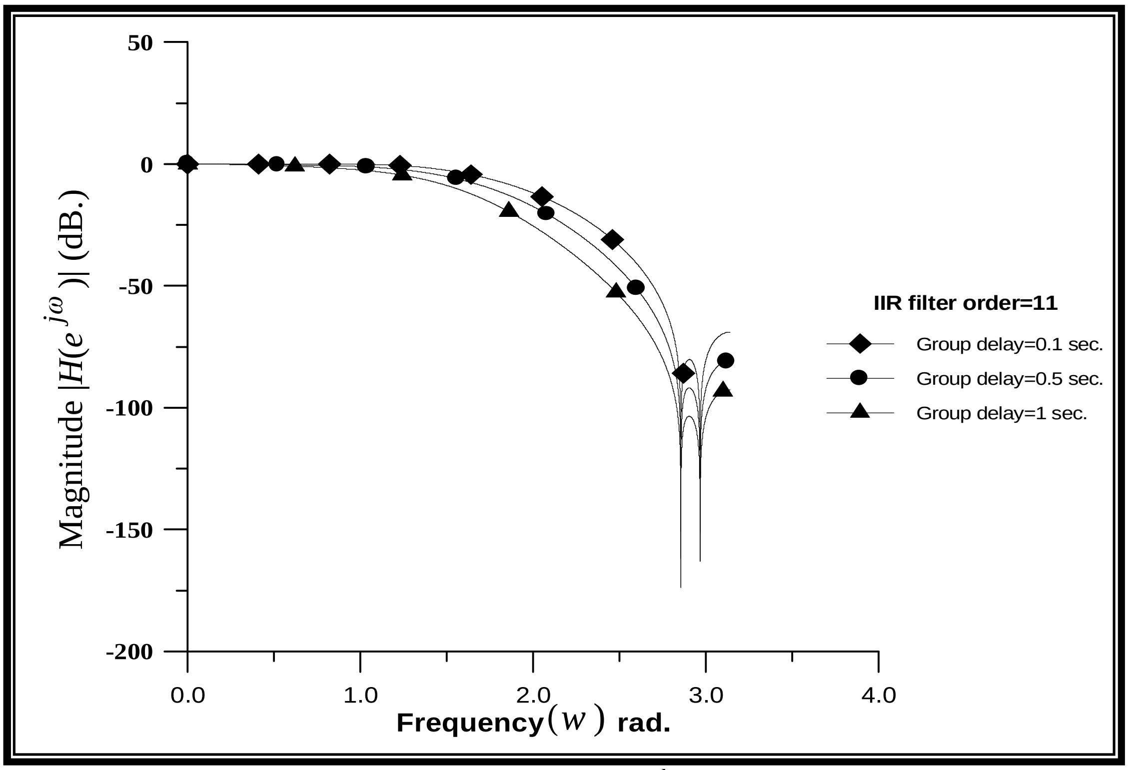

Fig. 4 Magnitude response for an 11th order LP IIR filter with a_{t1} =0.580273,a_{t2} =0.0983101,τ =0.1,0.5, and 1 sec.s.

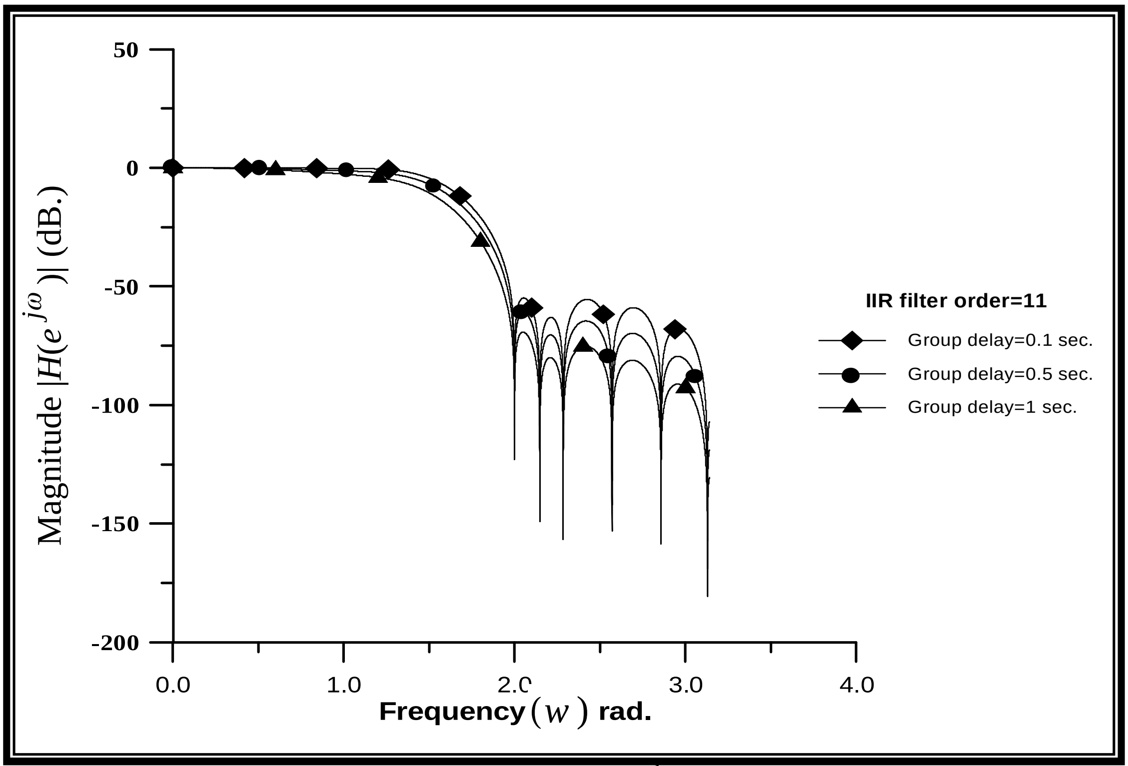

Fig. 5 Magnitude response for an 11th order LP IIR filter with a_{t1} =0.779858, a_{t2} =0.220142, τ =0.1,0.5, and 1 sec.s.

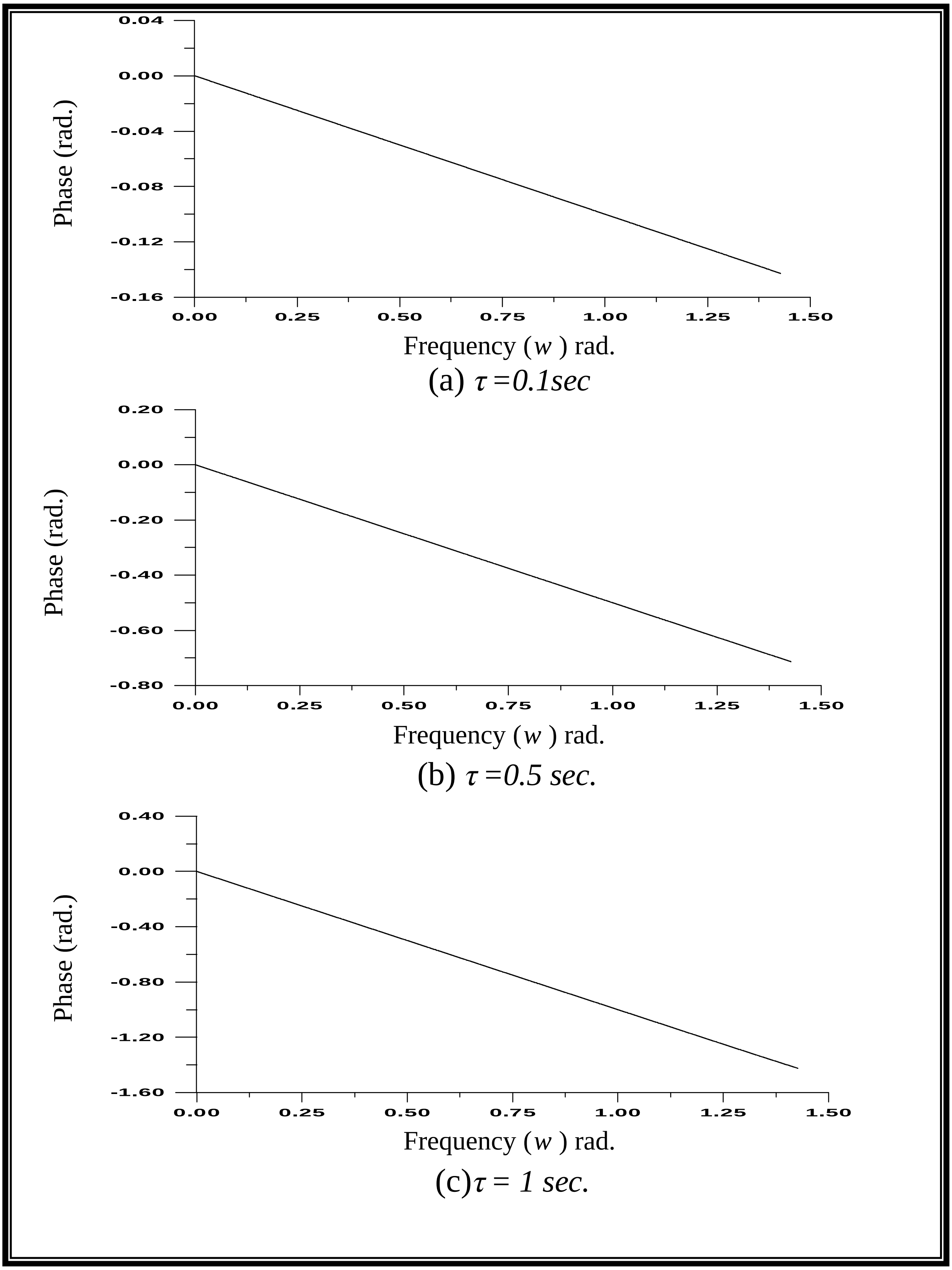

Fig. 6 Pass-band phase response for an 11^{th} order all-pole function with group delay values τ = 0.1,0.5,and 1.0 secs.

		Coefficient Number											
		0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
$t=0.1$	b_i	0.437581	0.272728	0.0	-0.0759297	0.0	0.0294854	0.0	-0.0071285	0.0	-0.0043429	0.0	0.003978
	a_i	1	-0.180328	0.0819672	0.0380974	0.0160423	-0.0058223	0.0017602	$-4.283*10^{-4}$	$8.0308*10^{-5}$	$-1.0869*10^{-5}$	$9.431*10^{-7}$	$-3.9395*10^{-8}$
$t=0.5$	b_i	0.26087	0.16259	0.0	-0.0452664	0.0	0.017578	0.0	-0.00424976	0.0	-0.00258912	0.0	0.00237154
	a_i	1	-0.846154	0.604396	-0.362637	0.181319	-0.0746606	0.0248869	-0.00654918	0.00130984	$-1.8171*10^{-4}$	$1.701*10^{-5}$	$-3.9395*10^{-7}$
$t=1$	b_i	0.141304	0.0880698	0.0	-0.0245193	0.0	0.0095214	0.0	-0.00230195	0.0	-0.00140244	0.0	0.00128458
	a_i	1	-1.57143	1.57143	-1.17847	0.93277	-0.323529	0.119195	-0.0340557	0.00729766	-0.00110571	$1.0576*10^{-4}$	$-4.8074*10^{-6}$

Table 1 IIR digital filter coefficients for *Case* **1**

		Coefficient Number											
		0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
$t=0.1$	b_i	0.437581	0.272728	0.0	-0.0759297	0.0	0.0294854	0.0	-0.0071285	0.0	-0.0043429	0.0	0.003978
	a_i	1	-0.180328	0.0819672	0.0380974	0.0160423	-0.0058223	0.0017602	$-4.283*10^{-4}$	$8.0308*10^{-5}$	$-1.0869*10^{-5}$	$9.431*10^{-7}$	$-3.9395*10^{-8}$
$t=0.5$	b_i	0.26087	0.16259	0.0	-0.0452664	0.0	0.017578	0.0	-0.00424976	0.0	-0.00258912	0.0	0.00237154
	a_i	1	-0.846154	0.604396	-0.362637	0.181319	-0.0746606	0.0248869	-0.00654918	0.00130984	$-1.8171*10^{-4}$	$1.701*10^{-5}$	$-3.9395*10^{-7}$
$t=1$	b_i	0.141304	0.0880698	0.0	-0.0245193	0.0	0.0095214	0.0	-0.00230195	0.0	-0.00140244	0.0	0.00128458
	a_i	1	-1.57143	1.57143	-1.17847	0.93277	-0.323529	0.119195	-0.0340557	0.00729766	-0.00110571	$1.0576*10^{-4}$	$-4.8074*10^{-6}$

Table 2 IIR digital filter coefficients for *Case* **2**

Coefficient Number													
11	**10**	**9**	**8**	**7**	**6**	**5**	**4**	**3**	**2**	**1**	**0**		
0.003978	0.0	-0.0043429	0.0	-0.0071285	0.0	0.0294854	0.0	-0.0759297	0.0	0.272728	0.437581	b_i	$t=0.1$
$-3.9395*10^{-8}$	$9.431*10^{-7}$	$-1.0869*10^{-5}$	$8.0308*10^{-5}$	$-4.283*10^{-4}$	0.0017602	-0.0058223	0.0160423	0.0380974	0.0819672	-0.180328	1	a_i	
0.00237154	0.0	-0.00258912	0.0	-0.00424976	0.0	0.017578	0.0	-0.0452664	0.0	0.16259	0.26087	b_i	$t=0.5$
$-3.9395*10^{-7}$	$1.701*10^{-5}$	$-1.8171*10^{-4}$	0.00130984	-0.00654918	0.0248869	-0.0746606	0.181319	-0.362637	0.604396	-0.846154	1	a_i	
0.00128458	0.0	-0.00140244	0.0	-0.00230195	0.0	0.0095214	0.0	-0.0245193	0.0	0.0880698	0.141304	b_i	$t=1$
$-4.8074*10^{-6}$	$1.0576*10^{-4}$	-0.00110571	0.00729766	-0.0340557	0.119195	-0.323529	0.93277	-1.17847	1.57143	-1.57143	1	a_i	

Table 3 IIR digital filter coefficients for *Case* **3**