Restoration of Medical Images Using One Pass Kalman Filter

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Abstract

This work introduces a new image restoration algorithm based on one-pass Kalman filter, which is one of the most successfully implemented methods to recover the original image, This work involved many useful procedures to enhance the final results and to obtain the optimum estimates of unaffected image.

The introduced filter is applied on medical images that were degraded by 2-D Gaussian point spread function (PSF) with different STD.

Finally, we illustrate the effectiveness of the present Method by quantitative simulation results.

الخلاصة

العمل الحالي تضمن تقديم خوارزمية ترميم جديدة مبنية على مرشح كالمن ذي التكرار الواحد حيث يعد مرشح كالمن واحدة من ابرز طرق التطبيق الناجحة في استرجاع الصورة الاصلية .تضمن العمل الحالي تبني العديد من المعاملات لتحسين الصورة الناتجة والحصول على اعظم تخمين للصورة غير المتأثرة بالتردي . طبقت الطريقة المقدمة على صور طبية متردية بدوال غشاوة كاوسية ذات قيم انحراف معياري مختلفة . اظهرت المقاييس الكمية للنتائج كفاءة الطريقة المقدمة .

1-Introduction

Digital processing of images has become in recent years a very active field of computer applications and research. One of the more active topics in this field has been digital restoration of images, this subject has received a great deal of attention because of the many interesting application areas, medical imaging, satellite imagery ,space research implementations … etc. a variety of digital techniques has been proposed and developed for the removed of degradation and to recover the original image from degraded images. Kalman filter is one of a recursive spatial domain estimator .several authors have been proposed different 2-D kalman filtering schemes for restoring images that have been degraded by both blur and noise [Suresh, 1981; Azimi, 1987; Koch, 1993; Kondo,1994].

Kalman filtering has been used in various formulations in the field of image restoration. The fundamental principle of this well established algorithm consists in determining recursively the causal least mean square error estimate with a two-step procedure. first, a prediction of the state variables is formed on the basis of the previous state of the system .then, the prediction is updated on the basis of the observed image data to form the estimate of the present state of the system .the update operation is a achieved by means of a kalman gain, which allows a linear correction of the previous state. This gain is computed at each iteration by a least mean square error minimization [Mattavelli, 1996; Welch,2006]. In our present work, an adaptation is suggested to develop a one-pass kalman filter. The introduced method is based on a single –predication instead of the long iterative procedures.

2-Image model

In order to use kalman Filter to restore the image we first have to know the correlation between points of the original image when the previous image points are estimated, the current point can be predicted based on the correlation. It must be noted that the Fourier transform of the auto-correlation function is represent the image power spectrum, i.e.

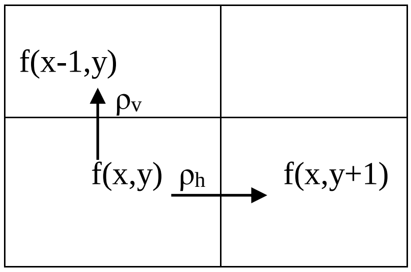
$$\Im R_{ff}(x,y) = |F(u,v)|^2 \ldots\ldots\ldots\ldots\textbf{(1)}.$$

$$R_{ff}(x,y) = \Im^{-1}|F(u,v)|^2 \ldots\ldots\ldots\textbf{(2)}$$

where R_{ff} (x,y)=autocorrelation function and $\Im, \Im^{-1}$ are forward and inverse Fourier transform respectively .

Where R (0,0)=maximum(the correlation of pixel with itself), R(0,1)=ρ_h ,R(1,0)=ρ_v

Where ρ_h, ρ_v are horizontal and vertical adjacent image point correlations .as shown in Fig(1)



Fig(1) illustrates the horizontal and vertical adjacent image point

A discrete random field with this autocorrelation can be generated by the recursive formula.

$$f(x+1,y+1)= \rho_h f(x+1,y)+ \rho_v f(x,y+1)- \rho_v \rho_h f(x,y)+ \sqrt{(1-\rho_h^{2})(1-\rho_v^{2})}\, U(x,y)\ldots\ldots \textbf{(3)}$$

Where U(x,y) is an un-correlated random field with the same variance as elements of the image [Rosenfeld, 1976 ;Pratt, 1978]

3-The Discrete Kalman filter

Kalman filter is a set of mathematical equations that provides an efficient computational (recursive) means to estimate the state of process, in a way that minimizes the mean of the squared error .Kalman filter requires two distinct models an observation model (4) and an image model (5) which can be given by the flowing equations.

$$\mathbf{Z_{(k)}=H\ X_{(k)}} \quad \ldots\ldots\textbf{(4)}$$

$$\mathbf{X_{(k)}=A\ X_{(k-1)}+B\ U_{(k)}} \quad \ldots\ldots\textbf{(5)}$$

Where $X_{(k)}$=original image ,$Z_{(k)}$=observation image ,$U_{(k)}$=random process that generates the image signal, the matrices A,B and H are the system ,drive and observations (PSF) ,respectively [see Appendix] . $U_{(k)}$=is uncorrelated white Gaussian zero mean noise sources with variance

$$\boldsymbol{\sigma_u^2=(1-\rho_v^2)(1-\rho_h^2)} \ldots\ldots\ldots\ldots \textbf{(6)}$$

The kalman filter equations for this model are: (for more details see [Brown, 1992 ; Jacobs, 1993 ; Welch, 2006]

$$\mathbf{X_{b\ (k)}= A\ X_{a\ (k-1)}} \quad \ldots\textbf{(7)}$$

$$\mathbf{P_{b\ (k)}= A\ P_{a\ (k-1)}A^T+BQB^T} \quad \ldots\textbf{(8)}$$

$$\mathbf{K_{(k)}= P_{b\ (k)}\ H^T(H\ P_{b\ (k)}\ H^T)^{-1}} \quad \ldots\textbf{(9)}$$

$$\mathbf{X_{a\ (k)}= X_{b\ (k)}+K_{(k)}[Z_{(k)}- H\ X_{b\ (k)}]} \quad \ldots\textbf{(10)}$$

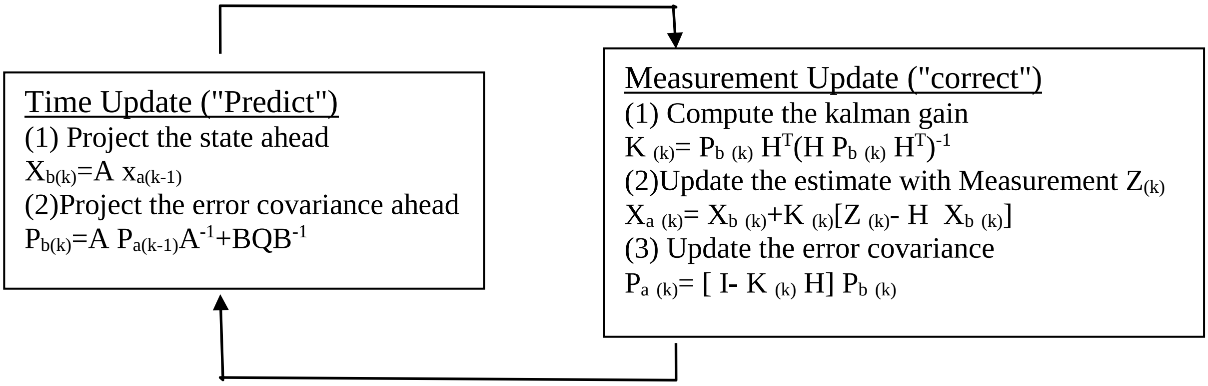
$$\mathbf{P_{a\ (k)}= [\ I- K_{(k)}\ H]\ P_{b\ (k)}} \quad \ldots\textbf{(11)}$$

Here $P_{i(k)}=E\,[(X_{(k)}-X'_{(k)})_I\,(X_{(k)}-X'_{(k)})_i^T]$ i=a,b

Is the error covariance matrix , the subscripts a and b indicate "after " and "before " updating respectively and $K_{(k)}$ the gain matrix of Kalman filter that minimize the error covariance , T indicates transposition process , Q_u= covariance matrix of $U_{(k)}$.

The equations for Kalman filter fall into two groups: *time update* equations and *measurement update* equations .the time update equations are responsible for projecting forward (in time) the current state and error covariance estimates to obtain the *a priori* estimates for the next time step. The measurement update equations are responsible for the feedback –i.e. for incorporating a new measurement into the *a priori* estimate to obtain an improved *a posteriori* estimate.

The time update equations can also be thought of *as predictor* equations , while the measurement update equations can be thought of *as corrector* equations (see Fig (2)).



Fig(2) a complete picture of the operation of the kalman filter .

4-Restoration algorithm

We shall adopt this filter to become a one-pass algorithm as described by the following steps:

1. Replace the matrix A of eq.(2) by one coefficient a(i,j) the relationships between neighboring points is, only adopted(either horizontally or vertically), these approaches are based on the hypothesis that a pixel has relevant correlation only for certain neighboring pixels.
2. for obtaining a better result, the image is rotated through different angle Kalman filter is applied on each rotated image, the final restored image will be their average.
3. A good initial guess $X^0_{(k)}=Z_{(k)}$ should be adopted.

5-Experimental results

Our adaptive one pass kalman filter has been implemented on the medical images that were degraded by 2-D Gaussian point spread function (PSF) with different STD (0.5,1,1.5,2).

Simulation results are illustrated to show the effectiveness of the proposed algorithm, Fig (3) reports examples of the restoration for x-ray image and MR image using introduced method. Table (1) contains the PSNR values for images degraded by Gaussian with different STD values and implementing one-pass Kalman filter. This table illustrates a high efficiency of one-pass Kalman filter at the large values of STD.

6-Discussion and Conclusions

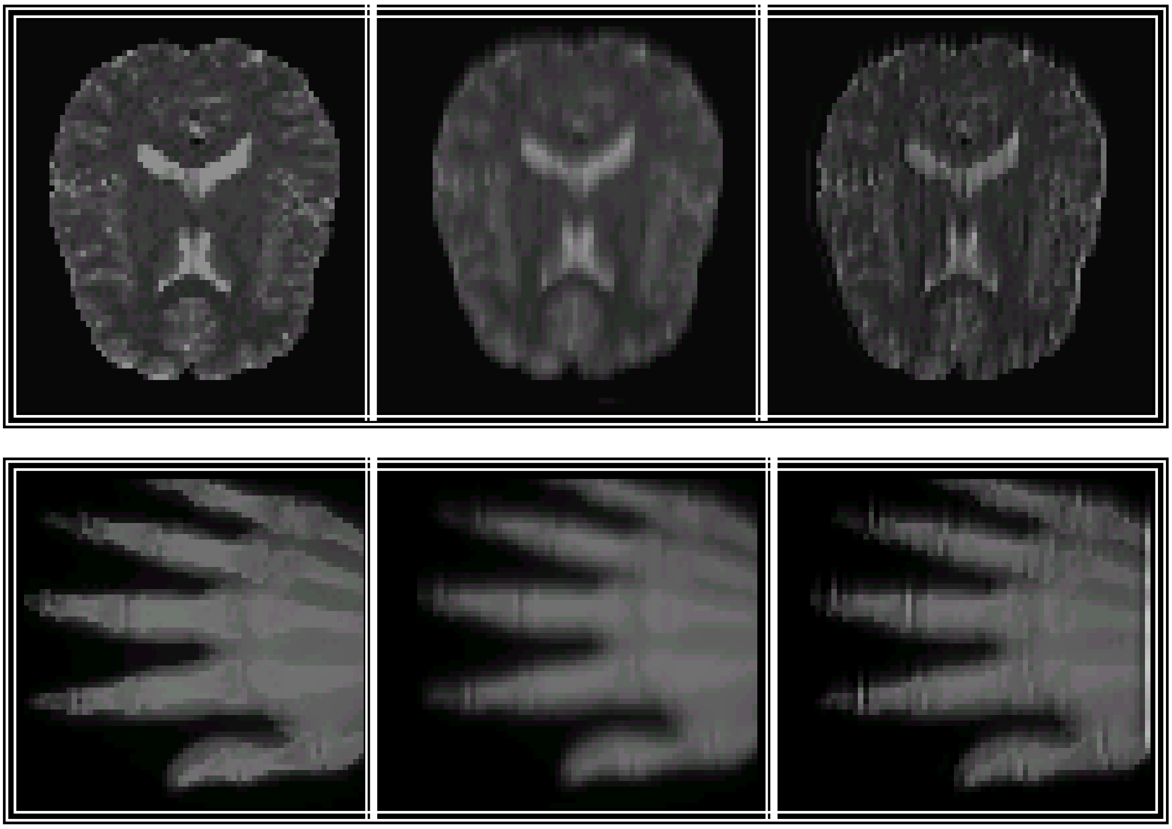
In this work , adaptation is suggested to develop a one-pass kalman filter . The introduced method is based on a single prediction. We have presented some results of applying the proposed method to image restoration.

1.The proposed method gives a better restored image than the conventional restoration techniques (inverse and wiener filter)

2.This method can be applied to other kinds of degradations.

3.This method can be applied not only to the monochromatic image but also to the color images.

4.The disadvantage associated with implementation of this method is that, it involves a huge numbers of multiplication operations, inversion of large a mount of computer memory and along processing time order required for alleviation process.



Original image **Blurred image** **Restored image**

Fig (3) Original image(left),blurred image (center) and restored image (right) by one- pass Kalman filter .the images degraded by 2-d blurring

Table (1) PSNR results and improvement for two images

The results of x-ray image (64*64)			
STD OF BLURRING FUNCTION	PSNR BLURRED IMAGE	PSNR RESTORE D IMAGE	PSNR GAIN
PSF std=0.5	29.05	34.02	5.03
PSF std=1.0	18.8	23.26	5.18
PSF std=1.5	14.08	19.43	5.35
PSF std=2.0	11.85	17.35	5.5
The results of MR image (64*64)			
PSF std=0.5	34.6	35.07	1.53
PSF std=1.0	21.8	23.99	2.19
PSF std=1.5	18.38	21.12	2.74
PSF std=2.0	16.69	19.59	2.9

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Appendix

As was shown in section (2-image model),such a random field is modeled by

$$f(i,j) = \rho_v f(i-1,j) + \rho_h f(i,j-1) - \rho_v \rho_h f(i-1,j-1) + n(i,j) ....(a-1)$$

or, equivalently by

$$f(i,j) - \rho_h f(i,j-1) = \rho_v [f(i-1,j) - \rho_h f(i-1,j-1)] + n(i,j) ....(a-2)$$

Where ρ_v and ρ_h are the vertical and the horizontal correlation coefficients of the random field, respectively, and n(i,j) is an array of zero –mean uncorrelated random variables with variance $\sigma_n^2 = (1-\rho_v^2)(1-\rho_h^2)$ .let $\vec{s}(i)$ be a column vector $[f(i,1), f(i,2)....,f(i,N)]^t$ and $\vec{N}(i)$ be a column vector $[n(i,1), n(i,2)....,n(i,N)]^t$, where t denotes matrix transpose. Equation (a-2) can now be written as

Jth col. Nth col.

$$[0 \quad 0 \quad ... \quad -\rho_h \quad 1 \quad 0 \quad ... \quad 0 \quad]\vec{s}(i)$$

Jth col. Nth col.

$$= \rho_v [0 \quad 0 \quad -\rho_h \quad 1 \quad 0 \quad ..... \quad 0]\vec{s}(i-1)$$

$$+ [0 \quad ... \quad 0 \quad 0 \quad ... \quad 1 \quad 0.... \quad 0 \quad]\vec{N}(i) ....(a-3)$$

Jth col. Nth col.

A vector equation like (a-3) can be written for each value of j=2,3,…N.
When j=1,eq.(a-2) reduces to the following:

$$f(i,1) = \rho_v f(i-1,1) + n(i,1), \qquad 1 \le i \le M ......(a-4)$$

equation (a-4) can be written as

Nth col. Nth col. Nth col.

$$[1 \quad 0 \quad .... \quad 0 \quad]\vec{s}(i) = \rho_v [1 \quad 0 \quad .... \quad 0 \quad]\vec{s}(i-1) + [1 \quad 0 \quad ..... \quad 0 \quad]\vec{N}(i) ......(a-5)$$

equations (a-5) for j=1 and N-1 equations like (a-3),all these N equations can be combined into the following vector-matrix form:

$$[G]\vec{s}(i) = \rho_v [G]\vec{s}(i-1) + [I]\vec{N}(i) ......(a-6)$$

Where [I] is the NxN identity matrix. [G] is the NxN matrix

$$G = \begin{bmatrix} 1 & 0 & .... & 0 \\ -\rho_h & 1 & 0..... & 0 \\ 0.. & -\rho_h & 1.. & 0 \\ 0.. & .... & -\rho_h & 1 \end{bmatrix} ....(a-7)$$

premultiplying by $[G]^{-1}$ on both sides of (a-6), we get

$$\vec{s}(i) = \rho_v \vec{s}(i-1) + [G]^{-1} \vec{N}(i) .....(a-8)$$

or $\vec{s}(i) = \rho_v \vec{s}(i-1) + [B] \vec{N}(i) .....(a-9)$

Where $[B]=[G]^{-1}$ is the drive matrix

- **The covariance matrix of a random vector N(i)**

$$[Q_n(k)] = E\{\vec{N}(i)\vec{N}^t(i+k)\} = \sigma_n^2 [I]\Delta(k) ......(b-1)$$

where $\Delta(k)$ is the kronecker delta,i.e., $\Delta(k) = 0$ when $k \neq 0$ and $\Delta(0) = 1$

- **System matrix**

By adopted two neighbors the system matrix is:

$$A = \begin{bmatrix} 1 & \rho_h .... & .... & ....0 \\ \rho_h & 1 & \rho_h .... & ....0 \\ 0.. & \rho_h & 1. & \rho_h ....0 \\ 0.. & .... & \rho_h & 1 \end{bmatrix} ....(b-2)$$

- **Observed matrix**

$$h(x,y) = \frac{1}{\sqrt{2\pi\sigma}} e^{-(x^2+y^2)/2\sigma^2} ....(b-3)$$