

On Feebly –Closed Mappings In Bitopological Spaces

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Abstract

This search discusses the α -set and feebly –closed sets in bitopological space, and these concepts define the feebly-closed function , semi-closed function and pre-closed function also we defines an α -closed function and study the relation between these concepts.

Introduction

let S be a subset of a bitopological spaces (X, t_1, t_2) , we denote the closure of S and the interior of S with respect to t_1, t_2 by $cl_{t_1}(S)$, $int_{t_1}(S)$ and $cl_{t_2}(S)$, $int_{t_2}(S)$ respectively.

[O.Njastad,1965] introduced the concept of α -set in a topological space (X, t) . A subset S of (X, t) is called an α -set if $S \subseteq \text{int}(cl(\text{int}(S)))$. the notion of semi open set , per -open set were introduced by [N.Levine, 1963]. a subset A is said to be Feebly-open set [S.N.Maheshwari and U.D.Tapi,1978] in (X, t) if there exist an open set U such that $U \subseteq A \subseteq scl(U)$, the complement of Feebly- open set is called Feebly closed set.

In this search , we shall define the α -set and Feebly -open set in bitopological space (X, t_1, t_2) . A subset S of a bitopological (X, t_1, t_2) is said to be α -set if S is α -set with respect to t_1 or t_2 , that is if $S \subseteq \text{int}_{t_i}(cl_{t_i}(\text{int}_{t_i}(S)))$, $i=1$ or 2 , we shall define a Feebly open set in a bitopological space (X, t_1, t_2) if there exist a open set U with respect to t_1 or t_2 such that $U \subseteq A \subseteq scl_{t_i}(U)$, $i=1$ or 2 , the complement of Feebly- open set is called Feebly- closed set .

Feebly closed and α -closed mapping

The concept of α -closed and Feebly- closed mapping have been introduced by [Mashhour A.S. , etral,1983] and [Maheshwari S.N. and Tapi U.D.,1978] respectively.

Definition (2-1) [Greenwood S.G. and Reilly I.L. , 1986]

Let (X, t) and (Y, σ) are two topological spaces , a function $f: (X, t) \rightarrow (Y, \sigma)$ is said to be:

- 1- Feebly –closed if the image of each closed set in X is Feebly- closed in Y .
- 2- α -closed if the image of each closed set in X is α -closed set in Y .

Lemma: (2-2) [Greenwood S.G. and Reilly I.L. , 1986]

Let A be a subset of (X, t) then $\text{sint}(cl(A)) = cl(\text{int}(cl(A)))$.

Proposition(2-3) [Greenwood S.G. and Reilly I.L., 1986]

Let (X, t) be a topological spaces , a subset A of (X, t) is Feebly closed if and only if A is α -closed set.

Definition (2-4) [Greenwood S.G. and Reilly I.L. , 1986]

Let (X, t) and (Y, σ) are two topological spaces , a function $f: (X, t) \rightarrow (Y, \sigma)$ is said to be:

- 1- semi-closed if the image of each closed set in X is semi -closed set in Y .
- 2- pre-closed if the image of each closed set in X is pre- closed in Y .

Proposition (2-5)[Greenwood S.G. and Reilly I.L. , 1986]

Let (X, τ) and (Y, σ) are two topological spaces , a function $f: (X, \tau) \rightarrow (Y, \sigma)$ is α -closed if and only if it is semi- closed and pre-closed .

Feebly-closed and α -closed mapping in bitopological space.

Definition(3-1)

Let (X, τ_1, τ_2) and (Y, σ_1, σ_2) are two bitopological spaces , a function

$f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ said to be:

1- Feebly- closed if the induced maps $f: (X, \tau_1) \rightarrow (Y, \sigma_1)$ or $f: (X, \tau_2) \rightarrow (Y, \sigma_2)$ is Feebly-closed.

2- α -closed the induced maps $f: (X, \tau_1) \rightarrow (Y, \sigma_1)$ or $f: (X, \tau_2) \rightarrow (Y, \sigma_2)$ is α - closed.

Proposition (3-2):

Let (X, τ_1, τ_2) be a bitopological space and A be a subset of X then $\text{int}_i(A) = \text{cl}_i(\text{int}_i(A))$, $i=1$ or 2 .

Proof: since we have $\text{cl}_i(\text{int}_i(\text{cl}_i(A)))$ is semi open set with respect to τ_i and $\text{cl}_i(\text{int}_i(\text{cl}_i(A))) = \text{cl}_i(\text{int}_i(\text{cl}_i(\text{int}_i(A))))$ and $\text{cl}_i(\text{int}_i(\text{cl}_i(A))) \subseteq \text{cl}_i(A)$ then $\text{cl}_i(\text{int}_i(\text{cl}_i(A))) \subseteq \text{int}_i(\text{cl}_i(A)) \dots (1)$.

Now if V is any τ_i -semi-open set contained in $\text{cl}_i(A)$ then $U \subseteq \text{cl}_i(\text{int}_i(V)) \subseteq \text{cl}_i(\text{int}_i(\text{cl}_i(A)))$ and hence $\text{int}_i(\text{cl}_i(A)) \subseteq \text{cl}_i(\text{int}_i(\text{cl}_i(A))) \dots (2)$. By (1) and (2) we get the result.

Proposition(3-3)

Let (X, τ_1, τ_2) be a bitopological space a subset A of X is Feebly- closed set in X if and only if A is α -closed.

Proof :

It follows from the definition of an α -set and α -closed set in bitopological space . That a subset A of (X, τ_1, τ_2) is α -closed set if and only if $\text{cl}_i(\text{int}_i(\text{cl}_i(A))) \subseteq A$, $i=1$ or 2 , since $\text{cl}_i(\text{int}_i(\text{cl}_i(A))) \subseteq A$ if and only if $\text{int}_i(\text{cl}_i(A)) \subseteq A$ by lemma (2-2) in bitopological space the result exist.

Definition(3-4)

Let (X, τ_1, τ_2) and (Y, σ_1, σ_2) are two bitopological spaces , a function

$f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ said to be:

1- semi-closed if the induced maps $f: (X, \tau_1) \rightarrow (Y, \sigma_1)$ or $f: (X, \tau_2) \rightarrow (Y, \sigma_2)$ is semi-closed.

2- pre -closed the induced maps $f: (X, \tau_1) \rightarrow (Y, \sigma_1)$ or $f: (X, \tau_2) \rightarrow (Y, \sigma_2)$ is pre- closed.

Proposition (3-5):

Let (X, τ_1, τ_2) and (Y, σ_1, σ_2) are two bitopological spaces , a function

$f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is α -closed if and only if it is semi-closed and pre-closed.

Proof: since $f: (X, \tau_1) \rightarrow (Y, \sigma_1)$ is α -closed if and only if it is semi closed and pre-closed [theorem (3), I.L.Reilly and M.R.VamanaMurthy] , similarly $f: (X, \tau_2) \rightarrow (Y, \sigma_2)$ is α -closed if and only if it is semi-closed and pre-closed [theorem (3), I.L.Reilly and M.R.VamanaMurthy] and hence the result.

This example show that if $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is pre closed then f not to be α -closed.

Example (3-6): let $X=\{1,2,3\}$ and defined τ_1 to be the discrete topology and $\tau_2=\{X, \emptyset, \{1\}, \{3\}, \{1,3\}\}$, $\sigma_1=\{X, \emptyset, \{1\}, \{3\}, \{1,3\}\}$ and σ_2 be the discrete topology. Define $f: (X, \tau_1) \rightarrow (Y, \sigma_1)$ by $f(1)=f(2)=f(3)=1$ then f is pre closed but not α -closed since $\{1\}$ is pre closed in (Y, σ_1) but not α -closed in (X, τ_1) and define $f:$

$(X, t_2) \rightarrow (Y, \sigma_2)$ by $f(1)=f(2)=f(3)=1$ then f is pre closed but not α -closed since $\{1\}$ is pre closed in (Y, σ_2) but not α -closed in (X, t_2) .

This example show that if $f: (X, t_1, t_2) \rightarrow (X, \sigma_1, \sigma_2)$ is semi- closed then f not to be α -closed.

Example(3-7)

let $X=\{1,2,3\}$ and defined t_1 and t_2 to be the discrete topologies and σ_1 and σ_2 be the indiscrete topology. Define $f: (X, t_1) \rightarrow (Y, \sigma_1)$ by $f(1)=f(2)=f(3)=1$ then f is semi closed but not α -closed and $f: (X, t_2) \rightarrow (Y, \sigma_2)$ defined by $f(1)=f(2)=f(3)=1$ then f is semi closed but not α -closed since $\{1\}$ is

Conclusion:

From this paper we can conclude that the type of a bitopological space (X, t_1, t_2) is depend on the type of a topological space (X, t_1) and (X, t_2) since the subset of these topologies will induce the subset of (X, t_1, t_2) and because these subset is the same in (X, t_1, t_2) , hence any definition and proposition which is true in (X, t_1) and (X, t_2) will be true in (X, t_1, t_2) in this search.

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