

A Dynamic Economic Dispatch Solution Method using Hopfield Neural Network

Q.M.Alias (B.Sc., MSc., Ph.D.)*

F. Benhamida (BSc. , M.Sc.)**

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Abstract

This paper analyzes and tests a proposed neural network (NN) to solve the Dynamic Economic Dispatch (DED) as part of the unit commitment problem. The proposed NN is a fast and direct computation solver using a Hopfield model for solving the dynamic economic dispatch problem of thermal generators which is a dynamic optimization problem taking into account the constraints imposed on system operation by generator ramping rate limits. Formulations for solving the ED and DED problems are explored. Through the application of these formulations, direct computation instead of iterations for solving the problem becomes possible. Not like the usual Hopfield network, which select the weighting factors of the energy function by trials, the proposed network determines the corresponding factors by calculations and employs a linear input-output model for the neurons. The effectiveness of the developed neural network is identified through its application to the New England test system. Computational results manifest that the model has a lot of excellent performances.

الخلاصة

يعرض البحث تحليل واختبار طريقة مقترحة لحل مسألة التحميل الديناميكي الامثل للاحمال على وحدات توليد القدرة الكهربائية كجزء من مسألة جدولة عمل الوحدات التوليدية .
الطريقة المقترحة تعتمد الحل المباشر والسريع بشبكة هوب فيلد العصبية في التحميل الامثل لوحدات توليد حرارية فقطاخذة بنظر الاعتبار المحددات المفروضة على عمل المنظومة الكهربائية من قبل معدل تحميل الوحدات. تم مناقشة وعرض الاسلوب التقليدي للتحميل الامثل للوحدات والاسلوب الديناميكي للتحميل . في الطريقة المقترحة يتم حساب معاملات الاوزان الداخلة بطريقة مباشرة وليس اختياريًا من محاولات كما في طرق شبكة هوب فيلد الاعتيادية وذلك باعتماد نموذج دخل /خرج خطي للـ
neurons
العصبية للطريقة المقترحة من خلال تطبيقها على منظومة اختبارية قياسية واثبتت التجربة دقة وسرعة الحصول على النتائج المطلوبة .

List Of Symbols And Abbreviations

a_i, b_i and c_i are the cost coefficients,

(constants of the i -th generating unit) .

D : Total demand (MW).

F_T : Total generation cost (\$).

$F_i(P_i)$: Operating cost of unit- i (\$ /hr).

I_i : external input to neuron – i .

L : System transmission losses (MW) .

N : total number of units.

Dept. of Elect. & Electronic Engg. Univ.of Technology, Baghdad- Iraq *

Dept. of Elect. & Electronic Engg. Univ.of Sidi Bel Abbès, SBA, Algeria **

P_i : Real power output of the i -th unit (MW).

P_i^{\min}, P_i^{\max} : unit - i (MW) limits .

R_i^{up} : ramp-up constraint [MW/h].

R_i^{down} : ramp-down constraint [MW/h].

t : time interval (1hr) in the EDP & UCP .

T : total number of time intervals (horizon of the study) .

T_{ij} : The interconnection conductance from the Output of neuron- j to the input of neuron - i .

T_{ii} : The self-connection conductance of neuron - i

U_i : input of neuron - i .

V_i, V_j : The output of neuron - i, j respectively.

DED: Dynamic Economic Dispatch.

DEDP: Dynamic Economic Dispatch Problem.

DP: Dynamic Programming.

ED: Economic Dispatch.

EDP: Economic Dispatch Problem.

EP: Evolutionary Programming.

GA: Genetic Algorithm.

LP: Linear Programming.

NN: Neural Network.

QP: Quadratic Programming.

SA: Simulated Annealing.

UCP: Unit Commitment Problem.

1. Introduction

The subject of this paper is the construction and implementation of an exact method that solves both the economic dispatch problem (EDP) and the dynamic economic dispatch (DED). The performance of such methods with

respect to time and solution quality is a crucial part in the solution process of solving the unit commitment problem. For that reason, a part of this paper discusses a possible alternative to the heuristic approach and an argument for the selection made is given.

2. Problem Formulation

Economic dispatch (ED) is defined as the process of allocating generation levels to the generating units in the mix, so that the system load is supplied entirely and most economically [1] [2]. The objective of the ED problem is to calculate, for a single period of time, the output power of every generating unit so that all demands are satisfied at minimum cost, while satisfying different technical constraints of the network and the generators. Figure 1 shows the configuration that represent the ED problem. The system consists of N generating units connected to a single bus-bar serving a received electrical load D . The input to each unit shown as F_i , represent the cost rate of the unit. The output of each unit P_i is the electrical power generated by that particular unit. The total cost of the system is the sum of the costs of each of the individual units. The essential constraint on the operation is that the sum of the output powers must equal the load demand .

The standard ED problem can be described mathematically as an objective with two constraints as :

$$\min F_T = \sum_{i=1}^N F_i(P_i) \quad ..(1)$$

Subject to the following constraints:

$$\sum_{i=1}^N P_i = D + L \quad ..(2)$$

$$P_i^{\min} \leq P_i \leq P_i^{\max} \quad ..(3)$$

The fuel cost function or input-output characteristic of the generator maybe obtained from design calculations or from heat rate tests. Many different formats are used to represent this characteristic. The data obtained from heat rate tests or from the plant design engineers may be fitted by a polynomial curve. In some cases, quadratic characteristic have been fit to these data. A series of straight-line segments may also be used to represent the input-output characteristic [1]. The fuel cost function of a generator that is usually used in power system operation and control problems is represented by a second-order polynomial:

$$F_i = a_i + b_i P_i + c_i P_i^2 \quad ..(4)$$

3. Possible Solution strategy

The economic dispatch problem (EDP) was defined in section 2 and its objective is to minimize production cost while satisfying demand and working area constraints for a given combination of active units. The solutions of the EDP (combination of units with the least production cost), are used to guide the solution method that solves the combinatorial part of the unit commitment problem (UCP). Figure 2 shows the connection between the

combinatorial part and the EDP for the unit commitment problem. When the combinatorial part is solved, solutions from the EDP are used to estimate the quality of different unit combinations.

If the unit commitment problem is solved by exact methods the EDP should also be solved by exact methods. If not, there is a risk that the method might not converge to the optimal solution. Since the EDP is formulated here as nonlinear problem, the mathematical method of choice is the quadratic programming (QP).

Many practical problem instances of the UCP can be solved either by exact methods or heuristic methods. The number of iterations a heuristic method needs to converge is quite high. An iteration number between 10000 and 100000 is for example not unusual. In those cases the use of QP can be an expensive function evaluation with respect to time. The reason being that for moderately sized systems, the EDP is relatively large quadratic programming problem which has hundreds of decision variables and constraints. The effort used in solving the EDP with a fast solver can therefore lead to impractical computation times and there is a trade off between the quality of the EDP solution and the available computation time for the heuristic that solves the combinatorial part.

When exact methods are used to solve the UCP, the number of feasible solutions that the methods must go through to converge, is so large that the

computation time becomes impractical, and to converge to the optimal solution in a reasonable computation time, an optimal solution to the EDP is necessary and a good estimate is often insufficient. The use of the proposed Hopfield neural network methods to solve the EDP is therefore justifiable if the method produces optimal solutions and outperforms a near-optimal solver with respect to computation time.

4. Economic Dispatch using A Hopfield Neural Network [3]

The ED has been widely studied and reported by several authors in books and journals on power system analysis. Many techniques have been applied to solve this problem. For example, the lambda-iterative method, gradient technique, Interior Point, Lagrange technique, linear programming (LP), Quadratic Programming (QP), Dynamic Programming (DP), Simulated Annealing (SA), Genetic algorithm (GA) [4], Evolutionary Programming (EP), Neural Network and a hybrid between the above methods [5] [6]. Most of the aforementioned often suffers from large amount of computational requirement or give just a good estimate of the optimal (near optimal) solution to the EDP.

In the next section a fast Hopfield Neural Network method to solve the ED problem is proposed. The method employs a linear input-output model for neurons. formulations for solving the

ED problems are explored. Through the application of these formulations, direct computation instead of iterations for solving the problem becomes possible. Not like the usual Hopfield methods, which select the weighting factors of the energy function by trials, this method determines the corresponding factors by calculation.

In neural networks, the Hopfield model [7] has been employed to solve the ED problems for units having continuous or piece wise quadratic fuel cost function [8][9], and even for units having prohibited zones constraint [10] [11].

The conventional Hopfield model belongs to the kind of continuous and deterministic model, and the input-output relationship for its neurons can be described by a modified sigmoidal function. Due to the use of sigmoidal function in the conventional Hopfield model, to solve the ED problems, the numerical iteration method is inevitably applied, though the numerical iteration method often suffers from large amount of computational requirement. Adopting a modified sigmoidal function causes two other problems. The first, it incurs unreasonable or incorrect generation dispatch, which is attributable to the serious saturation phenomena existed in the input-output relationship represented by the sigmoidal function. The second, it is troublesome to select shape constant of the sigmoidal function. To avoid the aforementioned problems, a linear model describing the input-output relationship is adopted [4].

The Hopfield model is a mutual coupling neural network and of non-hierarchical structure. The dynamic Characteristic of each neuron can be described by the following differential equation [9] [12].

$$\frac{dU_i}{dt} = \sum_j T_{ij} V_j + I_i \quad ..(5)$$

The energy function of the continuous Hopfield model can be defined as:

$$E = -\frac{1}{2} \sum_i \sum_j T_{ij} V_i V_j - \sum_i I_i V_i \quad ..(6)$$

The time derivative of the energy function can be proved to be negative [13]. Therefore, in the computation process the model state always moves in such a way that the energy function gradually reduces and converges to a minimum.

To solve the ED problem using the Hopfield method, the energy function including both power mismatch, P_m and total fuel cost F is defined and as follow:

$$E = \frac{A}{2} \left[(D+L) - \sum_{i=1}^N P_i \right]^2 + \frac{B}{2} \sum_{i=1}^N (a_i + b_i P_i + c_i P_i^2) \quad ..(7)$$

$$= (A/2)P_m^2 + (B/2)F$$

A and B introduce the relative importance of their respective associated terms.

With the application of the conventional Hopfield method to the ED problem, we can represent the power output value by

using the output V_i of neuron i with a modified sigmoidal function [8] [10], described as follows:

$$P_i = f_i(U_i) = (1/2) (1 + \tanh(U_i/u_0)) \cdot (P_i^{max} - P_i^{min}) + P_i^{min} \quad ..(8)$$

where ,

u_0 : the shape constant of the sigmoidal function.

Comparing eq (6) with eq (7), we get :

$$T_{ii} = -A - Bc_i \quad ..(9)$$

$$T_{ij} = -A \quad ..(10)$$

$$I_i = A(D+L) - B b_i/2 \quad ..(11)$$

Substituting eq (9), eq (10) and eq (11) in eq (5), the dynamic equation becomes,

$$\frac{dU_i}{dt} = AP_m - \frac{B}{2} \left(\frac{dF_i}{dP_i} \right) \quad ..(12)$$

To avoid the problems resulting from curve saturation, a linear model shown in figure 3 is used to describe the input-output relationship for the neuron instead of the sigmoidal function. Figure 3 depicts three neurons as an example. Linear transfer function of its neurons is defined as follow :

$$P_i = f_i(U_i) = \begin{cases} \frac{U_i - U_{min}}{U_{max} - U_{min}} \cdot (P_i^{max} - P_i^{min}) + P_i^{min} & U_{min} \leq U_i \leq U_{max} \\ P_i^{max} & U_i \geq U_{max} \\ P_i^{min} & U_i \leq U_{min} \end{cases} \quad ..(13)$$

Substituting eq (13) in eq (12) the dynamic equation becomes:

$$\frac{dU_i}{dt} = AP_m - \frac{B}{2} [b_i + 2c_i(K_{1i}U_i + K_{2i})] \quad ..(14)$$

Where:

$$K_{1i} = \frac{(P_i^{\max} - P_i^{\min})}{U_{\max} - U_{\min}}$$

$$K_{2i} = -K_{1i}U_{\min} + P_i^{\min}$$

Solving eq (14) the neuron's input function, $U_i(t')$ is obtained as:

$$U_i(t') = [U_i(0) + \frac{K_{4i}}{K_{3i}}]e^{K_{3i}t'} - \frac{K_{4i}}{K_{3i}} \quad ..(15)$$

With,

$$K_{3i} = -Bc_i K_{1i} \quad ..(16)$$

$$K_{4i} = AP_m - \frac{B}{2}b_i - Bc_i K_{2i} \quad ..(17)$$

From eq (13), the neuron's output function, $P_i(t')$, is obtained as:

$$P_i(t') = \left\{ K_{1i}U_i(0) + K_{2i} - \left[\frac{2K_{AB}P_m - b_i}{2c_i} \right] \right\} e^{K_{3i}t'} + \left[\frac{2K_{AB}P_m - b_i}{2c_i} \right] \quad ..(18)$$

Where, $K_{AB} = A/B$

The first term in eq (18) decays exponentially, finally becomes very small and eventually setting $t' = \infty$, eq (18) gives,

$$P_i(\infty) = (2K_{AB}P_m - b_i)/(2c_i) \quad ..(19)$$

Here, $P_i(\infty)$ represents the optimal generation level for unit - i , which is the required solution.

Back substituting eq (19) in eq (18), a more simple formula for unit - i generation level is obtained as:

$$P_i(t') = [P_i(0) - P_i(\infty)]e^{K_{3i}t'} + P_i(\infty) \quad ..(20)$$

For convenience, imagine t' as the time, but, t is a dimensionless variable.

Using the power mismatch definition and eq (19) we obtain:

$$P_m = \frac{D + \frac{1}{2} \sum_{i=1}^N \frac{b_i}{c_i}}{1 + K_{AB} \sum_{i=1}^N \left(\frac{1}{c_i} \right)} \quad ..(21)$$

The expressions given by eq (19), eq (20) and eq (21) together make the Hopfield model for the economic dispatch problem and a direct computation (not iterative) process becomes possible. The flow chart in figure 4 presents the solution steps.

5. Dynamic Economic Dispatch Using Hopfield NN

The ED problem assumes that the amount of power to be supplied by a given set of units is constant for a given interval of time and attempts to minimize the cost of supplying this energy subject to constraints on the static behavior of the generating units. However, plant operators, to avoid shortening the life of their equipment, try to keep thermal gradients inside the turbine within safe limits. This mechanical constraint is usually translated into a limit on the rate of increase/ decrease of the unit power output. Such ramp rate constraints

distinguish the dynamic economic dispatch (DED) from the traditional, static economic dispatch. Since these ramp rate constraints involve the evolution of the output of the generators, the DED cannot be solved for a single value of the load. Instead it attempts to minimize the overall cost of power generation.. It is a method to schedule the online generator outputs with the predicted load demands over a certain period of time so as to operate an electric power system most economically while the system is operating within its security limits [14] [15]. This problem is a dynamic optimization problem taking into account the constraints imposed on system operation by generator ramping rate limits. The DED is not only the most accurate formulation of the ED problem but also the most difficult to solve because of its large dimensionality. The problem can be mathematically formulated as follows:-

$$\min F_T = \sum_{t=1}^T \sum_{i=1}^N F_i^t(P_i^t) \quad ..(22)$$

subject to the following constraints :

(a) Power balance constraint

$$\sum_{i=1}^N P_i^t - D^t = P_m^t \quad ..(23)$$

(b) Unit capacity constraints

$$P_i^{\min} \leq P_i^t \leq P_i^{\max} \quad ..(24)$$

(c) Ramp rate constraints

$$P_i^t - P_i^{t-1} \leq R_i^{up} \quad ..(25)$$

$$P_i^{t-1} - P_i^t \leq R_i^{down} \quad ..(26)$$

for $i = 1, 2, \dots, N$ and $t = 1, 2, \dots, T$

Since the introduction of the DED problem, several optimization methods have been used to solve this problem. However, all of those methods may not be able to provide an optimal solution, and usually getting stuck at a local optima. Recently, stochastic optimization techniques [16] [17] such as SA, GA and EP have been given much attention by many researchers due to their ability to seek for the near global optimal solution.

In this work, the drawbacks of the conventional sigmoidal based Hopfield model were treated via the adoption of a linear input/ output neuron relationship, thence, one calculation process is required (no iterations). This led to a very short computing time, therefore, suitability for on-line usage.

The detailed approach explained in section 4 for the ED problem solution was easily extended to solve the DED problem. The extension is self explanatory and depicted in the flow chart of figure 5.

6. Case Study and Results

To test and demonstrate the performance of the proposed Hopfield based DED solver, the New England Test System [18] is considered for study. The system consists of ten generating units, their cost data, limits and ramp rate limits are given in Table 1.

Simulations were carried out on Pentium III 500 MHz processor, and the results are shown in Table 2, which

illustrates the generators optimal power outputs for each hour including their corresponding fuel costs. Total production cost for the 12 hour intervals is \$2196534.40. The problem was solved using the proposed Hopfield model with $U_{min} = -0.5$, $U_{max} = 0.5$, $P_m = 0.001$ (the power mismatch). The computation time for each interval was about 0.004 sec. and the total computation time was about 0.048 sec. A comparison results regarding computation time of the proposed Hopfield NN model, LP [19] and EP [20] are given in Table 3, where, the same previous test system is used.

According to Table 3, the result obtained from the proposed model is better than those obtained from LP and EP. It is seen that the developed approach can provide a better solution within a very short time compared with other approaches.

The computation time for the EP presented in [20] varies with the population size (20 - 80). Increasing the population size may provide a cheaper total production cost but a longer computation time is required. Therefore, in order to obtain a high quality solution in EP, long computing time is necessary, which is not the case in the Hopfield Neural Network model proposed here, where the solution was obtained in a very short time due to the nonhierarchical structure of the model.

7. Conclusions

The subject of this paper was the construction and implementation

of an exact method that solves both, the EDP and the DEDP. For this purpose a very fast solver (a Hopfield neural network based model) is explained and constructed in section 4 and modified in section 5 to include the DED problem which is a dynamic optimization problem taking into account the constraints imposed on system operation by generator ramping rate limits. The developed model overcomes the drawbacks of the conventional sigmoidal function based Hopfield neural network one. This is done by adopting a linear input/output neurons relationship, which resulted in a superior Hopfield neural network as one calculation process is required (i.e. no iterations). This led to a very short computing time and suitability for on-line usage.

The model was constructed in such a way that the determination of the energy function weighting factors is not necessary, also, its connective conductances and external input can be determined directly using the system data. The non hierarchical structure of the developed model offers the benefits of, simplicity, efficiency no requirement for training and relative ease of application.

The results obtained using the proposed NN model on a standard test system verify that the proposed approach is a very fast solver compared to other powerful approaches such as LP and EP, and give the sought optimal results.

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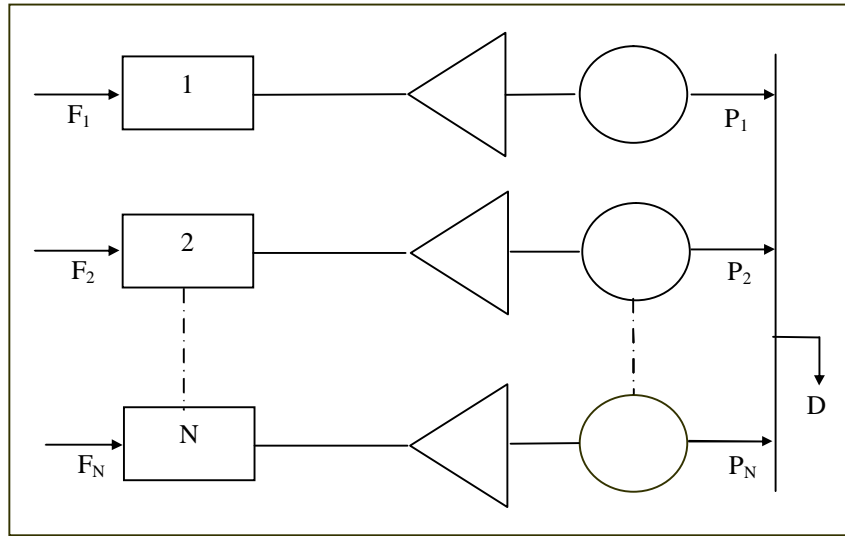


Fig 1: Economic dispatch (symbolic)

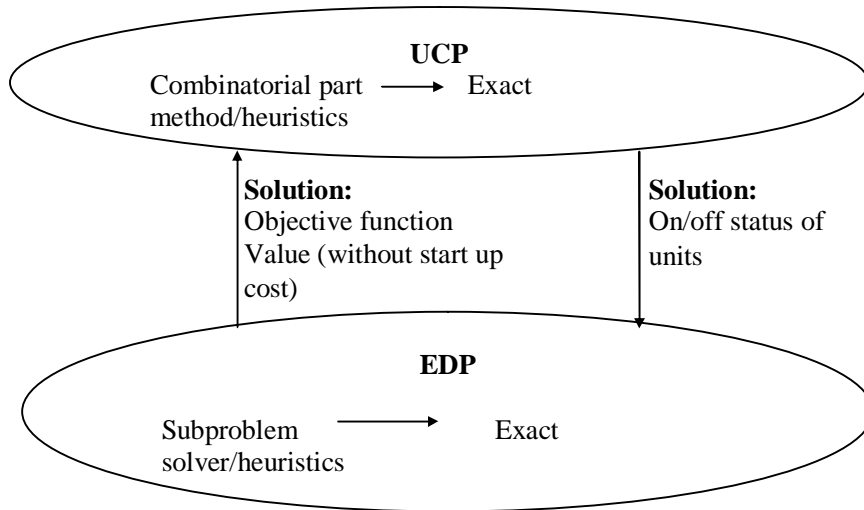


Fig 2: The connection between the combinatorial part of the UCP and the EDP.

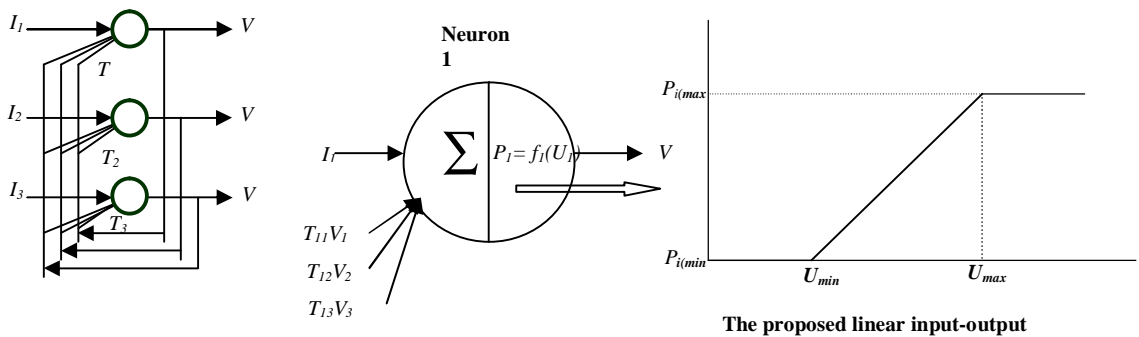


Fig 3: Topology of the Hopfield neuron networks.

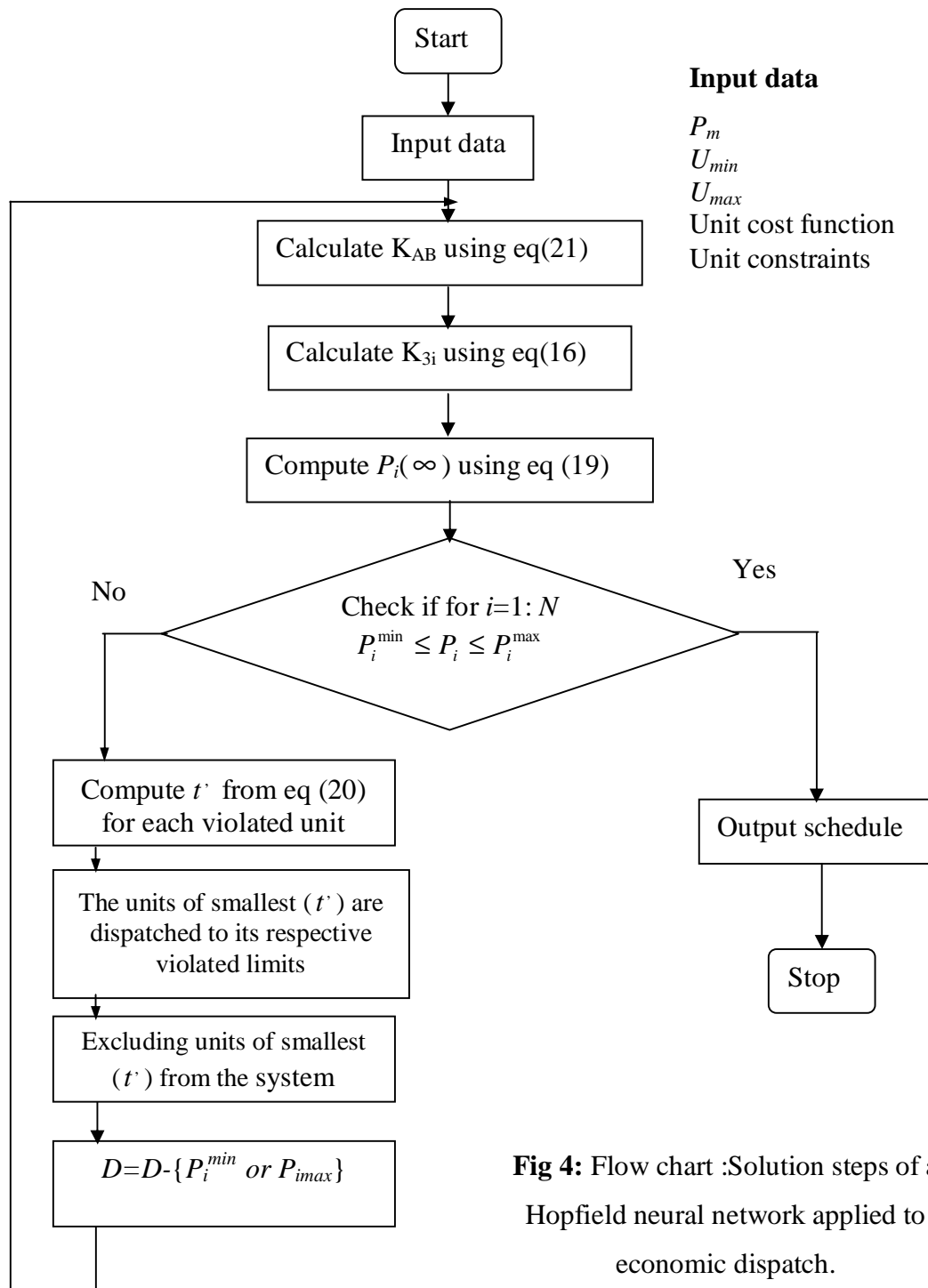


Fig 4: Flow chart :Solution steps of a Hopfield neural network applied to economic dispatch.

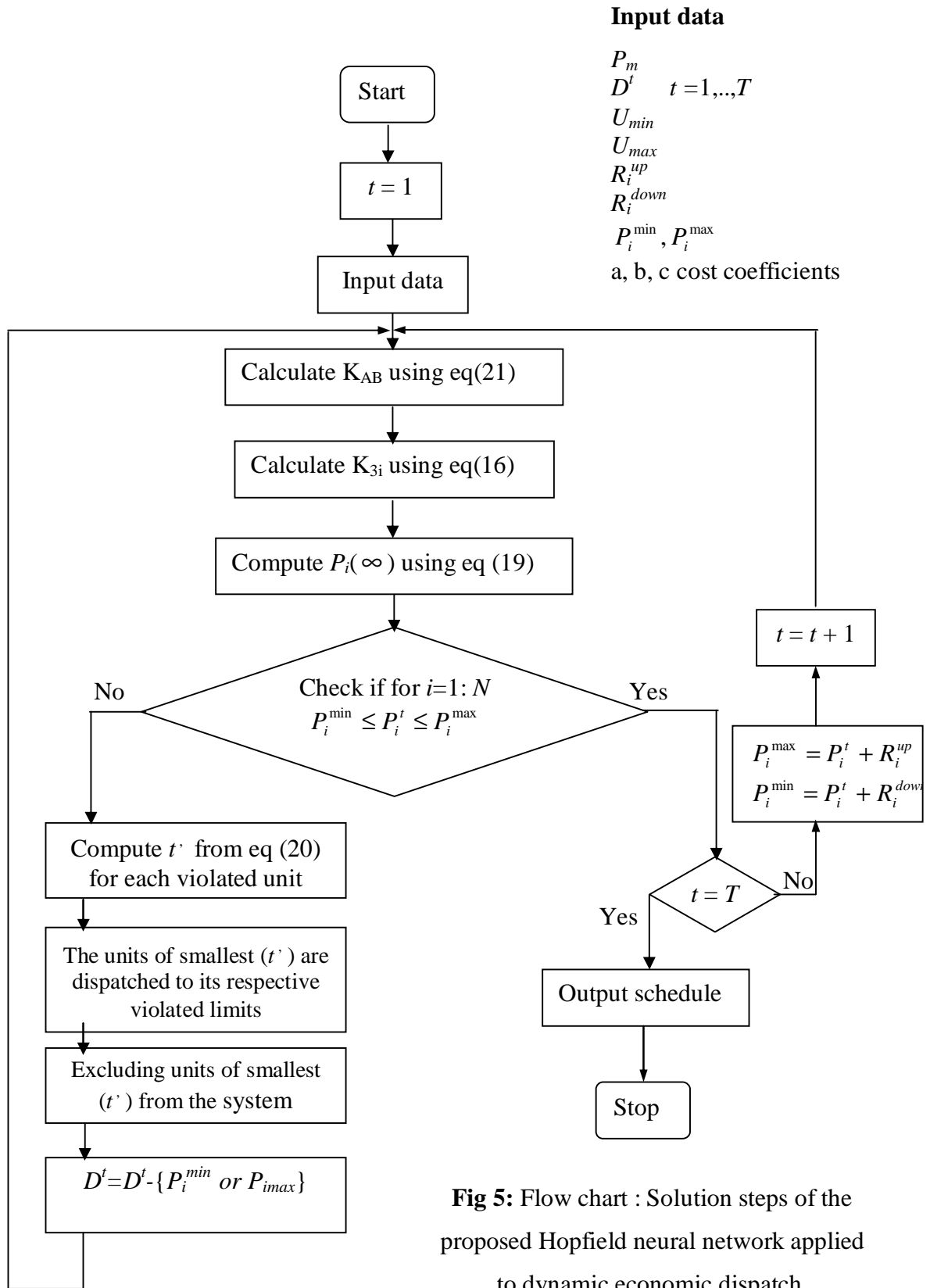


Fig 5: Flow chart : Solution steps of the proposed Hopfield neural network applied to dynamic economic dispatch.

Table 1 Unit data of the New England test system

	P_{max} (MW)	P_{min} (MW)	a (\$/h)	b (\$/MWh)	c (\$/MW2h)	R^{up} (MW/h)	R^{down} (MW/h)
Unit 1	360	155	180	26.4408	0.03720	20	25
Unit 2	680	320	275	21.0771	0.03256	20	25
Unit 3	718	323	352	18.6626	0.03102	50	50
Unit 4	680	275	792	16.8894	0.02871	50	50
Unit 5	600	230	440	17.3998	0.03223	50	50
Unit 6	748	350	348	21.6180	0.02064	50	50
Unit 7	620	220	588	15.1716	0.02268	100	100
Unit 8	643	225	984	14.5632	0.01776	100	150
Unit 9	920	350	1260	14.3448	0.01644	100	150
Unit 10	1050	450	1200	13.5420	0.01620	100	150

Table 2 DED results of the New England test system

Unit o/p (MW)	Time period						
	1	2	3	4	5	6	
Unit no .	1	238.61	243.55	261.72	238.61	280.94	286.65
	2	354.98	360.62	380.62	354.98	402.45	409.87
	3	411.52	417.44	439.23	411.52	462.29	469.13
	4	475.51	481.91	505.46	475.51	530.37	537.76
	5	415.66	421.36	442.33	415.66	464.52	471.11
	6	546.88	555.78	588.53	546.88	623.18	633.47
	7	620.00	620.00	620.00	620.00	620.00	620.00
	8	643.00	643.00	643.00	643.00	643.00	643.00
	9	907.80	918.97	920.00	907.80	920.00	920.00
	10	946.03	957.36	999.09	946.03	1043.24	1050.00
PD (MW)	5560.00	5620.00	5800.00	5560.00	5990.00	6041.00	
Total cost (\$)	173395.19	176057.81	184199.24	184658.67	193057.36	195481.77	

Unit o/p (MW)	Time period (contd.)						
	7	8	9	10	11	12	
Unit no .	1	281.96	260.63	249.51	236.96	250.52	256.59
	2	404.51	380.14	367.43	353.10	368.59	375.52
	3	463.51	437.93	424.59	409.55	425.80	433.08
	4	531.69	504.05	489.63	473.38	490.94	498.81
	5	465.70	441.08	428.24	413.76	429.41	436.41
	6	625.02	586.57	566.53	543.92	568.35	579.28
	7	620.00	620.00	620.00	620.00	620.00	620.00
	8	643.00	643.00	643.00	643.00	643.00	643.00
	9	920.00	920.00	920.00	904.08	920.00	920.00
	10	1045.59	996.60	971.06	942.25	973.38	987.31
PD (MW)	6001.0	5790.00	5680.00	5540.00	5690.00	5750.00	
Total cost (\$)	193578.49	183740.53	178744.55	172512.55	179194.97	181913.28	

Table 3 Comparison of simulation results

Method	Tot. Cost (\$)	Comp. time (s)
Linear Programming	2196939	20
Evolutionary Programming	2196608	700-2000
Hopfield Neural Network	2196534.40	0.048