Calculation The Cross Sections and Neutron Yield for $^{56}Fe(p,n)^{56}Co$ Reaction and Reverse Reaction

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Abstract:

In this study intermediate elements ^{56}Fe , ^{56}Co for $^{56}Fe(p,n)^{56}Co$ reactions as well as proton energy from (6.8) MeV to (18) MeV with threshold energy (5.60569) MeV were implemented according to the available data of reaction cross sections. The more recent cross sections data of $^{56}Fe(p,n)^{56}Co$ reaction were reproduced in fine steps of (200) keV and by using (Matlab-7.6) program we got the equation from 8-degree for plotted. By using inverse reaction principle we got mathematical equation to calculate the cross section of $^{56}Co(n,p)^{56}Fe$ reaction. We deduced that the high probability to produce ^{56}Fe by bombarding ^{56}Co by neutron. These cross sections altogether with the stopping powers were calculated from the Zeigler formula and used to measure the n-yield for reaction.

حساب المقاطع العرضية والحصيلة النيوترونية لتفاعل $^{56}Fe(p,n)^{56}Co$ والتفاعل المعاكس له

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الخلاصة

في هذه الدراسة حسبت المقاطع العرضية للنوى المتوسطة ^{56}Co , ^{56}Fe للتفاعل $^{56}F(p,n)^{56}Co$ للبيانات المتوفرة في الادبيات العالمية وللمدى الطاقي من (6.8) MeV الى (18) MeV وبخطوات طاقية مقدارها (200) keV وبطاقة عتبه مقدارها (5.60569) MeV كدالة للمقاطع العرضية .بأستخدام نظرية التعاكس تم اشتقاق معادلة لحساب المقاطع العرضية لتفاعل $^{56}Co(n,p)^{56}Fe$ وذلك بالاعتماد على المقاطع العرضية لتفاعل $^{56}Fe(p,n)$ ^{56}Co . اشتقت النتائج ورسمت وجدولت بالاضافة الى مناقشة النتائج وتحديد نوع النيوترون لأنتاج النظير ^{56}Fe .أستخدمت هذه المقاطع العرضيه المستحدثة وبالأعتماد على برنامج (SRIM) لحساب قدرة الايقاف من معادلات Zeigler تم حساب الحصيلة النيوترونية للتفاعل المذكور.

Key word: 1-Cross Section 2- Reverse Reaction 3- Stopping Power 4-Neutron Yield

1. Introduction

When two charged nuclei, overcoming their Coulomb repulsion, a rearrangement of the constituents of the nucleus may occur. Similar to the rearrangement of atoms in reacting molecules during a chemical reaction this may result as a nuclear reaction. Nuclear reactions are usually produced by bombarding a target nucleus with a nuclear projectile in most cases a nucleon (neutron or proton) or a light nucleus such as a deuteron or an α-particle [1].

At low excitation energies (< 10 MeV), the majority of nuclear reactions involve the formation of two nuclei, one nearly equal in charge and mass number to the target nucleus. Such reactions are represented by an equation of the type :

$$X + a \rightarrow Y + b \quad \text{------ (1)}$$

Or X (a,b) Y

Where a is the light projectile nucleus (proton , neutron, deuteron, ^{3}H, ^{3}He, or ^{4}He) and X is the target nucleus at rest in the laboratory system. Y is the produced nucleus and b is a light nuclear particle which carries away the major share of the kinetic energy. If the product nucleus Y is left in an excited state after the emission of the light particle b, it usually subsequently decays by radiating one or more gamma rays. Alternatively if Y is beta unstable, it decays at some later date by electron or positron emission followed by gamma emission [2].

Nuclear reactions of low excitation energies include the following types : (n,γ) , (n,p) , (n,α) , (α,n) , (p,γ) , (p,n) , (d,n) , (d,p) , …..etc.

$$X + a \longrightarrow \begin{cases} X + a, \\ X^* + a, \\ Y + b, \\ Y^* + b, \end{cases} \quad \text{------ (2)}$$

In the first two reactions of the set (2) the outgoing particle is of the same kind as the incident particle, and the process is called scattering. The first reaction represents elastic scattering and the second reaction represents inelastic scattering in which the target nucleus X is raised into an excited state (X*). The other reactions of the set represent different possible nuclear transmutations in which the product nuclei may be found in their ground states or, more often, in excited states. The excited product nucleus usually decays very quickly to the ground state with the emission of γ-rays[2].

2. Cross Sections Of Nuclear Reactions:

To characterize the probability that a certain nuclear reaction will take place, it is customary to define an effective size of the nucleus for that reaction, called a cross section [1]. The reaction cross section data provides information of fundamental importance in the study of nuclear systems. The cross section is defined by [3]:

$$\sigma = R / I \quad \text{------ (3)}$$

where σ : is the cross section,

R : is the number of reactions per unit time per nucleus.

I : is the number of incident particles per unit time per unit area,

The cross section has the units of area and is of the order of the square of nuclear radius. A commonly used unit is the barn:

$$1 \text{ barn} = 10^{-24} \text{ cm}^2$$

In general, a given bombarding particle and target can react in a variety of ways producing a variety of light reaction products per unit time. The total cross section is then defined as [4]:

$$\sigma_{tot} = \sum_i \sigma_i \quad \text{------- (4)}$$

Where σ_i is the partial cross section for the ith process.

3. Reverse Reaction:

If the cross-sections of the reaction A(p,n)B are measured as a functions of T_p (T_p = Kinetic energy of incident proton), the cross–sections of the inverse reaction B(n,p)A can be calculated as a function of T_n (T_n = Kinetic energy of neutron) using the reciprocity theorem [5] which states that :

$$\frac{\sigma_{(p,n)}}{g_{(p,n)}\lambdabar_p^2} = \frac{\sigma_{(n,p)}}{g_{(n,p)}\lambdabar_n^2} \quad \text{------ (5)}$$

Where $\sigma_{(p,n)}$ and $\sigma_{(n,p)}$ represent cross-sections of A(p,n)B and B(n,p)A reactions respectively, $g_{(p,n)}$ and $g_{(n,p)}$ represent a statistical factors of A(p,n)B and B(n,p)A reactions respectively $\lambdabar$ is the de–Broglie wave length divided by 2π and is givens by [6]

$$\lambdabar = \frac{\hbar}{M\nu} \quad \text{--------- (6)}$$

Where ħ is Dirac constant ($h/2\pi$), h is Plank constant, M and ν are mass and velocity of p or n .

From eq.(6) ,we have

$$\lambdabar^2 = \frac{\hbar^2}{2MT} \quad \text{------ (7)}$$

The statistical g-factors are givens by [5]

$$g_{(p,n)} = \frac{2J_c+1}{(2I_A+1)(2I_p+1)} \quad \text{------ (8)}$$

and

$$g_{(n,p)} = \frac{2J_c+1}{(2I_B+1)(2I_n+1)} \quad \text{---- (9)}$$

The conservation law of the momentum and parity implies that :

$$I_A + I_p = J_c = I_B + I_n \quad \text{------ (10)}$$

and

$$\pi_A \cdot \pi_p (-1)^{\ell p} = \pi_c = \pi_B \cdot \pi_n (-1)^{\ell n} \quad \text{---- (11)}$$

J_c and π_c are total angular momentum and parity of the compound nucleus .

I_A and π_A are total angular momentum and parity of nucleus A.

I_B and π_B are total angular momentum and parity of nucleus B.

I_p and π_p are total angular momentum and parity of proton.

I_n and π_n are total angular momentum and parity of neutron .

$$\pi_p = \pi_n = +1 \quad \text{------ (12)}$$

$$I_p = s_p + \ell_p \quad \text{------ (13)}$$

where

s_p is spin of proton = ½

ℓ_p is the orbital angular momentum of proton

and

$$I_n = s_n + \ell_n \quad \text{---------- (14)}$$

where

I_n is the total angular momentum of the neutron

s_n is spin of neutron = ½

ℓ_n is the orbital angular momentum of neutron

From eq.(10) ,we have :

$$|J_c - I_A| \le I_p \le J_c + I_A \quad \text{---- (15)}$$

and

$$\left|J_c - I_B\right| \le I_n \le J_c + I_B \quad \text{---- (16)}$$

The reactions A(p,n)B and B(n,p)A can be represented with the compound nucleus C as in the following schematic diagram. It is clear that there are some important and useful relations between the kinetic energies of the neutron and proton.

$$E = S_p + \frac{M_A}{M_A + M_p} T_p \quad \text{---- (17a)}$$

$$E = S_n + \frac{M_B}{M_B + M_n} T_n \quad \text{----- (17b)}$$

One can calculate the separation energies of proton (S_p) and neutron (S_n) using the following relations:

S_p = 931.5 [M_A + M_p - M_c]------- (18)

S_n = 931.5 [M_B + M_n - M_c] ----- (19)

Combining (17a) , (17b) , (18) , (19)

And the equation of Q- value of the reaction A(p,n)B which is given by :

Q = 931.5 [M_A + M_p − M_B – M_n] ---- (20)

We got that :

$$Q = \frac{M_B}{M_B + M_n} T_n - \frac{M_A}{M_A + M_p} T_p \quad \text{---- (21)}$$

Or :

$$T_n = \frac{M_B + M_n}{M_B}\left[\frac{M_A}{M_A + M_p} T_p + Q\right] \text{----- (22)}$$

Then the threshold energy E_{th} is :

$$E_{th} = \left|-Q\frac{M_A + M_P}{M_A}\right| \quad \text{------- (23a)}$$

Or

$$Q = -\frac{M_A}{M_A + M_p} E_{th} \quad \text{------ (23b)}$$

Then

$$T_n = \frac{M_B + M_n}{M_B} \times \frac{M_A}{M_A + M_p}(T_p - E_{th}) \quad \text{-----(24)}$$

Thus eq . (5) can be written as follows:

$$\sigma_{(n,p)} = \frac{g_{(n,p)} M_p T_p}{g_{(p,n)} M_n T_n}\, \sigma_{(p,n)} \quad \text{-----(25)}$$

It is clear from this equation that the cross sections of reverse reaction are related by a variable parameters which can be calculated if the nuclear characteristics of the reactions are known.

4. Stopping Power:

Is a measure of the effect of a substance on the kinetic of a charged particle passing through it. Stopping power is often quoted relative to that of a standard substance, usually air or aluminum [7].

5. Proton Stopping Power :

For hydrogen projectiles, the nuclear stopping power is very small for all energies of interest [8]. The electronic stopping power is found to be proportional to projectile velocity, the specific dependence [9] being given by:

$$S_e = Z_1^{1/6} \times 8\pi e^2 a_o \frac{Z_1 Z_2}{(Z_1^{2/3} + Z_2^{2/3})^{3/2}} \times \frac{\upsilon}{\upsilon_o}, \quad \text{-- (26)}$$

Where $\upsilon \langle \upsilon_o Z_1^{2/3}$

where Z_1 and Z_2 are the atomic numbers of projectile and target, respectively,

υ is the projectile velocity,

a_o , υ_o are the Bohr radius of the hydrogen atom and the Bohr velocity.

In the present work, we used the formulas proposed by Varelas and Biersack sited in Ziegler [8]

$$S_e = \frac{S_{Low} S_{High}}{(S_{Low} + S_{High})} \quad \text{------ (27)}$$

where S_{Low} (Low energy stopping) is

$$S_{Low} = B_1 E^{1/2} \quad \text{------ (28)}$$

and S_{High} (High energy stopping) is

$$S_{High} = \frac{B_2}{E} \ln(1 + \frac{B_3}{E} + EB_4) \quad \text{----- (29)}$$

where B_1, B_2, and B_3 are fitting constants

$$B_4 = 4\ m / I\ M$$

where m is the electron mass,

I is the mean ionization potential,

M is the projectile mass

6. Neutron Yields :

For an accelerating beam traversing a target, the occurred nuclear reactions produce N light product particles per unit time. Referring to Fig. (1) the yield is given by

$$Y(x) = I_o N_d \sigma x \quad \text{------ (30)}$$

Experimentally, the yield of neutrons detected per incident particle, Y_n , for an ideal, thin and uniform target and mono-energetic beam of energy E is given by

$$Y_n = (N_d x) \sigma(E_b) \eta(E_b) \quad \text{----- (31)}$$

where (N_d x) is the density of target atoms, and η is the neutron-detection efficiency.

For a target which is not infinitesimally thin, the beam loses energy as it passes through the target, and the yield is then given by [10]

$$Y_n = \int_{E_t}^{E_b} \frac{\sigma(E`)\eta(E`) f dE`}{\frac{dE}{dx}(E`)} \quad \text{--- (32)}$$

in which $E_t = E_b - \Delta E$, where ΔE is the energy loss of the beam in the target, f is the number of target atoms in each target molecule, and $\frac{dE}{dx}(E`)$ is the stopping power per target molecule, If the target is sufficiently thick, and there exist one atom per each molecule (i.e., f = 1) and taking η(E`) = 1, then the resulting yield is called the thick-target yield which is given by

$$Y(E_b) = \int_{E_{thr}}^{E_b} \frac{\sigma(E) dE}{dE/dx} \quad \text{------ (33)}$$

Where E_{thr} is the reaction threshold energy.

Thus, by measuring the yield at two closely spaced energies E_1 and E_2, one can determine the average value of the integrand over this energy interval as follows [11]:

$$[\frac{\sigma(E)}{dE/dx}]_{E_b} = \frac{Y(E_2) - Y(E_1)}{E_2 - E_1} \quad \text{---- (34)}$$

where E_b is the average of E_1 and E_2 . If σ(E) are available in the literature as a function of projectile energy E_b for natural elements, then the neutron yield can be calculated using eq.(34). If neutron yield is available as a function of projectile energy E_b, then eq. (34) can be used to calculate σ(E) as a function of E_b. Thus, consequently the neutron yield can be calculated using eq. (34).

For natural elements and if only one stable isotope is available in nature, then [12]

$Y_o = Y(E)$ ------ (35)

where Y_o is the neutron yield per 10^6 bombarding particle for the natural element. If σ(E) is calculated for a certain isotope whose concentration (enrichment) is C %, then [12]

$$Y_o = \frac{a}{c} Y(E) \quad \text{------ (36)}$$

where a is the abundance of the isotope in the natural element. If there exist more than one isotope that can be involved in the nuclear reaction and the cross sections are calculated as a function of incident energy for each isotope, then [10].

$$Y_o = \frac{a_1}{c_1} Y_1(E) + \frac{a_2}{c_2} Y_2(E) + ... \quad \text{---- (37)}$$

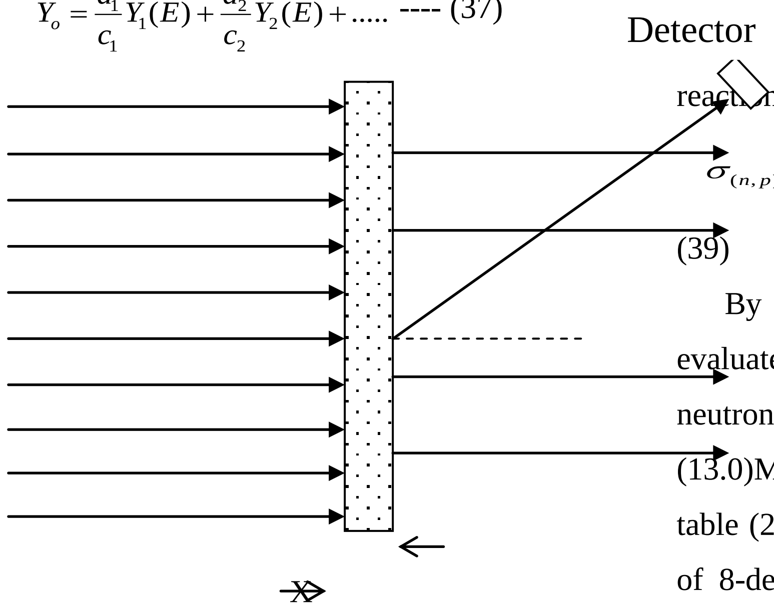

Figure (1): A schematic diagram illustrating the definition of total cross section in terms of the reduction of intensity[10]

I_o : is the number of incident particles per unit time per unit area

I : is the number of out particles per unit time per unit area

7. Results and Discussion

These data have been fitted, spline interpolated and recalculated in fine steps of (200 keV) for proton energy from (6.8) MeV to (18.0) MeV by using Matlab program as shown in table (1). The reproduced cross sections by authors Wenrong Z. , Hanlin LU. and Weixiang Y. [13] and declared by EXFOR-Library , we got 8-degree polynomial for plotted shown in Fig.(2) as follows:

$Y = 0.00016*x^8 – 0.019*x^7 + 0.92*x^6 – 25*x^5 + 4.2*10^2*x^4 – 4.4*10^3*x^3 + 2.8*10^4*x^2 – 9.7*10^4*x + 1.4*10$ ----------- (38)

Bv using the compound theory we derive mathematical formula for $^{56}Co(n,p)^{56}Fe$ reaction for first exited state :

$$\sigma_{(n,p)} = 1.42772 \frac{M_p T_p}{M_n T_n} \sigma_{(p,n)} \quad \text{----- (39)}$$

By using semi empirical formula the evaluated cross sections as a function of neutron energy from (1.2)MeV to (13.0)MeV of present work are listed in table (2). Theses were fitted to polynomial of 8-degree and we got the mathematical equation representing the cross section distribution in the indicated range of neutron energy Fig.(3) as follows :

$$Y = 0.00036*x^8 - 0.026*x^7 + 0.75*x^6 - 11*x^5 + 99*x^4 - 4.9*10^2*x^3 + 1.4*10^3*x^2 - 1.6*10^3*x + 9.5*10^2 \quad ------------ (40)$$

These cross sections altogether with the stopping powers calculated from the Zeigler formula (33) have been used to measure the n-yield for reaction as shown in Fig.(4).

Table (1):The cross sections of $^{56}Fe(p,n)\,^{56}Co$ reaction as a function of proton energy with threshold energy (5.60569)MeV

p -energy (MeV)	Cross-sections (mbarn)	p -energy (MeV)	Cross-sections (mbarn)	p -energy (MeV)	Cross-sections (mbarn)
6.8	125.968	10.4	348.945	15.0	319.178
7.0	141.135	10.6	350.997	15.2	294.613
7.2	156.301	10.8	353.048	15.4	267.477
7.4	171.468	11.0	357.837	15.6	250.172
7.6	186.585	11.2	366.301	15.8	234.809
7.8	200.849	11.4	369.309	16.0	218.571
8.0	215.114	11.6	372.318	16.2	207.351
8.2	229.379	11.8	375.326	16.4	196.858
8.4	243.643	12.0	378.334	16.6	186.508
8.6	257.176	12.2	381.959	16.8	176.429
8.8	270.691	12.4	385.219	17.0	168.338
9.0	284.205	12.6	387.915	17.2	160.248
9.2	297.719	12.8	390.612	17.4	146.014
9.4	311.233	13.0	393.308	17.6	124.992
9.6	324.748	13.2	392.189	17.8	103.969
9.8	338.262	13.4	388.148	18.0	102.211
10.0	344.841	13.6	381.611	----	----
10.2	346.893	14.8	331.625	----	----

Table (2): The cross sections of ^{56}Co (n,p) ^{56}Fe reaction as a function of neutron energy.

n-energy (MeV)	Cross-sections (mbarn)	n -energy (MeV)	Cross-sections (mbarn)	n -energy (MeV)	Cross-sections (mbarn)
1.2	286.333	5.2	799.899	9.2	750.452
1.4	320.690	5.4	811.194	9.4	722.257
1.6	355.046	5.6	829.981	9.6	665.684
1.8	389.403	5.8	836.796	9.8	604.213
2.0	423.589	6.0	843.611	10.0	565.753
2.2	455.903	6.2	850.426	10.2	530.952
2.4	488.217	6.4	857.241	10.4	494.428
2.6	520.531	6.6	865.492	10.6	469.012
2.8	552.839	6.8	872.816	10.8	445.296
3.0	583.452	7.0	878.924	11.0	421.851
3.2	614.066	7.2	885.032	11.2	399.161
3.4	644.680	7.4	891.140	11.4	380.833
3.6	675.294	7.6	888.199	11.6	362.505
3.8	705.907	7.8	878.867	11.8	329.452
4.0	736.521	8.0	864.058	12.0	281.829
4.2	767.135	8.2	849.249	12.2	234.206
4.4	781.308	8.4	834.440	12.4	231.670
4.6	785.956	8.6	821.016	12.6	236.312
4.8	790.604	8.8	806.842	12.8	237.156
5.0	795.251	9.0	778.647	13.0	225.110

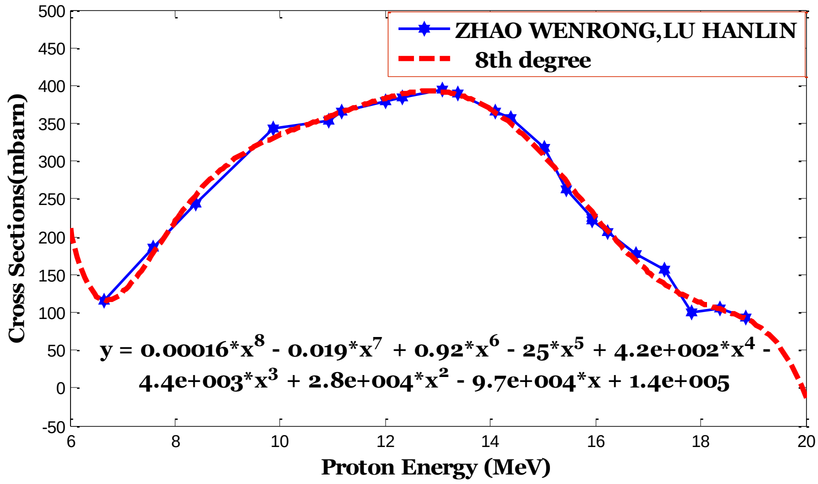

Figure (2): Cross sections of $^{56}Fe(p,n)^{56}Co$ reaction

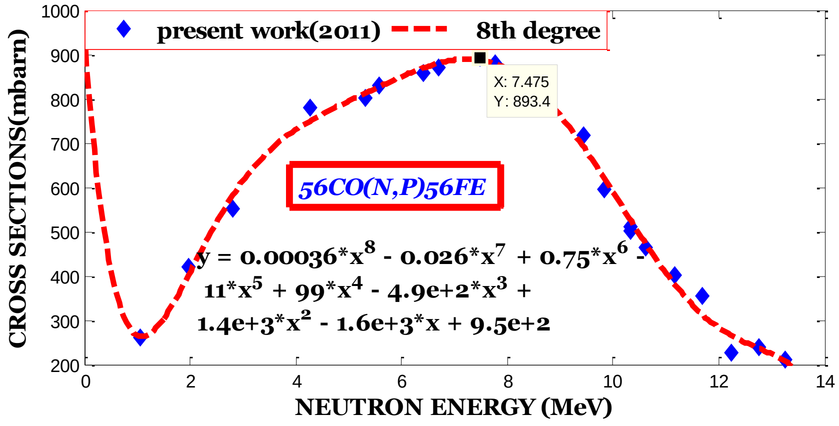

Figure (3): Cross sections for $^{56}Co(n,p)^{56}Fe$ reaction

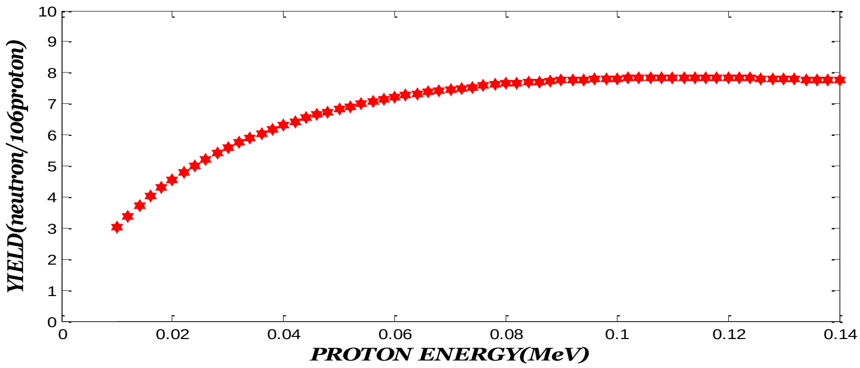

Figure (4): Neutron yield for $^{56}Fe\ (p,n)^{56}Co$ reaction

Reference:

[1] Meyerhof W. E., (1967) "Elements of Nuclear Physics" , Mc Graw- Hill Book Inc.

[2] Smith C. M. H., (1964) , "Nuclear Physics " .

[3] cottingham W. N. and Greenwood D. A., (2001)," An Introduction to nuclear physic", 2nd ed. S Cambridge Univ . press.

[4] Jean L .B., James R., and Michel S., (2005) ,"Fundamentals in Nuclear physics" springer, p.14- 6 .

[5] Macklin R. L. and Gibbons J. H. (1968); phys. Rev. 165 ,1147.

[6] Abrahaim S. A. (2011) ; " Calculation the Cross Sections of (n,α) and (n,p) Reactions by Using the Reciprocity Theory for the First Exited State" P-6 , .

[7] Arora E. D. , (2003) ; "Dictionary of Physics" ," Low Cost Text Book Publishers", Delhi, India.

[8] Anderson H. H. and Ziegler J. F., (1977) , "Hydrogen: Stopping Powers and Ranges in all elements " vol.3, Pergamon Press, Oxford .

[9] Linhard J. and Scharff M., (1961) , Phys. Rev. 124, 128 .

[10] Wrean P. R., Ph.D. Thesis, California Institute of Technology, (1998).U.S.A.

[11] Norman E. B., Chupp T. E., Lesko K. T., Grant P.J., & Woodruff G. L. (1984) : Phys. Rev. C 30, 1339.

[12] Abdulla R. H., (1999), Ph.D. thesis, University of Baghdad .

[13] Wenrong Z., Hanlin Lu. and Weixiang Yu. (1993); "measurement of cross sections by bombarding Fe with protons up to 19 MeV" , J, CNP , 15, (4), 337,.