Optimal Firing Angle for d.c. Series Motor

Nabeel E. Adam

University of Technology ,Department of Electrical &Electronic Engineering ,Bagdad

Abstract

The input voltage to the direct current series motor is controlled by means of an optimal regulator, for changing the firing angle of the thyristors using pulse width modulation technique. An input output linearization technique is introduced to linearize the non–linear mathematical model of the d.c series motor system .The open-loop and closed-loop control systems hve been analyzed theoretically. A computer program written in a MATLAB is used for computing the system dynamics. The closed-loop control system shows a significant modification in the system parameters like armature current,peak overshoot,settling time……..etc,and an optimal firing angle is achieved.

Key words : firing angle,d.c. motor ,regulator,linearization**,**converter**.**

زاوية القدح المثلى لمحرك تيار مستمر توالي

نبيل انويا ادم

قسم الهندسة الكهربائية والالكترونية

الجامعة التكنولوجية- بغداد –العراق

الخلاصة:

يعرض البحث نظام يتكون من محرك تيار مستمر نوع توالي مجهز من مغير احادي الطور. الفولتية الداخلة الى المحرك مسيطر عليها بواسطة منظم مثالي من خلال تغيير زاوية القدح للثايرسترات باستعمال تقنية تضمين عرض الموجة. وضع نظام تحويل مناسب لتحوير النظام اللاخطي الى اخر خطي .تم تحليل الدارة المفتوحة والدارة المغلقة للنظام نظريا . وضع برنامج في الماتلاب بكج (MATLAB package) لايجاد متغيرات النظام. اظهر نظام الدارة المغلقة نقصان كبير في تيار منتج المحرك ونتائج مقبولة للمتغيرات الاخرى, وتم ايجاد زاوية القدح المثلى للمغير الاحادي الطور المغذي للمحرك.

List Of Symbols

V_a : Applied voltage (average output of converter)(V)

R_a : Armature resistance (Ω)

K_a : Back induced e.m.f constant (V.sec./rad)

I_a : Armature current (A)

C : Motor constant (H)

T_e : Electromagnetic torque (N.m)

T_l : Load torque (N.m)

β : Friction coefficient (N.m/(rad/sec))

ω : Motor speed (rad/sec)

ϕ : Flux per pole (weber)

J : Moment of inertia (N.m)

K : Feedback gain matrix

i_a: Instantaneous armature current (A)

i_f: Instantaneous field current (A)

V_m : Maximum A.C. voltage (V)

L_a : Armature motor inductance (H)

L_f : Series field motor inductance (H)

a : Firing angle (degree)

ω_r: Reference speed (rad/sec)

A : System matrix

B : Input matrix

u : Input vector

D : Output matrix

x : State vector

r_o: Reference input

f : Frequency (Hz)

e : Error (rad/sec)

u_o : Controlled signal

1- Introduction

Direct current (d.c) motors have and still being widely used in industry. They can provide high starting torque and wide range of speed control, both below and above base speed.The speed control methods of a d .c motor is simpler and less expensive than those of a.c motor[1].Although commutators prohibits their use in certain applications, such as high speed drives and operation in hazardous atmospheres which d .c motors play a significant role in many industrial drives application

The steady state speed equation of the d .c motor is given by :

$$\omega = \frac{V_a - i_a R_a}{K_a \phi}$$

From equation above the rotating speed can be controlled by :

i- Changing the applied voltage (V_a) to control the speed in the range below the nominal speed .

ii- Adjustment of the flux , usually by means of field current control to control the speed in the range above the nominal speed . Some times it is necessary to use both of them.

There are two basic methods to obtain variable voltage, these are :

i-Converter control[a.c.to d.c.]

ii-Chopper control [d.c-d.c converter]

The majority of industrial variable – speed d.c drives are permanent – magnet and separately excited d.c motors [2-3]. In industrial applications the d.c shunt and series motors are used, because the performance of the motors were improved by adding compensating winding. Unfortunately the non – linearities which appear in the mathematical models of the d.c series and shunt connected motors complicate their application when closed loop control system must be designed [4].

A state space technique has emerged in the last forty years as a powerful tool in modern control theory. This technique which uses vector and matrices for system representation , permits a simple notation , that is easily accepted and processed by digital computers . The wide use of digital computers now a-days makes this technique popular compared to conventional techniques .

2- State Space Concept :

A dynamic system consists of a finite number of chosen state variables may be described by a set of ordinary differential equations as follows[5]:

$$x^{\bullet} = Ax + Bu \qquad \ldots\ldots\ldots(1)$$

$$y = Dx + Eu \qquad \ldots\ldots\ldots(2)$$

A block diagram representation of the system defined by equations 1&2 is shown in figure (1).

The objective of modern control system design can be described by two words : optimal control , in other words a system is to be designed so that it is optimum in a prescribed sense[6]. Feedback control can be achieved by two main techniques , for each one there are different algorithms and approaches[7].

i- Pole – assignment technique.

ii- Optimal regulator technique.

The dynamic processes of a system is characterized by vector matrix – differential equations.

The problem is to get a linear control law[8].

$$u_o = -Kx(t) \qquad \ldots\ldots\ldots.(3)$$

Where K is the feedback matrix , which minimizes the specified performance index J (or cost function) i.e. :

$$J = \int_0^{\infty} (x^T Qx + u^T Ru) dt \ldots \quad (4)$$

Where Q : is a positive definite (or positive semi definite) real symmetric weighting matrix normally chosen to be a diagonal one ,and

R : is a positive – definite real symmetric weighting matrix .

The value of $u_0(t)$ is said to be optimum if the feed back matrix K is given by :

$$K = R^{-1}B^{T}P \quad \ldots\ldots\ldots\ldots\ldots(5)$$

Where P is the solution of the algebraic matrix Riccati equation

$$A^{T}P + PA - PRP^{T} + Q = 0 \quad \ldots\ldots\ldots (6)$$

The MATLAB package is used in the present work to solve the algabric matrix Ricatti equation equation (6) for P and (5) equation for K , considering different arbitrary values for the weighting matrices R and Q to get the optimally controlled process.

3-MachineMathematical Model

The d.c motor mathematical model can be given by the following differential equations.

$$v_a = E_b + i_a R_a + L_a \frac{di_a}{dt} \quad \ldots\ldots\ldots (7)$$

$$Te = T_L + B * \omega + J\frac{d\omega}{dt} \quad \ldots\ldots\ldots (8)$$

For a d.c series motor,

$$i = i_a = i_f \quad \ldots\ldots\ldots (9)$$

For a single phase fully controlled converter with free wheeling diode , the average output d . c voltage is:

$$Va = \frac{Vm}{\pi}(1 + \cos a) \quad \ldots\ldots\ldots\ldots (10)$$

which is the input voltage of the system

Therefore, the d . c series motor equations will be:

$$\omega^{\bullet} = -\frac{\beta}{J}\omega + \frac{C}{J}i_a^2 - \frac{1}{J}T_L \ldots\ldots\ldots (11)$$

$$i_a^{\bullet} = -\frac{R_a}{L}i_a - \frac{C}{L}i_a\omega + \frac{Vm}{\pi L}(1 + \cos a) \ldots. (12)$$

These are non – linear equations the non-linearity is not taken into acount assuming the operation in the linear region of the magnetization curve . But the non – linearity is still present , because of the multiplication term in equation 11 and 12 of two state variables [4]. Therefore, differential equations (11) &(12) have been linearized using input – output linearization technique[9].

4- Optimal Control Design

The system under investigation is a d.c series motor fed from a.c to d.c fully controlled converter by which the speed of the motor is kept constant if the system is subjected to sudden change in the applied torque. The transformed feedback voltage (v) which is a controlling input signal to the firing circuit of the thyristors, controls the magnitude of the output voltage of the converter.

Fig (2) shows the schematic diagram of the system with controller. The transformed input signal (v) is given

$$v = K_1\omega + K_2 Te + K_3 \int_0^\infty (\omega_r - \omega) dt \quad \ldots.(13)$$

The modified d.c drive system described by equations (11&12) can be put in state space form as follows

$$\frac{dx}{dt} = A\,x + Bv + DT_l \quad \ldots.\ \ldots\ldots (14)$$

where

$$x = [y_1 \ \ y_2]^T$$

now the states of the new system are motor speed , electromagnetic torque and integral of the error between reference speed and motor speed .

Let $\quad p = \omega_r - \omega \quad$ then

$$\begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \\ \dot{p} \end{bmatrix} = \begin{bmatrix} -\frac{B}{J} & \frac{1}{J} & 0 \\ 0 & \frac{-2R}{L} & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ p \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} v + \begin{bmatrix} -\frac{1}{J} \\ 0 \\ 0 \end{bmatrix} Tl + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \omega r.$$

…. (15)

Substituting the values of the parameters of the d . c motor given in appendix(I) yields :-

$$A = \begin{bmatrix} -0.001 & 1 & 0 \\ 0 & -23.2 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

Using the command K= lqr (A , B , Q , R) , in the MATLAB package to solve the linear quadratic Ricatti – equation with weighting matrices R & Q of ,

R = [1]

and

$$Q = \begin{bmatrix} 10^3 & 0 & 0 \\ 0 & 10^6 & 0 \\ 0 & 0 & 10^6 \end{bmatrix}$$

The following optimal feedback matrix gains obtained as.

$$K_1 = -1414.8, \quad K_2 = -978.5, \quad K_3 = +1000$$

To study the open-loop system performance, a computer program is written in MATLAB package using Rung – Kutta method of the fourth order to solve the state equations.

In order to obtain the closed loop response,the resulting contineous–time feed back laws must be applied at distinct instant with varying sampling time as follows [10] :

$$\Delta t = \frac{1}{2\pi f}\left(\frac{\pi}{18} + \Delta a_i\right) \quad \ldots\ldots (16)$$

where Δa_i is the changing in the firing angle at instant i with respect to the previous firing angle i– 1

To determine the discrete–time feed back law , with respect to time . The original input u=cos(a) is differentiated as follows :

$$Let\ \ q=\frac{\pi L}{2V_m Ci}$$

$$u=\frac{\pi L}{2V_m Ci}\left(\frac{2C^2i^2\omega}{L}-\frac{2ciV_m}{\pi L}+k_1\omega+k_2ci^2+k_3\int(\omega_r-\omega)dt\right)$$

$$\frac{du}{d\omega}\Delta\omega=q(k_1+\frac{2C^2i^2}{L})\Delta\omega$$

$$\frac{du}{di}\Delta i=q\left(\frac{4C^2i\omega}{L}-\frac{2CV_m}{\Pi L}+2K_2Ci\right)\Delta i$$

$$\frac{du}{dt}\Delta t=q[K_3(\omega_r-\omega]$$

$$\Delta t=\frac{1}{2\pi f}(\frac{\pi}{18}+\Delta a)$$

$$\Delta t=\frac{1}{36f} \qquad \text{when } \Delta a=0$$

$$\frac{du}{dt}\Delta t=q[k_3(\omega_r-\omega)]\Delta t=q[k_3(\omega_r-\omega)]\frac{1}{2\pi f}(\frac{\pi}{18}+\Delta a)$$

$$\frac{du}{dt}\Delta t=q[k_3(\omega_r-\omega)]\frac{1}{36f} \qquad \text{at } \Delta a=0$$

$$\frac{du}{dt}\Delta t=q[k_3(\omega_r-\omega)]\frac{1}{2\pi f}\Delta a \qquad \text{at } \Delta a\neq 0$$

$$\frac{du}{da}\Delta a=-\sin a\Delta a$$

$$\frac{du}{da}\Delta a+\frac{du}{dt}\Delta t=\frac{du}{d\omega}\Delta\omega+\frac{du}{di}\Delta i+\frac{du}{dt}\Delta t$$

$$\Delta a=\frac{-(k_1+2C^2i^2)\Delta\omega-(\frac{4C^2i\omega}{L}+2k_2Ci)\Delta i-k_3(\omega_r-\omega)\frac{1}{36f}}{\frac{2V_mCi}{\pi L}\sin a-k_3(\omega_r-\omega)\frac{1}{2\pi f}} \quad ....(17)$$

Equation(17) is added to the program of open-loop to achieve the closed-loop response of the motor under different load conditions .

5- Theoretical Results

The closed-loop response is a chieved by adding equation (17)to the program of open-loop system in order to improve the system behavior , In each iteration the firing angle (a) is computed such that :

$$a_i=a_{i-1}+\Delta a$$

where a_i : is the new firing angle

a_{i-1} : is the previous firing angle .

Δa : is the changing of the firing angle .

Fig.s (3&4) show the open-loop responses for the system under the load condition (T_L=0.5 p.u & T_L=1.0 p.u) respectively.

Fig.s (5&6) shows the closed-loop responses for the system under the load condition (T_L=0.5 p.u & T_L=1.0 p.u) respectively.

The state variables and firing angle will vary to reach their steady state values,which are depending on the reference speed1500 r.p.m or 157.08 rad / sec .

6- Conclusions:

Optimal operation of a d.c series motor has been done as follows:

The dynamic response of a d . c series motor is represented by a set of non – linear differential equations in state space form choosing the motor speed and armature current as state variables. An optimal regulator has been designed for the system using a program written in MATLAB implementing Rung – Kutta method of the fourth order to solve the equations, and obtain the time response for the state variables.Significant improvments in the system performance were observed these include:-

i- peak overshoot reduction.

ii- good tracking to reference speed for different loads

iii-optimal firing angle convergance with feedback gains.This is primarly achieved through the linearization technique adapted for the system equations.

Appendix I Motor specifications are 250 V , 15 kW [10].

Parameter	**Value**	**Unit**
R_a+R_f	*0.58*	*Ohm*
$L=L_a+L_f$	*0.05*	*Henry*
J	*1.0*	*Kg.m^2*
B	*0.001*	*N.m.sec/rad*
C	*0.0183*	*Henry*

References

[1] SEN,P.C, " Thyristor DC Drives ",John Wiley,U.S.A ,1981.

[2]EL-SHARKAWI,M.A.and HUANG,C.H.,"Variable Structure Tracking of DC Motor for High PerformanceApplication",IEEE Trans.,Vol.4 ,Desember 1989.

[3]JIANG,J.P.,CHEN.S and SINHA.P., "Optimal Feedback Control of Direct Current Motors",Trans., Indust.Elect 1990,37,pp. 269 274.

[4]ALEXANDRIDIS,A.T.&IRACLEOUS ," Non Linear Controllers for PEMC 1994", Warsaw, September, 1994, Vol.1 , pp.142-146.

[5] KATSUHIKO OGATA, "Modern Control Enginering" Prentic Hall,U.S.A,1970.

[6] BENGAMIN,C. KUO, "Automatic Control System",Fourth Edition,Prentic Hall,1975

[7] Ali,H.A. and PETKOV,H.P., "Algorithm for Pole Assignment of Time-Invariant Linear Control Systems",ScientificSession,VMEI,Leni-n, Sofia,1989..

[8] RICHARDS,R.J, "An Introduction to Dynamics and Control ",Longman group Limited ,p.479 .

[9]JIN-HOON KIM and ZEUNGNAM BIEN, "Some Bound of theSolution of the Algebric Riccati Equations",IEEE,Trans.onAutoCont.Vol.37,No.8,pp.1209-1210,August 1992.

[10] ALEXANDRIDS,A.T. and IRACLEOUS,D.P., "Optimal Non Linear Firing Angle Control of Converter Fed DC Drive System",IEE Proc. Electr.Power Application,Vol.145,No.3,May 1998.

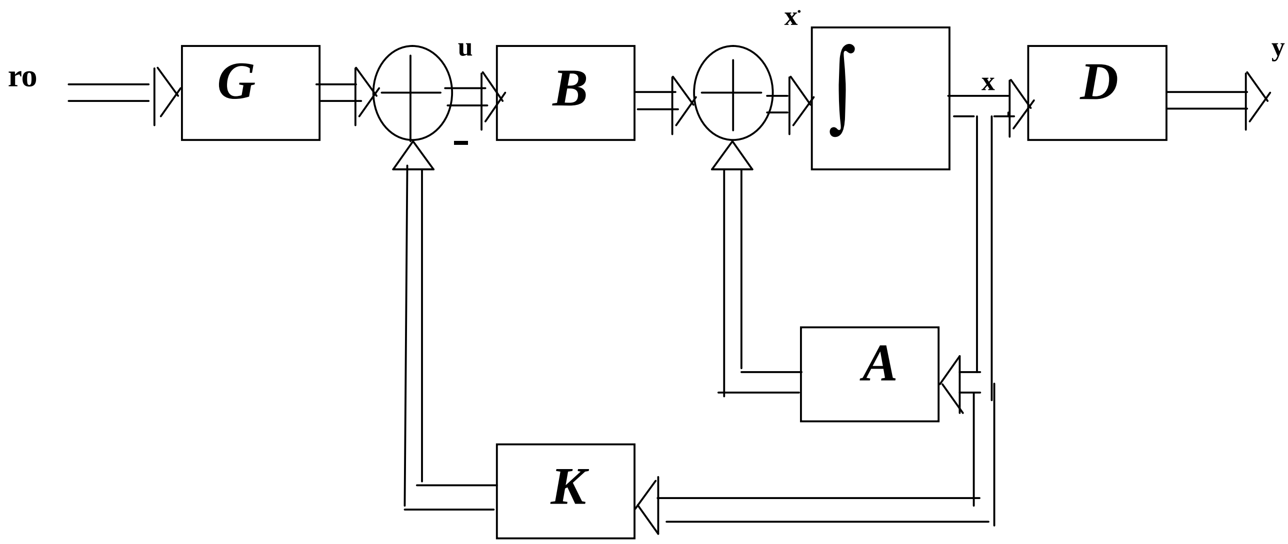

Fig. (1) : Closed-loop control system

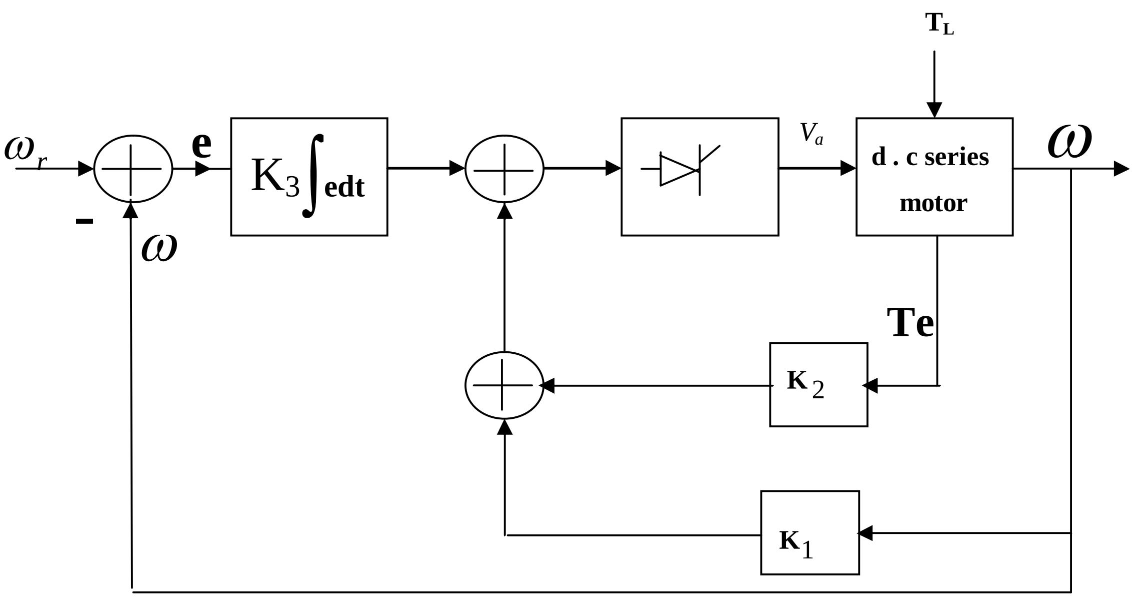

Fig. (2) : Schematic diagram of the system with controller

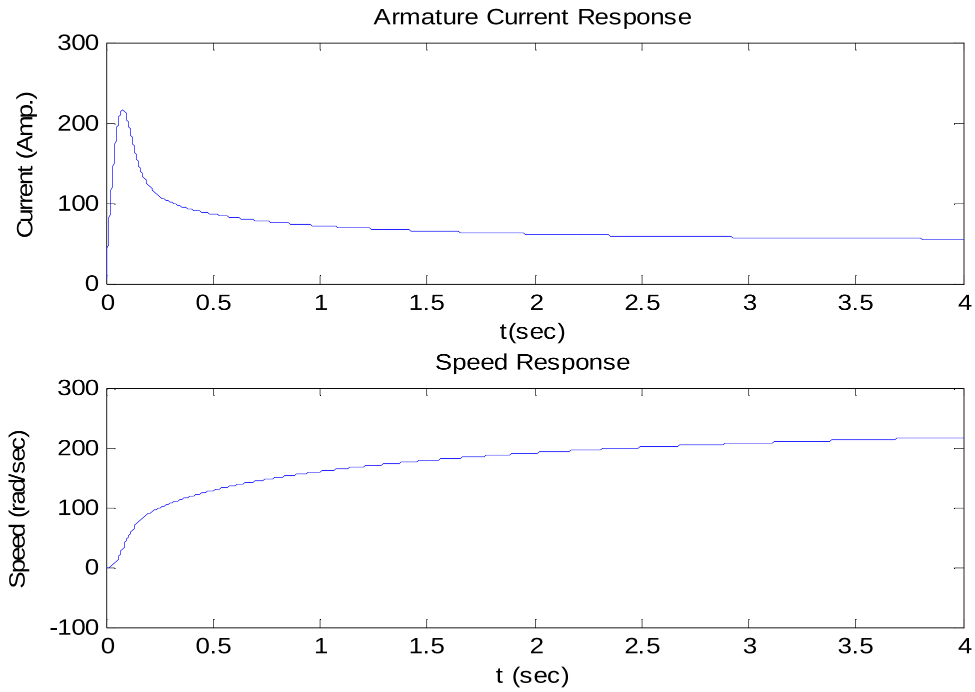

Fig.(3) : Open loop response for the system under load condition (T_L= 0.5 p.u).

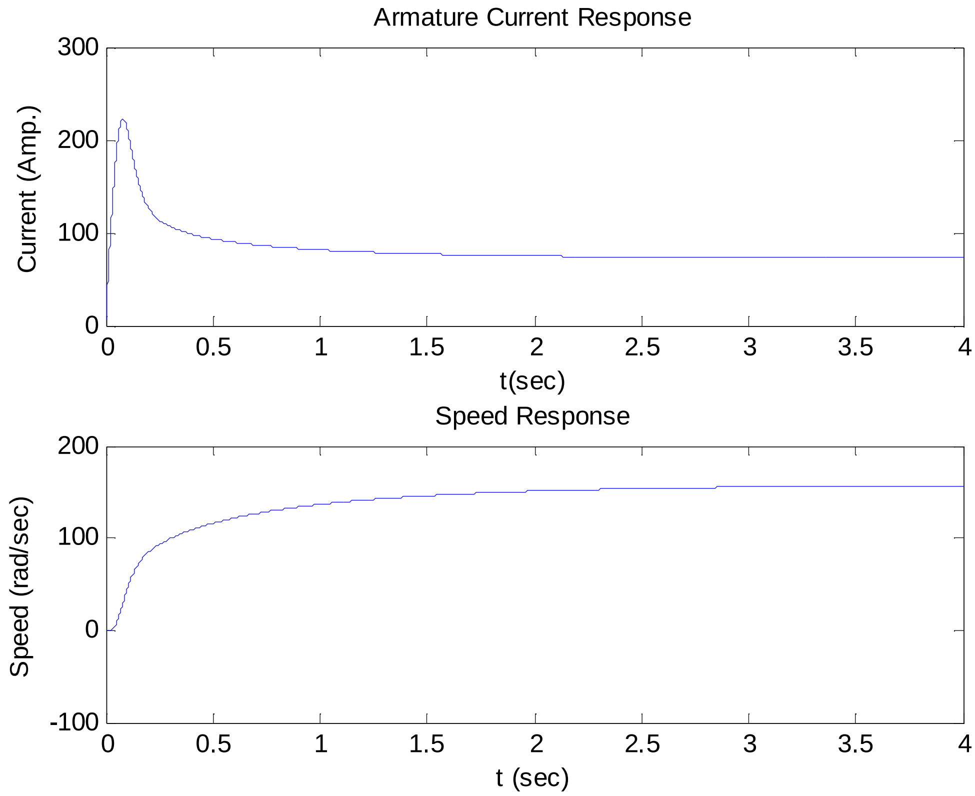

Fig.(4) :Open loop response for the system under load condition (T_L= 1 p.u).

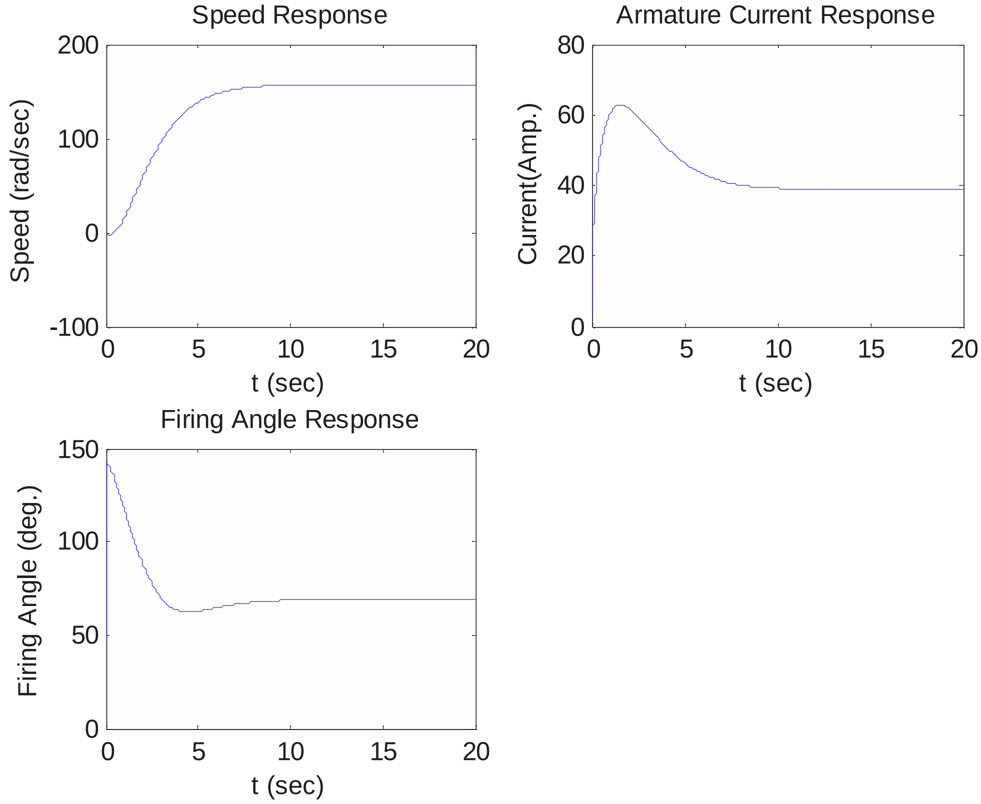

Fig.(5) Closed loop response for the system under load condition (T_L= 0.5 p.u).

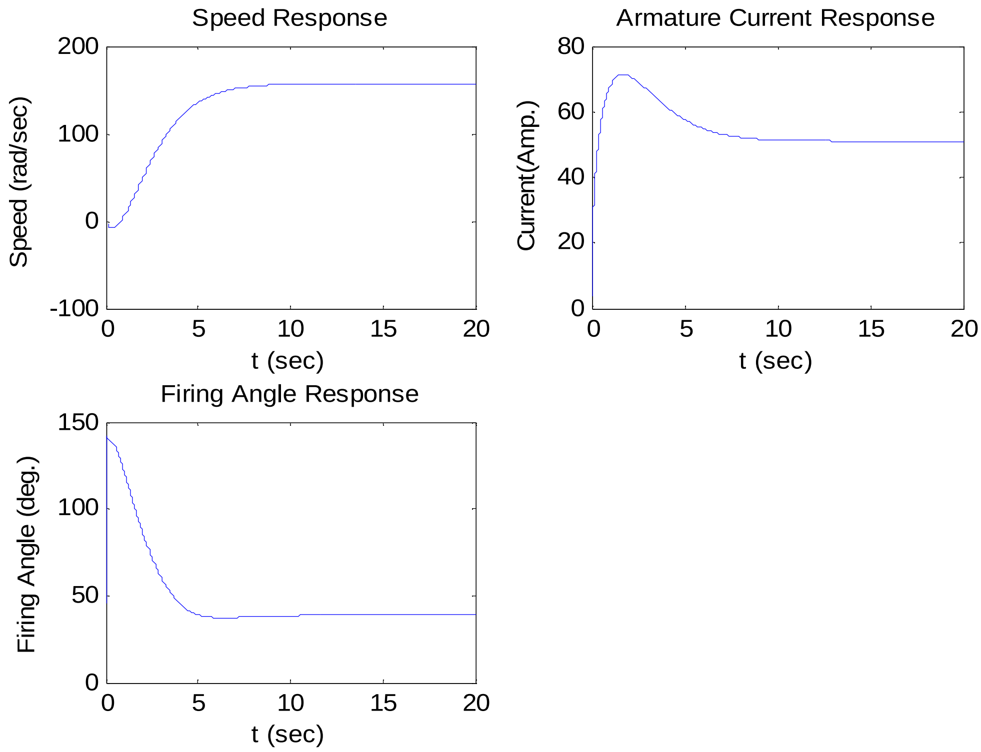

Fig.(6) Closed loop response for the system under load condition (T_L= 1.0 p.u).