State Estimation of Synchronous PM Motor Drive based on Pole Assignment

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Abstract

This paper presents an application of the Kalman filter to permanent magnet synchronous motor drive estimation. Such drive suffers from instability under certain operating conditions when operates open loop without rotor position sensor. Pole placement was implemented to improve the performance and stability of the system. Rotor velocity measurement of the PM drive is used by the Kalman filter to estimate the state of the controlled system. The Kalman filter is simple but requires a good dynamic model to give reliable results. Numerical evaluation showed that the estimated states converge well in time .The performance of estimator and control is robust in terms of unknown initial operating conditions and load variations

الخلاصة

يقدم هذا البحث تطبيقا لمرشح كالمن لتخمين حالات منظومة سوق محرك كهربائي متزامن ذي مغناطيس ثابت، حيث ان تلك المحركات تعاني من عدم استقرارية في ظروف تشغيل معينة ، حينما يشتغل المحرك في دائرة مفتوحة بدون تغذية خلفية من متحسس الجزء الدوار للمحرك. ولقد تمت السيطرة عليه باستخدام طريقة اسناد الاقطاب ، حيث بالامكان الحصول على اداء و استقرارية ذات مواصفات جيدة . يستعمل مرشح كالمن قياس سرعة الدوار للمحرك لتخمين حالات المنظومة المسيطر عليها ان المخمن المستخدم بسيط ولكنه بتطلب نموذجا ديناميكيا جيدا للحصول على نتائج ذات اعتمادية عالية، وبينت النتائج ان الحالات المخمنة تتقارب مع قيمتها الصحيحة في وقت مناسب جدا مما يوفر وقتا لاحتساب قيم المسيطر .ولم يتاثر عمل المخمن و المسيطر بعدم القدرة على معرفة القيم الأبتدائية الحقيقية لمتغيرات حالات منظومة سوق المحرك الكهربائي ولا بتغير الحمل خلال العمل.

List of Symbols

β	Viscous friction, N. m. s
I	electric current, A.
λ_m	Magnet stator flux linkage.
J	rotor inertia, Kg.m^2
L	electric inductance, H.
P	pole pairs.
R	electric resistance, Ohm
T_L	load torque, N.m.
Φ	State transition matrix
Γ	State input matrix
V	voltage, V.
δ	Load angle.
E_0	induced air gap voltage.
ω_s	Supply frequency.
ω_r	Rotor velocity.
A	dynamic matrix
B	input matrix
C	output matrix
D	cross coupling matrix
K	feedback gain matrix
U	input vector.
X	state vector
n	number of states
m	number of inputs
q	Number of outputs

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1. Introduction

Recently, permanent magnet (PM) synchronous machine is proved to be highly efficient machines due to its virtually loss free rotor and high power factor operation. It has a smaller frame size and lower inertia for a given output. [1]. It is widely used in AC motor drives, because of the constant field excitation supplied by permanent magnets. The permanent magnet (PM) synchronous machines also have different design, behavior and performance compared with the conventional wound-field synchronous machines .It is widely recognized that the permanent magnet (PM) synchronous machine can have positive performance advantages over all other machine equivalents [2].

Speed control of electric drives involving synchronous machines including PM is commonly achieved by using the rotor position of the machine as a feedback signal to synchronize the inverter switching with the shaft rotation and to ensure a stable operation. Speed control of PM machine drives without rotor position feedback can also be achieved precisely from an independent oscillator in a similar way to that of an induction motor in spite of the problems associated with instability.

The dominant complex poles of PM machines are very close to the imaginary axis of the complex plane, therefore, the motor losses its synchronism with the supply frequency under some operating conditions [2].

The stability of a buried magnet type of permanent magnet machine, when operated in a synchronous variable speed drive under an open loop control, was considered in [1,2].

A pole assignment method of multi-input was used in [2] to accomplish a control algorithm for a stable operation of the drive system over the entire range of speed independent of changes in operating conditions.

But since only the rotor velocity can be measured practically, the other state variables must be estimated using a proper technique. A so widely applied technique in industry [3, 4 and 5] that performs estimation of the state variables is Kalman filter. It consists of a practical set of procedures that can be used to process numerical data to obtain estimates of parameters and variables whose values are uncertain. It provides an estimate of the system at the current time based on all measurements of the system of the obtained up to and including the present time.

In this paper the Kalman filter is used to calculate the unmeasured changes of load angle and stator currents respectively. This estimation can be used for the performance of control system based on the calculated values of the unmeasured variables in the real system.

In the following sections a discrete time drive system model is described, then the design of pole placement controller is described as well, and the Kalman filter estimator formulation is developed and evaluated numerically. Finally the results obtained are discussed.

2. Drive System Model

A full model of the drive system that combines the major power components, the inverter and PM synchronous motor can be presented in state space model equations: [1, 2]

$$\dot{x} = \mathrm{f}(\mathbf{x}, \mathbf{u}) \tag{1}$$

Where ($\dot{x}$) are derivatives of the state variables corresponding to a state vector ($\mathbf{x}$) which consists of load angle, rotor frequency currents in the direct and quadrate axis, and f($\mathbf{x}$,$\mathbf{u}$) is a non-linear function of the state vector ($\mathbf{x}$) and an input control vector (**u**) . The input vector (**u**) comprises voltage, torque and frequency.

For a small enough perturbation $\Delta\mathbf{x}$ about a nominal state space operating point ($\mathbf{x}_0$), the behavior of drive system roots can be investigated by forming a linearised state equation around the nominal point ($\mathbf{x}_0$). A linearised fourth order model can be obtained by applying the linearization principles to the non-linear model of the drive system as given below:

$$\dot{x} = A\mathbf{x} + B\mathbf{u} \qquad (2)$$

Where

$$\mathbf{x}^T = [\Delta\delta \ \ \Delta\omega_r \ \ \Delta id \ \ \Delta iq],$$

A=

$$\begin{bmatrix} 0 & -1 & 0 \\ 0 & -\beta/J & 3p^2(L_d - L_q)i_q/2J \\ -V\cos(\delta)/L_d & L_q i_q/L_d & -R/L_d \\ -V\sin(\delta)/L_q & -(\lambda_m + L_d i_d)/L_q & -L_d\omega_r/L_q \end{matrix} \ldots\ldots\ldots\ldots$$

$$\ldots\ldots\ldots\ldots \begin{matrix} 0 \\ 3p^2[\lambda_m + (L_d - L_q)i_d]/2J \\ L_q\omega_r/L_d \\ -R/L_q \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -P/J & 0 \\ 0 & 0 & -1/L_d\cos(\delta) \\ 0 & 0 & -1/L_q\sin(\delta) \end{bmatrix}$$

And $\mathbf{U}^T = [\omega_s \quad T_L \quad V]$

3. Problem Statement

A multi-input system described by the state space equation is considered:

$$\dot{x}(t) = A\,\mathbf{x}(t) + B\,\mathbf{u}(t) \qquad (3)$$

$$y = C\mathbf{x}(t) + D\mathbf{u}(t) \qquad (4)$$

Where; **x** (t) is an nx1 state vector, **u** (t) is an mx1 input vector, A is an nxn system matrix, B is an nxm input matrix is a q*n output matrix and D is q*m cross coupling matrix.

It is assumed that the pair (A, B) is controllable. The discrete representation of eqs. (3) and (4) is given as:

$$\mathbf{x}(k+1) = \Phi\mathbf{x}(k) + \Gamma\mathbf{u}(k) \qquad (5)$$

$$\mathbf{y}(k) = H\mathbf{x}(k) + J\mathbf{u}(k) \qquad (6)$$

Where

$$\Phi = e^{AT} \qquad (7)$$

$$\Gamma = \int_0^T e^{At}\,dtB \qquad (8)$$

As for the continuous case, the control law is simply the feedback of a linear combination of all the state elements, that is

$$\mathbf{u} = -K\mathbf{x} = -[K_1 \ \ K_2 \ldots] \begin{bmatrix} x_1 \\ x_2 \\ . \\ . \\ \end{bmatrix} \qquad (9)$$

Substituting Eq. (9) in Eq. (5) results in:

$$\mathbf{x}(k+1) = \Phi\mathbf{x}(k) - \Gamma K\mathbf{x}(k) \qquad (10)$$

Therefore the z-transform of Eq.(10) is

$$(zI - \Phi + \Gamma K)\mathbf{x}(z) = 0$$

And the characteristic equation of the system with the hypothetical control law is

$$|zI-\Phi+\Gamma K|=0 \qquad (11)$$

4.Pole Placement

The approach which is followed here is pole placement [3] , that is taking a control law with enough parameters to influence all the closed-loop poles.

The control-law design perform the task of finding the elements of K so that the roots of eq.(11), that is , the poles of the closed-loop system, are in the desired locations.

Given desired pole locations,

$$z_i = p_1, p_2, \ldots, p_n$$

the desired control-characteristic equation is

$$\alpha_c(z)=(z-p_1)(z-p_2)\ldots(z-p_n)=0 \qquad (12)$$

Equations (11) and (12) are both the characteristic equation of the controlled system; therefore they must be identical, term by term. The calculation of the gains becomes rather tedious when the order of the system is greater than 2. Therefore it could be performed using a MATLAB function named (place.m), which is best for high order systems and can handle MIMO systems.

The specific difficulty that might be encountered in using the pole placement approach is brought about the necessity to pick n desired pole locations. It is helpful to move poles as little as possible in order to minimize the required amount of control effort and to avoid exciting the system any more than necessary [3].

5.Kalman Filter

The Kalman filter is the best linear estimator, which can produce an optimal estimate of the states of a linear dynamic system, subjected to the disturbances having Gaussian distribution. The optimality is in minimum estimation-error-variance sense. The function of a filter is to separate the signal from the noise corrupted data. The separation is possible only if the signal and the noise lie in different frequency ranges. When the signal and the noise occupy overlapping frequency bands, the kalman filter can be used to compute the optimal estimation of the signal [4].

In the Kalman formulation we desire the best linear estimate of the state vector $\mathbf{x}(k)$ as a function of the observation vectors $y(1),y(2),\ldots,y(k)$, where the best is assumed to be in the minimum mean squared error sense.[5]

Let $\hat{x}(k|k)$ represent the estimate of the state at time index k given observations $y(1),y(2),\ldots,y(k)$. the Kalman filter thus gives us a running estimate of the state at time index k as a function of the observations.

The system is precisely represented by a linear model exited by a white noise process $\mathbf{w}(k)$, which is given by:

$$\mathbf{x}(k+1)=\Phi(k+1,k)\mathbf{x}(k)+\Gamma(k+1,k)\mathbf{u}(k)+C(k+1,k)\mathbf{w}(k) \qquad (13)$$

The vector observation (measurements) is given by:

$$y(k+1)=H(k+1)\mathbf{x}(k+1)+\mathbf{v}(k+1) \qquad (14)$$

Where the process noise $\mathbf{w}(k)$ and measured noise $\mathbf{v}(k)$ are independent zero mean , white noise sequences with known statistics. The parameters of Φ, Γ and H are assumed to be known .

The optimal linear filtered estimate $\hat{x}(k+1|k+1)$ to be governed by the recursive matrix formula for

$$\hat{x}(k+1|k+1)=\Phi(k+1,k)\hat{x}(k|k)+K_m(k+1)[y(k+1)-H(k+1)*\Phi(k+1,k)\hat{x}(k|k)] \quad (15)$$

with initial conditions $\hat{x}(0|0)=0,\ k\geq 0$

where $K_m(k+1)$ is the Kalman gain which is given by :

$$K_m(k+1)=P(k+1|k)H^T(k+1)*[H(k+1)P(k+1|,k)H^T(k+1)+R(k+1)]^{-1} \quad k=0,1,\ldots \quad (16)$$

The single-step prediction error covariance matrix P(k+1|k) is given by

$$P(k+1|k)=\Phi(k+1,k)P(k|k)\Phi^T(k+1,k)+\Gamma(k+1,k)Q(k)\Gamma^T(k+1,k) \quad (17)$$

With initial known condition $P(0|0)=P(0),\quad k=0,1,\ldots$

And the error covariance matrix for filtered error is:

$$P(k+1|k+1)=[I-K_m(k+1)H(k+1)P(k+1|k), \quad k=0,1,\ldots. \quad (18)$$

To have optimal estimator knowledge of the initial estimation error (noise) covariance P(0), an initial estimate $\hat{x}(0|0)$ and a model are required. Use is made of the PM drive system model to generate such data (reference data).

6. PM Performance with Regular Kalman Filter

A regular Kalman filter scheme to estimate P.M. states is shown in fig. (1). the machine data for an open circuit at voltage 37.5v and 50 Hz is given in Table 1. The state space model and state output model are calculated by equations (13) and (14), respectively.

The choice of the covariance matrices for system disturbances and measurement noise is selected arbitrarily. This is appropriate since the model is simplified, the errors do not satisfy the statistical assumptions of normality and zero mean; it is hard to accurately quantify the covariance matrices [6]. In this work the covariance matrices needed are assumed to be diagonal.

An example is taken to demonstrate the proposed algorithm that represents sever case of instability of the drive system. The case of constant operating frequency at 50 Hz is taken as shown in fig. (2), the torque of 3N.m gives a positive real part of the closed loop poles. A leading power factor of 0.9 at 50Hz and 3N.m operation are taken. The calculated matrices of the system at this operation condition are:

$$A=10^4*\begin{bmatrix} 0 & -0.0001 & 0 & 0 \\ 0 & 0 & -0.0006 & 0.0055 \\ -1.8092 & 0.0014 & -0.028 & 0.0554 \\ -0.2347 & -0.0029 & -0.0178 & -0.016 \end{bmatrix}$$

$$B=\begin{bmatrix} 1 & 0 & 0 \\ 0 & -80 & 0 \\ 0 & 0 & 218.2821 \\ 0 & 0 & 28.3201 \end{bmatrix}$$

It is required to place the desired poles for the discrete system model at:

[0.134, 0.356, 0.28, 0.439].

The step responses for the selected poles of the controller are shown in fig. (3). the position of the desired locations of the poles depends upon the amplitude of the required response.
The simulated example shown in fig. (4) demonstrate the filter's capability in estimating the PM motor states to be used by the pole placement controller. The only information required for Kalman filter is the input variables (V, T_L, ω_s) and output variable ω_r. Gaussian noise is added to the rotor velocity to simulate measurement noise. It is clear that the estimated states follow that of the simulated reference system in less than 0.3 seconds. Better response can be obtained by tuning the parameters (covariance, measurement noise). The position of the selected desired poles is also very effective in smoothing the oscillation of the response.
The Kalman filter gains are dynamic and fast; they reach a steady state value in less than 0.3 seconds as shown in fig.(5). They are also very sensitive to any variations of load during simulation. The control signals that are generated using the estimated states are shown in fig. (6).Figure (7) shows the error percentage between the PM system states for comparison. This demonstrates the capability of the Kalman filter in estimating the PM drive system motor.
Figure (8) shows the robustness of the estimator using Kalman filter where the initial conditions used are further distant from the true states. The accuracy and speed of the estimator depends on the accuracy and value of the initial conditions in comparison with the actual motor conditions. The effect of changing the initial conditions is illustrated in Table 2, where the performance of the estimator has been evaluated by the performance index r defined by [6]:

$$R=\frac{1}{ni}\sum_{i=1}^{ni}| xest(i)-x(i)| \qquad (19)$$

Where x_{est}(i) is the estimated state value at i'th time point. x(i) is the reference value of the state and ni is the number of time points. The performance of the estimator is the average absolute deviation of the estimated states from the actual states. The estimator performance at the beginning of the simulation deviates substantially from the actual if initial condition of the estimator is different from real one but the quality of the estimator increased as the estimation proceeds, as shown in fig (8). Many numerical test runs were made and the value of the performance index r was computed to demonstrate the robustness of the estimator. In the test of table (2) four performance indexes are evaluated for each run. The table clearly shows the effect of changing the initial condition and in the same time the effectiveness of the kalman estimator.
Similarly test runs were made to study the effect of sudden changes (e.g. variation in load torque) during simulation run, the Kalman filter overcome this situation effectively as shown in fig.(9).

7. Conclusions

Based on the results obtained, a reliable estimation can be generated using Kalman filter. The implementation of advanced control techniques as the pole Assignment can be applied easily to improve the performance of the system.
It is also shown that the estimator part of the system gives robust estimates of the states (load angle and stator currents). Thus, the estimator seems to be general and powerful enough to handle all the situations in this application.
The estimator allows one to take into

account the changes occurring during the simulation run of the PM motor drive system like sudden load torque variations and violations of initial conditions.

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Table 1 machine data for an open circuit voltage of 37.5v at 50 Hz.

Rated torque N.m	λ_m mV.s/rad	L_q mH	L_d mH	R Ω	J N.mm.s²/rad	β N.mm.s/rad
3.0	119.36	9.23	3.43	1.28	0.025	0.0025

Table 2 Results of the evaluation of the Kalman estimator for different estimator initial conditions

Initial condition of kalman estimator	Performance index R_δ	Performance index $R_{\omega r}$	Performance index Rid	Performance index Riq
Xin=0.0	0.0157	0.514	0.6155	0.3164
Xin=5.0	0.0441	0.7579	1.3399	0.7261
Xin=10.0	0.0817	1.1318	2.5073	1.3146

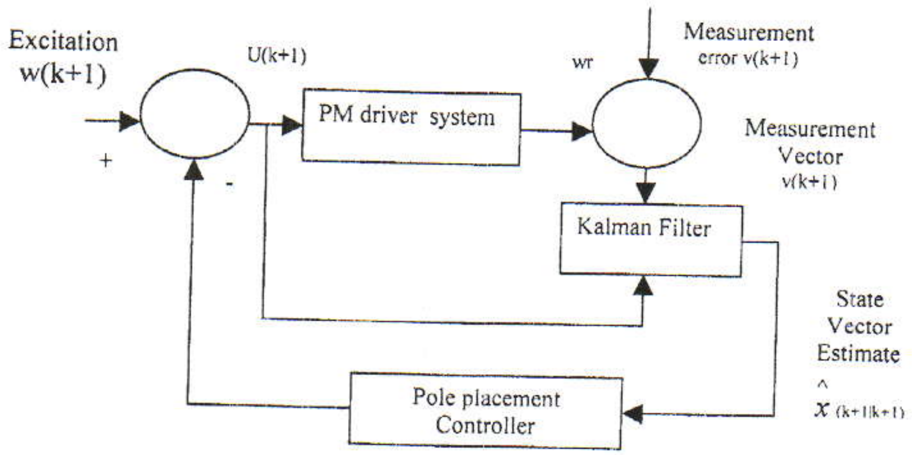

Fig. (1) Functional block diagram of the Estimator- controller algorithm.

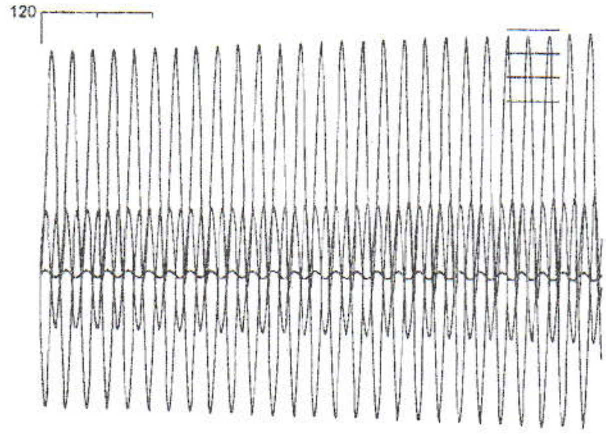

Fig. (2) The step response of the original PM drive system model.

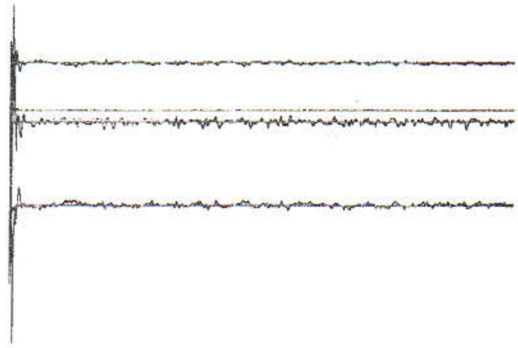

Fig. (4) The Step Response of the PM Drive System under Pole- Assignment Control using Kalman Filter.

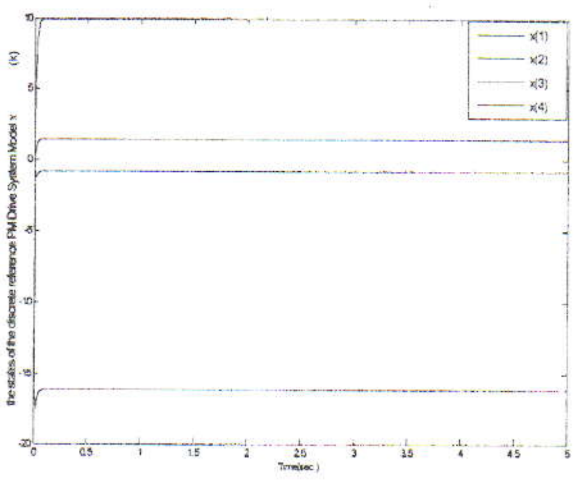

Fig. (3) The step response of the discrete system under pole placement control.

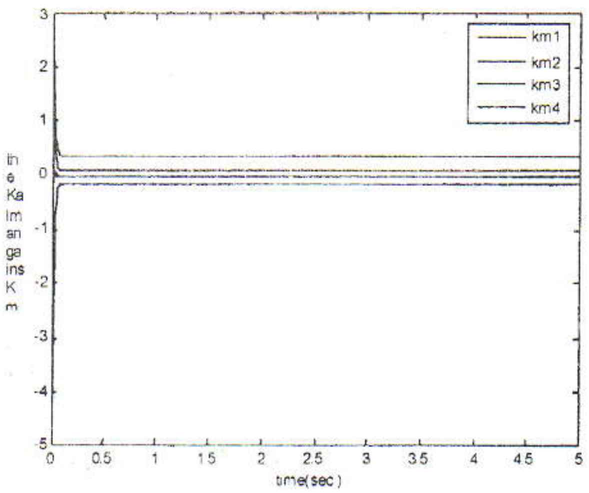

Fig. (5) The gains of the Kalman Filter.

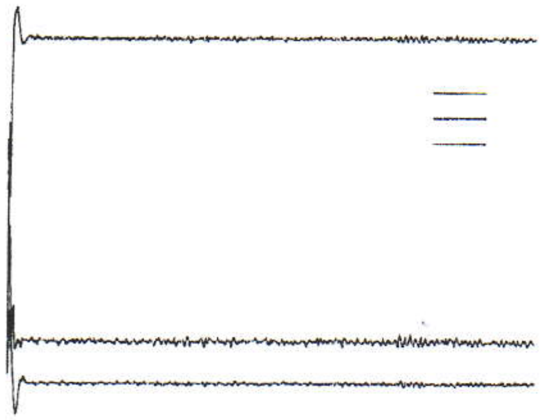

Fig. (6) The control signals of the controlled system.

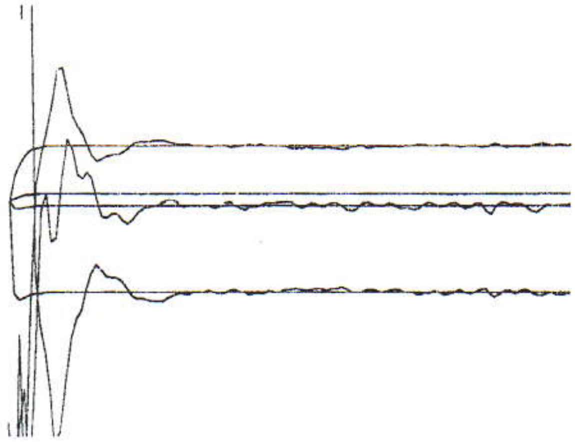

Fig. (8) PM drive system state estimation using Kalman Filter with initial conditions for estimator differs from the actual one.

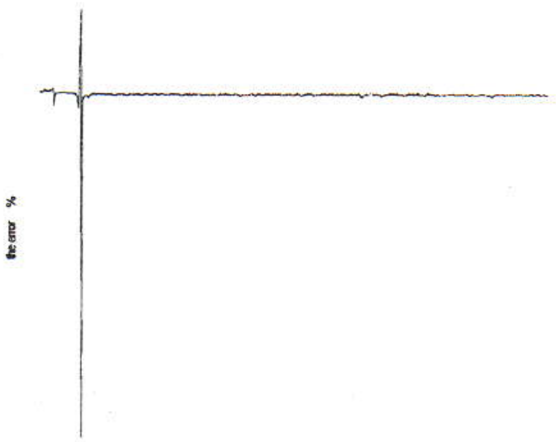

Fig. (7) Error Percentage between the reference and estimated rotor velocity for PM Drive System using Kalman Filter.

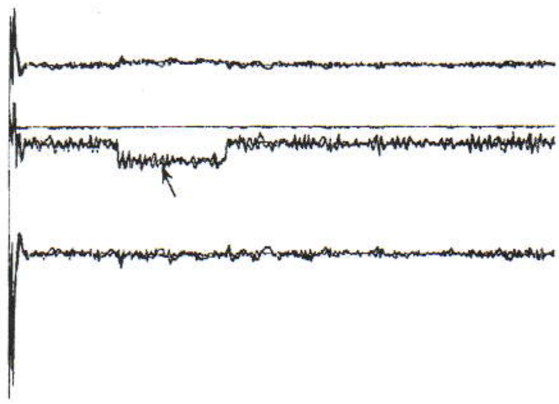

Fig. (9) PM Drive System state estimation using Kalman Filter with sudden changes in load torque.