

Density of nuclear-level parameter effect on the fission cross section that is induced using Gilbert–Cameron model

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ABSTRACT

Fission cross-sections exhibit a high degree of sensitivity to saddle point deformations and level density at equilibrium. In this research, optical and statistical models were used to examine the neutron-induced fission cross-section calculations for some americium isotopes ^{240,241,243,244}. Am (n, f) facing obstacles were effectively double humped considering the different level density parameters (LDPs) (a) of the Gilbert–Cameron Model (GCM). The EMPIRE-3.2 modular system computer code was used for this purpose, with energies ranging from 1 keV to 100 MeV for the incident neutron. Our calculations were compared with the available experimental data adopted from the EXFOR database. The LDP considered using the Arthur formula was the best parameter used in the GCM, and it showed a reasonable agreement with experimental results. The calculated cross-section of GCM was also compared with the results of different level density models.

Introduction

Measurements of fission cross-sections of heavy nuclei induced by particles within a broad energy range have long been a topic of interest [1]. Such data are necessary for applications, such as energy generation, accelerator-driven transmutation of nuclear waste, fundamental physics, etc[2].

One of the primary issues in the study of fission cross-sections is the statistical investigation of heavy actinide nuclei and nuclear level density (NLD) in the energy continuum[3]. NLDs are also required in the nuclear reaction statistical model in cases where discrete-level data are assumed to be lacking. [3]. Various phenomenological [4-9] and microscopic [10-15] approaches have been used to investigate NLDs, in which the shell effect, pairing correlation, and collective phenomena were incorporated in the microscopic versions applied to the generalized superfluid model developed during the preview decades [16]. All analytical phenomenological formulas derived from the basic relation of the Fermi gas model (FGM) [17], in which collective effects are excluded, mainly depend on a parameter called level density parameter (LDP; a).

This parameter plays an important role in the computations of fission cross-sections as an input parameter [18].

The present work mainly aimed to study the effect of LDP for the Gilbert–Cameron Model (CGM) on induced neutron reaction cross-section for some americium nuclei and face the problem of selecting the best value of LDP at equilibrium and saddle point deformations. Our calculations were compared with experimental data to adopt theoretical calculations using different NLD models. The calculation was conducted using the latest version of EMPIRE code.

Theory

The FGM basic expression, which mainly depends on the LDP (a), is the starting point of all phenomenological level density models [9].

The physical assumption of the FGM is that excited levels of the nucleus are equally spaced and constructed. According to this model, the density of intrinsic levels can be written in terms of excitation energy, spin, and parity as follows [19]:

$$\rho(E_x, J, \pi) = \rho(E_x) \rho(J, \pi) \quad (1)$$

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where the energy dependence term reads:

$$\rho(E_x) = \frac{\exp S}{\sqrt{Det}} \quad (2)$$

and

$$\rho(J, \pi) = \frac{1}{2\sqrt{8\pi\sigma^3}} \exp \left[-\frac{\left(J + \frac{1}{2}\right)^2}{2\sigma^2} \right] \quad (3)$$

is the spin and parity dependence term; σ^2 is the spin cutoff parameter, and S is the entropy.

These parameters and (Det) can be given by the following relations:

$$E_x = aT^2; S = 2aT; \sigma^2 = \mathfrak{I} T \quad Det = 144 \frac{a^3 T^5}{\pi} \quad (4)$$

where a denotes the LDP, $\mathfrak{I}$ is the moment of inertia, and T is the nuclear temperature.

Consider a coupled pair nuclei with a spin of zero. Pair separation is necessary for individual nucleon excitation. Thus, the excitation energy should be replaced by the effective excitation energy (U):

$$U = E_x - \Delta \quad (5)$$

where Δ is the pairing energy given by the following:

$$\Delta = n \frac{12}{\sqrt{A}} \quad (6)$$

with $n=0$ for even-even nuclei, $n=1$ for odd-A nuclei, and $n=2$ for odd-odd nuclei.

The total Fermi level density of randomly coupled of total angular momentum over all spin and parities [3]:

$$\rho_f^{tot}(E) = \frac{1}{\sqrt{2\pi\sigma}} \frac{\sqrt{\pi}}{12} \quad (7)$$

The energy dependence of the LDP is implied by the correlation between the a -parameter values calculated from the neutron resonance spacing and shell correction and by the disappearance of shell effects with the increase in excitation energy. Ignatyuk proposed the standard shape for this type of dependence [4]:

$$a(E_x) = \tilde{a} \left[1 + f(U) \frac{\delta W}{U} \right] \quad (8)$$

where $\tilde{a}$ is the asymptotic value of the a -parameter, δW is the shell correction, and

$$f(U) = 1 - \exp(-\gamma U) \quad (9)$$

γ is the damping parameter for shell effects.

Gilbert and Cameron (Gilbert & Cameron, 1965) proposed the total level density by combining the level density of a low-energy region with that of the high-energy region predicted above by GFM (Eq. 8) (Gilbert & Cameron, 1965):

$$\rho^{GC}(E_x) = \begin{cases} \rho^{CT}(E_x) & E_x \leq U_x \\ \rho^{FG}(E_x) & E_x > U_x \end{cases} \quad (10)$$

Thus, the model is called the Gilbert–Cameron Composite Model.

For the low-energy region, in which the temperature T was assumed to be constant, the level density ρ^{CT} is related to the number of the cumulative levels computed using the following:

$$\rho^{CT}(E) = \frac{dN(E)}{dE} = \frac{1}{T} \exp \left[(E - E_0)/T \right] \quad (11)$$

where E_0 and T are variables that can be freely adjusted through comparison with the experimental data.

In CGM, the energy constant a -parameter used in (Eq. 8) can be given by three systematic relations as follows:

Ignatyuk et al. (Ignatyuk, Smirenkin, & Tishin, 1975). $\tilde{a} = 0.154A + 6.3 \times 10^{-5}A^2$ and $\gamma = -0.054$ (12)

Arthur et al. (Young et al., 1989). $\tilde{a} = 0.1375A - 8.36 \times 10^{-5}A^2$ and $\gamma = -0.054$ (13)

Iljinov et al. (Iljinov et al., 1992). $\tilde{a} = 0.114A + 9.80 \times 10^{-2}A^{2/3}$ and $\gamma = -0.051$ (14)

The spin cut-off factor $\sigma(E_x)$ is calculated using the following equation:

$$\sigma^2(E_x) = 0.146A^{2/3} \sqrt{aU} \quad (15)$$

The collective effects is explicitly disregarded in the GCM. These effects are considered in $\tilde{a}$ when attempting to fit neutron resonance spacing. GGM suggests the nuclear-induced processes with compound nucleus excited up to 20 MeV. The Gilbert–Cameron (GC) level densities may be

the most accurate at excitation energies up to the neutron binding energy and slightly above.

Results and Discussion

Fission barriers and NLDs are important ingredients of nuclear reaction that can affect fission cross-section predictions. In this work, we focus on the NLDs calculated using different models to perform fission cross-section calculations for some americium isotopes.

Fission cross-sections were determined theoretically through additional penetrations through fission barriers and fission transition coefficient:

$$T(E, J, \pi) = \frac{T_A T_B}{T_A + T_B} \quad (16)$$

Given the large number of barriers, the so-called transition states have been introduced to each barrier top, which can be understood from physical predictions. Four models included in EMPIRE 3.2 code have been used for calculation of densities for these transition states, namely, phenomenological GCM, generalized superfluid model (GSM), enhanced GSM, and the microscopic Hartree–Fock–Bogoliubov Model.

The compared cross section results calculated using these models with the best systematic LDP using with the GCM is shown in the figures in the next pages of this study. In The fission cross-section can be calculated using the energy dependence LDP (a). Three systematic LDPs were determined for the GCM in our calculation for fission cross-section [19, 21, 22]. These calculations are in Figs. 1 (a, b, c, and d) for the americium isotopes under study along with experimental data available in the EXFOR database [23].

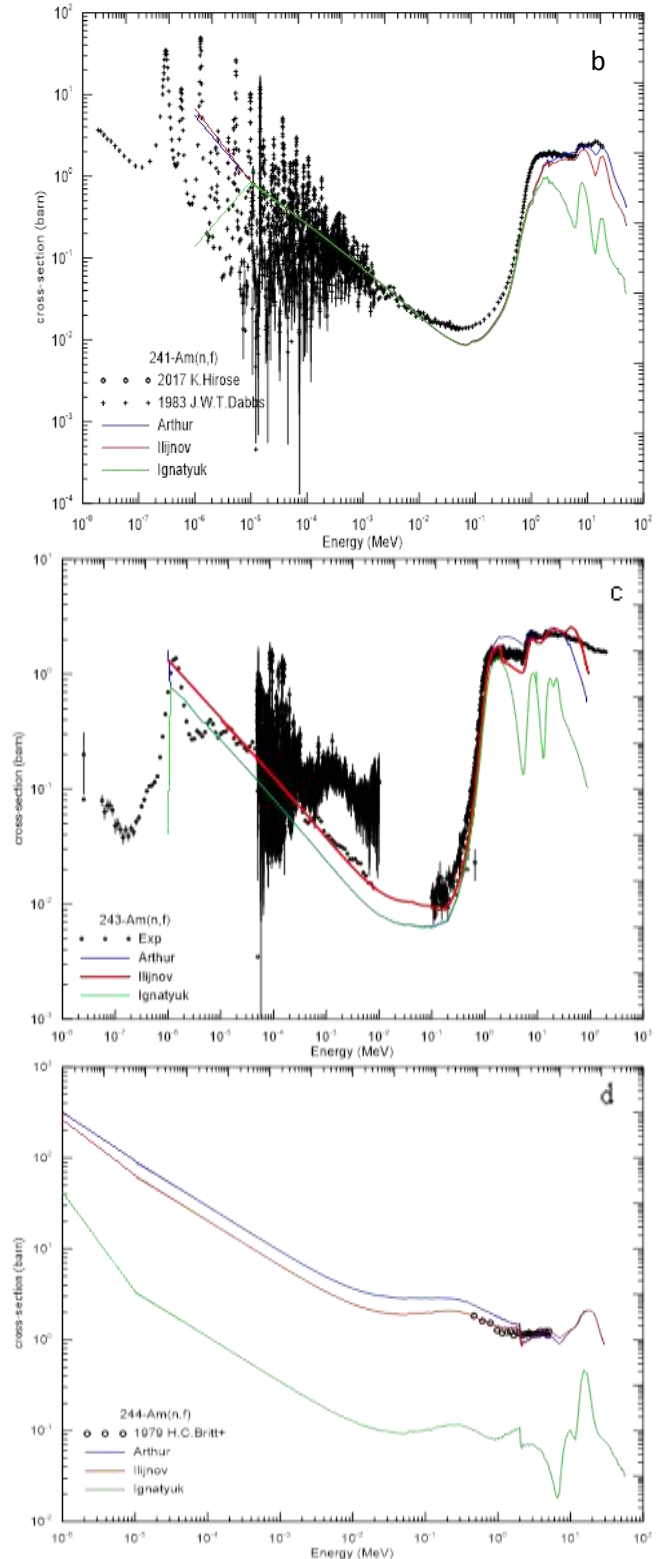
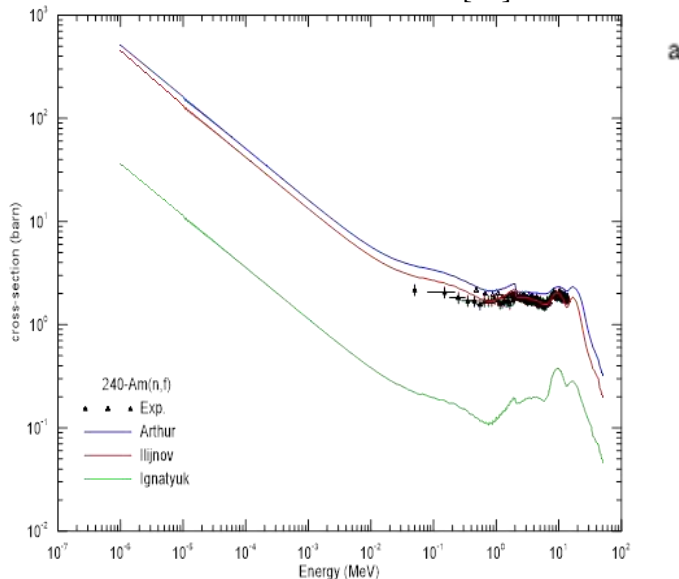


Fig. 1: Fission cross section for americium isotopes using different GCM LDP compadata [24]. (a) for ^{240}Am , (b) for ^{241}Am , (c) for ^{243}Am , and (d) for ^{244}Am .

At very low-energy region (up to 0.01 MeV) in which the cross section have high values and decreases as the incident neutron energy increase. Thus, we can say that the transition states on these two peaks (referred to as states in class II and III) are responsible for the resonance structure at the low-energy region. The results obtained for odd-A americium isotopes are in agreement with previous dataset without being to reproduce the resonance structure given that in our calculation, the transition states of class II/III are excluded (full damping for II/III states).

In the energy region above (0.01 MeV), the cross section increased with the increase in the incident energy for neutron due to the increased transition state number that belong to vibrational and rotational bands, especially for $^{241,243}\text{Am}$ isotopes.

Fig. 1 shows that the LDP obtained using Iljinov systematic are in a good agreement with experimental data for all isotopes under study. The other two systematic (Arthur and Ignatyuk) have values of the cross section underestimates and overestimation of experimental data.

The cross sections obtained with the GCM estimated with the best LDP systematic. The Iljinov expression is shown in Fig. 2, along with fission cross-sections calculated using the GS, EGS, and HFB level density models. The whole calculations were also compared WITH available experimental data for the four americium isotopes.

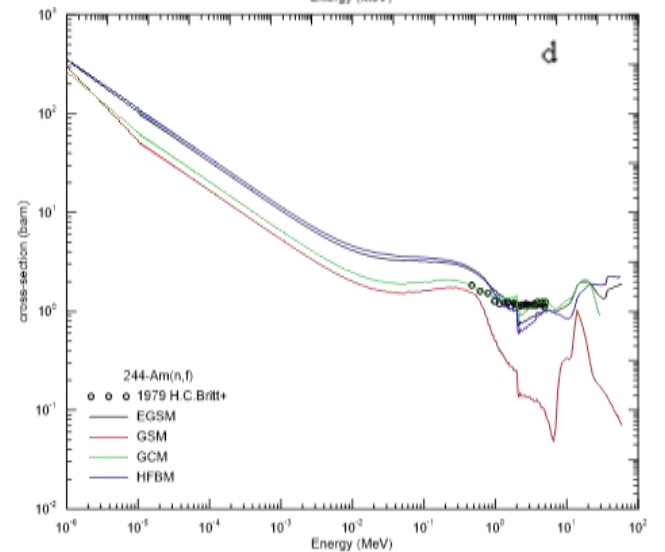
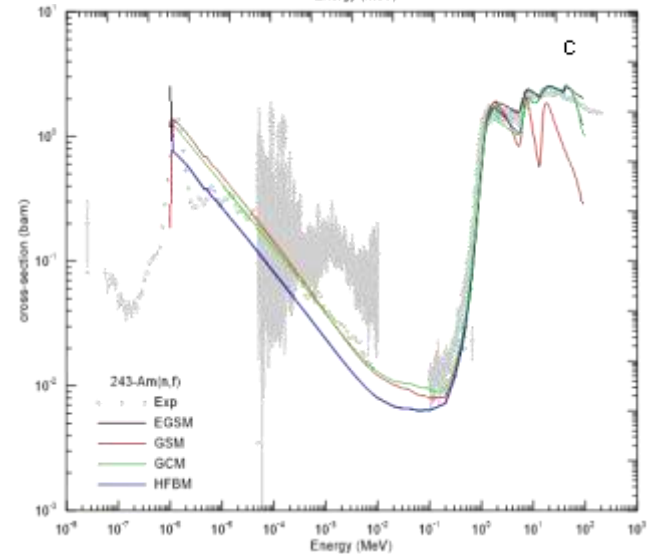
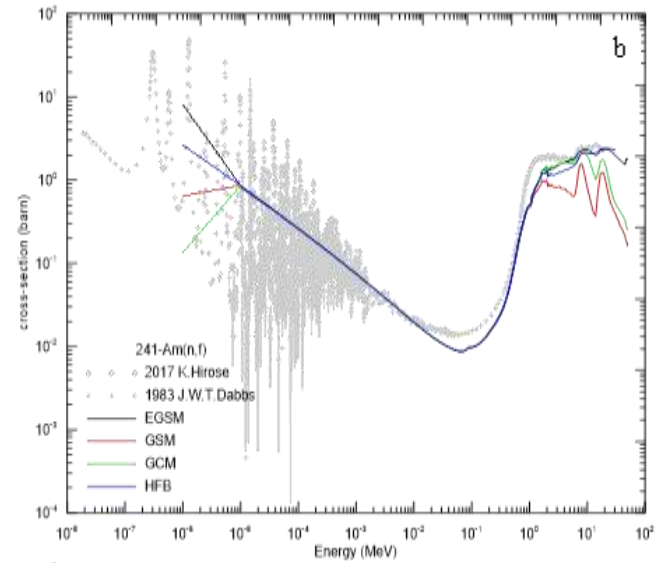
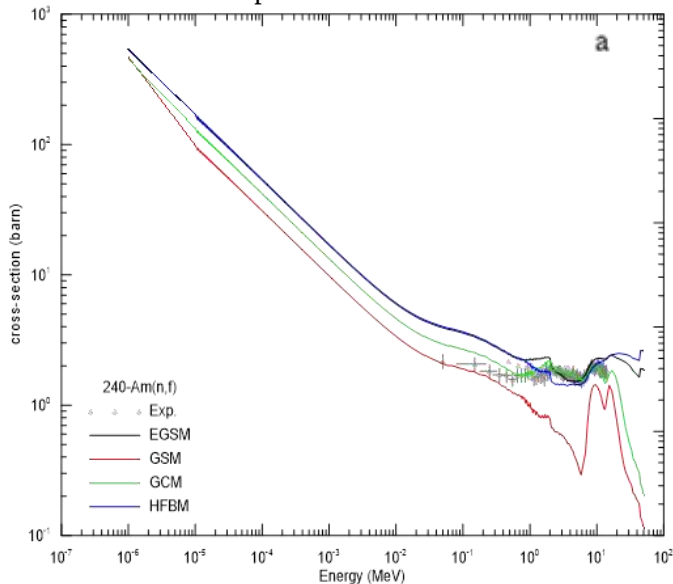


Fig. 2: Cross-section of neutron-induced fission, as measured using various NLD models (a) for ^{240}Am , (b) for ^{241}Am , (c) for ^{243}Am , and (d) for ^{244}Am .

This comparison indicates that the fission cross-section obtained using the GCM gives a good agreement and reproduce experimental result well than the other models included in Fig.2.

Conclusions

This research segment on induced neutron reactions for several isotopes of ammonium examined the effect of LDP and LDM on fission cross-section calculations. Compared with experimental data, the LDP chaise provided a satisfactory nuclear-level description for the calculation of fission cross sections. The transition states for the II/III (second and third grade) region must be incorporated in calculations to improve the reproducibility of experimental data across all incident energy regions, given the importance of these states.

Improving predictions of fission cross-sections requires further refinement of all nuclear properties pertinent to fission reaction models, such as the fission barrier shape and the description of deformed optical model potential.

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تأثير معلمة كثافة المستويات النووية على المقطع العرضي للأنشطار المستحث باستخدام نموذج جيلبرت كامبيرون

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الخلاصة:

تظهر المقاطع العرضية الانشطارية درجة عالية من الحساسية لكل من تشوهات نقطة السرج وكثافة المستوى عند التوازن. خلال هذا البحث، تم استخدام النموذج البصري والأحصائي، لدراسة حسابات المقاطع العرضية للأنشطار النيوتروني على بعض نظائر الأمريسيوم ($Am^{240,241,243,244}$) التي تواجه عوائق ذات حدة مزدوجة بشكل فعال مع الأخذ في الاعتبار معلمات مستوى الكثافة المختلفة (a) لنموذج جيلبرت كامبيرون (GCM) تم استخدام كود حاسوب النظام المعياري EMPIRE-3.2 لهذا الغرض بطاقة تتراوح من 1 كيلو إلكترون فولت إلى 100 ميكا إلكترون فولت. تمت مقارنة حساباتنا مع البيانات التجريبية المتاحة المعتمدة من قاعدة بيانات EXFOR. وجد أن معامل كثافة المستوى الذي تم اعتباره Arthur formula هو أفضل معامل مستخدم مع نموذج GC ويعطي توافقاً معقولا مع النتائج التجريبية. تمت أيضا مقارنة المقطع العرضي المحسوب لنموذج GC بنتائج نماذج كثافة المستويات المختلفة.

الكلمات المفتاحية: معلمة كثافة المستوى، نموذج جيلبرت كامبيرون الأمبير، المقاطع العرضية.