Reduced Order Strapdown Terrestrial INS Algorithm

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Received on: 10 / 1 / 2005

Accepted on: 28 / 12 / 2005

Abstract

Strapdown system algorithms are the mathematical definition of processes which convert the measured outputs of Inertial Navigation System (INS) sensors that are fixed to a vehicle body axis into quantities which can be used to control the vehicle.

In this work, a reduced and fast terrestrial strapdown INS algorithm was developed and implemented for three-degree of freedom (3DOF). The evaluation of the algorithm is based on the accuracy of the proposed algorithm with real data.

الخلاصة

خوارزميات نظامِ Strapdown هي التعريفَ الرياضيَ للعملياتِ التي تحوّلُ النواتجَ المقاسة لنظامِ ملاحةِ من نوع (INS) للمتحسّسات التي تُثبّتُ على محور جسمِ العربةِ إلى الكمياتِ التي يمكن أن تُستَعملَ للسَيْطَرَة على العربةِ.

في هذا العملِ، خفّضَ درجة خوارزميةَ strapdown INS من النوع الأرضي، وطُوّرَ وطُبّقَ لدرجةِ ثلاثة مِنْ الحريةِ (3DoF). يستند تقويمَ الخوارزميةِ إلى دقةِ الخوارزميةِ المُقتَرَحةِ بالبياناتِ الحقيقيةِ.

Keywords Vehicular navigation, inertial navigation, terrestrial algorithm.

1. Introduction

An Inertial Navigation System (INS) is a self-contained positioning and attitude device that continuously measures three orthogonal linear accelerations and three angular rates. By measuring vehicle acceleration and angular velocity in an inertial frame of reference, integrating it with respect to time and transforming it to the navigational frame, velocity, attitude and position components can be obtained [1]. Sensors used to implement such a system are accelerometers for the measurement of a vehicle's linear acceleration (specific force) and gyroscopes for monitoring vehicle rotation (angular velocity) with respect to an inertial frame. Since specific force measurements contain the effect of the Earth's gravity field, a gravity model is used to extract vehicle acceleration from the measurements. Because they employ three translational (accelerometers) and Three rotational (gyroscopes) sensors, Inertial Measuring Units (IMU) can be used as positioning and attitude monitoring devices [2].

In three-degree of freedom strapdown systems, two accelerometers are rigidly attached to the vehicle and aligned with the body frame X_b and Z_b axes (see Figure 1). Single gyro is also rigidly mounted on the vehicle.

The main task of this work is to develop the proposed algorithm described in [3] to minimize the Time and the number of equations required to solve the navigation problem.

2. Terrestrial Strapdown System Dynamic Equation

The differential equation of the relative quaternion between body coordinate and geographic coordinate is given by [4, 5]:

$$\dot{u} = \tfrac{1}{2}\Omega_{ib}^{b} \cdot u - \tfrac{1}{2}\Omega_{in}^{b} \cdot u \qquad \dots (1)$$

Where, the angular velocity skew-symmetric matrix W_{in}^{b} and W_{ib}^{b} are given by [3]:

$$\Omega_{in}^{b} = \begin{bmatrix} 0 & -w_D & w_E & w_N \\ w_D & 0 & -w_N & w_E \\ -w_E & w_N & 0 & w_D \\ -w_N & -w_E & -w_D & 0 \end{bmatrix} \qquad \dots (2)$$

$$\Omega_{ib}^{b} = \begin{bmatrix} 0 & w_Y & -w_P & w_R \\ -w_Y & 0 & w_R & w_P \\ w_P & -w_R & 0 & w_Y \\ -w_R & -w_P & -w_y & 0 \end{bmatrix} \qquad \dots (3)$$

and

$$\begin{bmatrix} w_N \\ w_E \\ w_D \end{bmatrix} = \begin{bmatrix} \left(|w_{ie}| + \dot{l}\right)\cos L \\ -\dot{L} \\ -\left(|w_{ie}| + \dot{l}\right)\sin L \end{bmatrix} \qquad \dots (4)$$

where

u: quaternion parameters

w_{ie}: Earth angular velocity (7.2921×10^{-5} rad / sec).

$[L,\ l,\ h]$: are geodetic positions (latitude, longitude, and height)

w_R, w_P, w_Y are the body angular velocities in the body coordinate (roll, pitch, and yaw), respectively.

w_N, w_E, w_D: navigation angular velocities in the navigation coordinate (North, East, and Down), respectively.

Body fixed coordinate to navigation coordinate (C_b^{n}) can be described in terms of the quaternion parameters [5]

$$C_b^n = \begin{bmatrix} u_0^2+u_1^2-u_2^2-u_3^2 & 2(u_1u_2-u_0u_3) & 2(u_1u_3+u_0u_2) \\ 2(u_0u_3+u_1u_2) & u_0^2-u_1^2+u_2^2-u_3^2 & 2(u_2u_3-u_0u_1) \\ 2(u_1u_3-u_0u_2) & 2(u_0u_1+u_2u_3) & u_0^2-u_1^2-u_2^2+u_3^2 \end{bmatrix} \qquad \dots (5)$$

The differential equations of the vehicle position in terms of latitude, longitude, and heading can be arranged in matrix form instead of set of equations as described in [5, 6]:

$$\begin{bmatrix} \dot{L} \\ \dot{l} \\ \dot{h} \end{bmatrix} = \begin{bmatrix} 1/(R_N+h) & 0 & 0 \\ 0 & 1/\left((R_N+h)\cos L\right) & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} V_N \\ V_E \\ V_D \end{bmatrix} \qquad \dots (6)$$

where

$[V_N\ V_E\ V_D] = V^{\,n}$: geodetic velocity vector (north, east, and down).

R_N and R_E: are the radii of curvature in the north and east directions and given by [9]:

$$R_N = \frac{r_e}{\left(1-e^2\sin^2(L)\right)^{1.5}} \qquad \dots (7)$$

$$R_E = \frac{r_e}{\sqrt{\left(1-e^2\sin^2(L)\right)}} \qquad \dots (8)$$

and e: eccentricity (= 0.0818)

The differential equations relating the second derivative of the geodetic position and velocities can be derived as [3]:

$$\begin{bmatrix} \dot{V}_N \\ \dot{V}_E \\ \dot{V}_D \end{bmatrix} = C_b^n \cdot f^b + \begin{bmatrix} -\left[\dfrac{V_E}{(R_E+h)\cos L} + 2w_{ie}\right]V_E\sin L + \dfrac{V_N V_D}{(R_N+h)} \\ \left[\dfrac{V_E}{(R_E+h)\cos L} + 2w_{ie}\right]V_N\sin L + \dfrac{V_E V_D}{(R_E+h)} + 2w_{ie}V_D\cos L \\ -\dfrac{V_E^2}{(R_E+h)} - \dfrac{V_N^2}{(R_N+h)} - 2w_{ie}V_E\cos L + g_e \end{bmatrix} \qquad \dots (9)$$

where

f^b : Specific force outputs in the body coordinate (accelerometers outputs) = $[f_x \; f_y \; f_z]^T$

g_e : Gravity force applied to down direction

Gravity force (g_e) can be found from initial gravity g_0 [7, 8]:

$$g_0 = 9.780327\ [1 + 0.0053024\ \sin^2(L) - 0.0000058\ \sin^2(2L)] \quad \ldots (10)$$

and

$$g_e = g_0 - [3.0877 \times 10^{-6} - 0.0044 \times 10^{-6} \sin^2(L)]h + 0.072 \times 10^{-12} h^2 \quad \ldots (11)$$

3. The Reduced Order Algorithm

Equations (1, 6, and 9), represent the mechanization equation for the terrestrial navigation system. These equations can be reduced to the following differential equations describing the mechanization of the navigation state for 3DOF only, as shown in Eq.(12).

$$\begin{bmatrix} \dot{L} \\ \dot{l} \\ \dot{h} \\ \dot{V}_N \\ \dot{V}_E \\ \dot{V}_D \\ \dot{P} \end{bmatrix} = \begin{bmatrix} \frac{1}{[R_N+h]} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{(R_N+h)\cos L} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos P & -\sin P & 0 & 0 \\ 0 & 0 & 0 & \sin P & \cos P & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_N \\ V_E \\ V_D \\ f_x \\ f_z \\ g_e \\ w_p \end{bmatrix} \quad \ldots (12)$$

where

P: is the pitch angle, as shown in Figure (1).

The following subsections will give the details of the developed algorithm to solve the above equation.

3.1 Initialization

Initial velocities and positions in Earth-Centered-Earth-Fixed frame (ECEF) coordinate data should be loaded to the vehicle navigation processor before start of moving.

Initial Position and velocity

Vehicle initial position in ECEF coordinate system is computed from [9]:

$$\left. \begin{aligned} X_e(0) &= (R_E + H_0)\cos l_0 \cos L_0 \\ Y_e(0) &= (R_E + H_0)\sin l_0 \cos L_0 \\ Z_e(0) &= \left[R_E\left(1 - e^2\right) + H_0\right]\sin L_0 \end{aligned} \right\} \quad \ldots (13)$$

while initial velocities are zero in ECEF frame.

Initial Euler pitch angle

By applying the transformation matrix [9]

$$C_b^n = \begin{bmatrix} 0 & \sin c & \cos c \\ 0 & \cos c & -\sin c \\ -1 & 0 & 0 \end{bmatrix} \quad \ldots (14)$$

Where

. χ : The horizontal shift of the Y_b-axis from the east direction

Euler pitch angles are directly obtained by applying the equation pitch angle only, given by [8]:

$$P = atan2(c_{21}, c_{11}) \quad \ldots (15)$$

Where

atan2 = is the four quadrant inverse tangent function

c_{ij}'s, $1 \leq i, \ j \leq 3$ are the (i, j)th elements of direction cosine matrix (C_b^n)

Initial gravity

Initial gravity, which affects the Z_e-axis in ECEF frame, is computed by executing Eq.(10)

3.2 Updating Algorithm

Accelerometers and Gyro outputs (f_x, f_z, and w_P) are used directly in Eq.(12) to obtain the position and velocity of the vehicle.

Vehicle position (latitude, longitude, and altitude) is used to find the curvature in the north and east directions using Equations (7) and (8), and gravity force from the Equations (10) and (11).

The developed algorithm can be summarized as a flow diagram described in Figure (2).

4. Simulation Results

The algorithm has been implemented on 3DOF missile flight simulator described in [3]. The data of Figures (3a-c) are used as inputs to navigation algorithm. The vehicle velocity and position are calculated using the computational procedure shown in Figure (2).

The study used MATLAB 6.5 computer-aided software, including Simulink toolboxes.

Figure (4a) shows a comparison between the velocity of the typical trajectory and the navigator velocity output. While the displacement was compared in Figure (4b).

The same simulation was repeated with additional error as in Table (1) [3]. Figure (5) shows the velocity vectors for the algorithm with measurement errors.

5. Conclusions

The paper offers a new method for 3DOF strapdown terrestrial algorithm. The proposed method reduces the total time required to execute the same data of the system without reduction to 50%. In addition, the kinematics of the terrestrial strapdown navigation states is reduced to one set of equations in state space model.

The proposed method was capable of providing reliable and accurate position information for the vehicle with free accelerometers and gyro errors.

6. References

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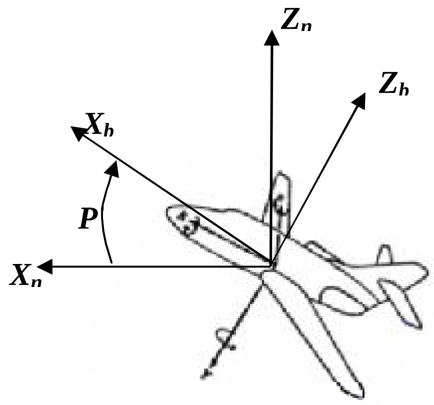


Figure (1): The Body Frame

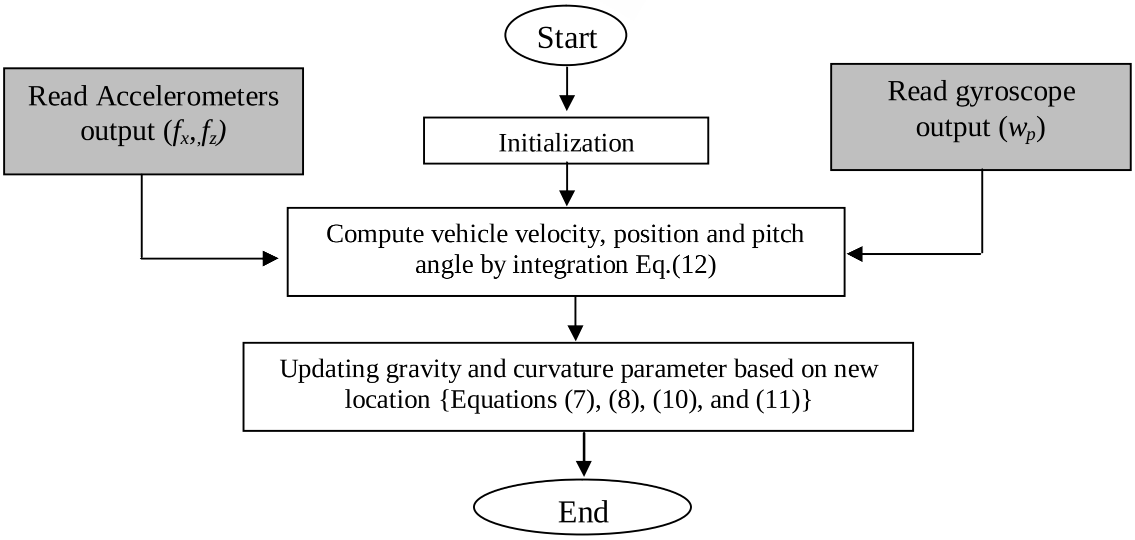
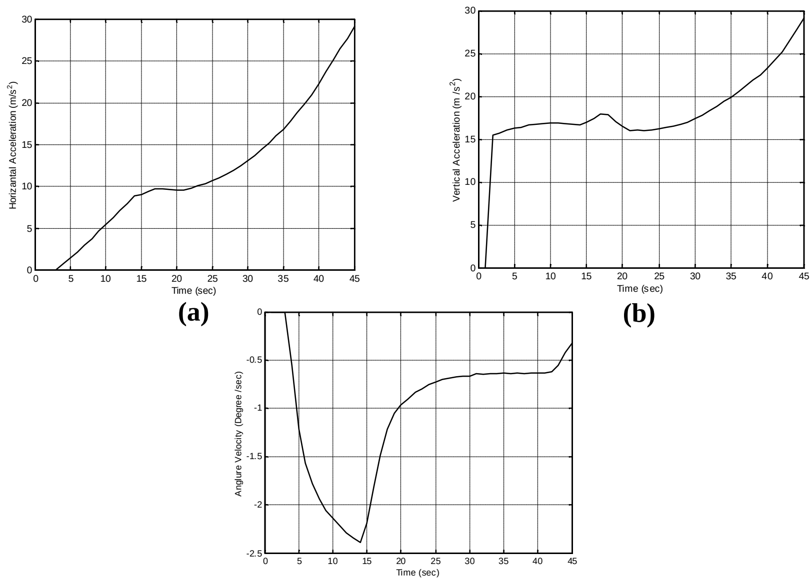
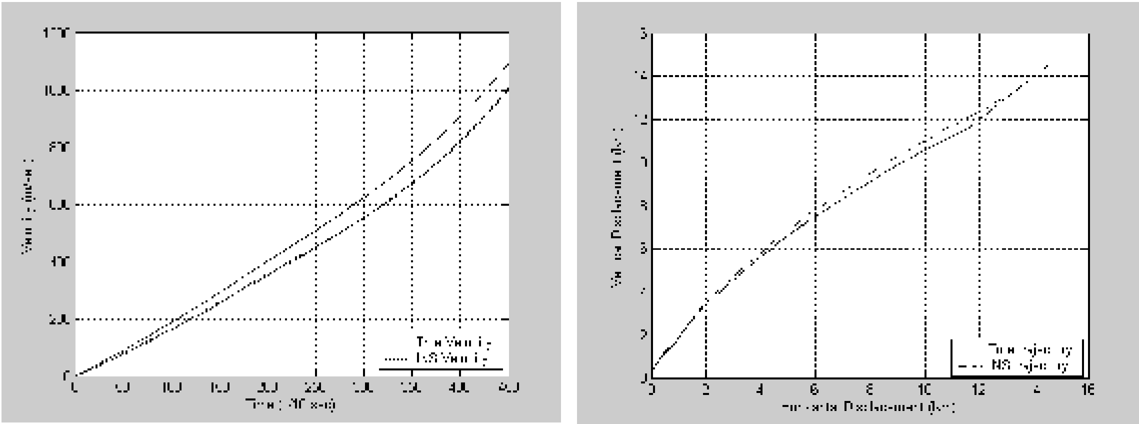


Figure (2): Flow diagram of reduced order 3DOF terrestrial navigation algorithm



(c)

Figure (3): Missile reference trajectory; horizontal acceleration (a), vertical acceleration (b), and pitch angular velocity (c)



(a) **(b)**

Figure (4): True and calculated trajectory for Terrestrial algorithm (a) velocity (b) position without measurement errors

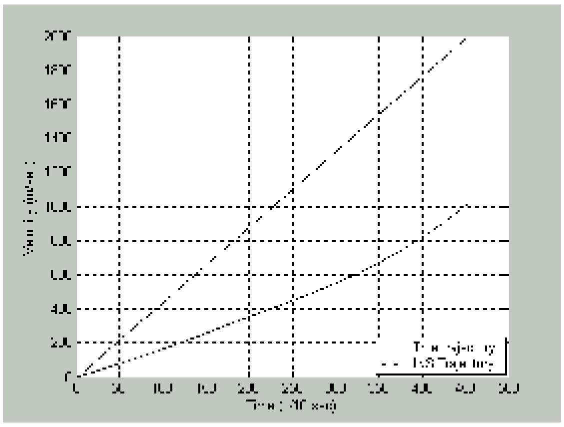


Figure (5): True and calculated velocities for terrestrial algorithms with measurement errors

Gyro Errors	Accelerometer Errors
Constant = $12^0/h$ g dependent = $2^0/h/g$ Random = $10^0/h$	Bias = $0.001\times g$ Scale factor = 5% Random = $2g\times10^{-5}$

Table (1): Gyros and Accelerometers errors