The Effect of Coma Aberration on Point Spread Function for Array Synthetic Circular Obscured Aperture

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Abstract

In this research we study the effect of coma aberration on point spread function (PSF) for array synthetic circular obscured aperture. The equation of point spread function (PSF) have been derived for this case, coma aberration is used in different angles for set of obscured circular synthetic apertures, addition to different obscuration ratios, by using MathCAD programs. The results show an increasing the resolving power by using array synthetic circular obscured aperture and coma aberration leads to decrease the secondary peaks for point spread function (Apodization).

تأثير الزيغ المذنبي على دالة الانتشار النقطية لفتحة مركبة تتألف من مصفوفة من الفتحات الدائرية المعاقة

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الخلاصة

تم في هذا البحث دراسة تاثير الزيغ المذنبي على دالة الانتشار النقطية لمصفوفة من الفتحات الدائرية المعاقة المركبة، واشتقت معادلة دالة الانتشار النقطية لهذه الحالة ، حيث أستخدم الزيغ المذنبي بزوايا مختلفة ولمجموعة من الفتحات المعاقة المركبة بالاضافة الى تغير نسب الاعاقة باستخدام برنامج MathCAD. ووجد من خلال النتائج زيادة قدرة التحليل بأستخدام مصفوفة من الفتحات الدائرية المعاقة المركبة وأن الزيغ المذنبي يؤدي الى تقليل القمم الثانوية لدالة الانتشار النقطية .

1. Introduction

Synthetic aperture techniques are commonly used to obtain high resolution from data acquired using low resolution sensors, these techniques are commonly used in modern sonar and radar systems, being designated by Synthetic Aperture Sonar (SAS) and by Synthetic Aperture Radar (SAR) systems, respectively; this kind of systems is presently used in civil and military applications [1].

It is a fact that obscuring the aperture center of an optical system which means using obscured circular aperture has important effects in the practical application, where the use of obscured aperture appears obvious in imaging by the optical systems and astronomical telescopes [2].

When a lens is corrected for spherical aberration, it forms a point image of a point object situated on the axis; But if the point object is situated off the principal axis, the lens, even corrected for spherical aberration, forms a comet-like image in place of point image, this defect in the image is called coma aberration [3]. In coma aberration, the wave front aberration varies linearly with field height [4]." Synthetic Aperture " Can be defined as a structure to separate optical systems of large individual aperture function sometimes called " Mosaic" or "Segmented Mirror", Synthetic aperture is an image system for independent optical system which are together sharing the image domain [5].

Comatic aberration is Asymmetric , it can be given by the polynomial [5] .

$$W=\sum_{m=odd} {}_1w_{m1}\sigma r^m\cos\phi = {}_1w_{31}\sigma r^3\cos\phi + {}_1w_{51}\sigma r^5\cos\phi + \ldots\ldots\ldots\ldots$$

In Cartesian coordinate (x , y): where
x= r sin ϕ, y= r cos ϕ

$$W(x_1,y_1) = {}_1w_{31}(x_1^{\,2}+y_1^{\,2})y_1 + {}_1w_{51}(x_1^{\,2}+y_1^{\,2})^2 y_1 + \ldots\ldots\ldots$$

When the Cartesian coordinates are rotated by an angle ψ , the new coordinates become:

$$x_1 = x\cos\psi - y\sin\psi$$
$$y_1 = x\sin\psi + y\cos\psi$$

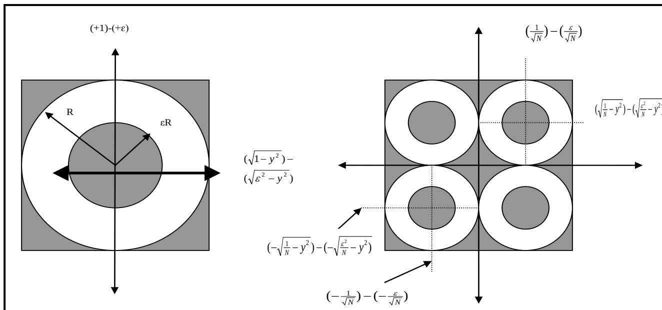
Where ψ is rotation angle of Cartesian coordinates, we get:

$$W(x,y) = {}_1w_{31}(x^2+y^2)(x\sin\psi + y\cos\psi)\ldots\ldots\ldots(1)$$

Comatic aberration produced by a single lens can also be corrected by properly choosing the radii of curvature of the lens surfaces. Coma can be altogether eliminated for a given pair of object and image points, whereas spherical aberration cannot be completely corrected [6]. Further, a lens corrected for coma will not be free of spherical aberration and the one corrected for spherical aberration will not be free of coma [6].

2. Deriving the Equation of Point Spread Function for Array Synthetic Circular Obscured Aperture with Coma Aberration.

The representation of array synthetic circular aperture with radius (R), having array synthetic circular obscuration with radius (ε R) is shown in Figure (1).



(a) Obscured Individual Circular Aperture

(b)Obscured Synthetic Aperture N=4, Where N is the Number of Obscured Aperture

Figure (1) The Integral Boundary for Array of Obscured Circular Synthetic Aperture

The ratio between the radii of outer to inner circles in the obscured aperture is called obscuration ratio (ε) [7,8].

The equation for array of circular synthetic aperture is given by the relation :

$$x^2 + y^2 = (\frac{R}{\sqrt{N}})^2 \quad ....(2)$$

and when $R = 1$, then:

$$x = \pm\sqrt{\frac{1}{N} - y^2}$$

and $$y = \pm\frac{1}{\sqrt{N}} \quad ....(3)$$

The equation for array of obscured circular synthetic aperture is given by the relation

$$x'^2 + y'^2 = (\frac{\varepsilon R}{\sqrt{N}})^2 \quad .... (4)$$

$$x' = \pm\sqrt{\frac{\varepsilon^2}{N} - y'^2}$$

and

$$y' = \pm\frac{\varepsilon}{\sqrt{N}} \quad ....(5)$$

where x' and y' is the Cartesian coordinate of individual synthetic aperture.

The intensity in point spread function for array of obscured circular synthetic apertures, can be calculated by the relation [8].

$$PSF = |F|^2 = |F_1 - F_2|^2 \quad ....(6)$$

where

F_1: is the complex amplitude function for array of circular apertures.

F_2: is the complex amplitude function for array of circular obscurations.

We can express the complex amplitude in the point (u', v') in the image plane by using Fourier transform to pupil function [5,7,9,10,11,12,13].

$$f(x', y') = \tau(x', y')e^{ikw(x'.y')} \quad ....(7)$$

where

$\tau(x', y')$: represents the real amplitude distribution in exit pupil and is called " pupil transparency " or " transmission function " and often chooses equal one unit .

$e^{ikw(x',y')}$: Wave front aberration function.

$w(x', y')$: Aberration function

(x', y') : Exit pupil coordinates [4,8,14].

We can express point spread function for a single circular aperture in its integral form, by

Putting the exit pupil coordinates of form (x, y) instead of (x', y') , for simplicity.

$$F_1(u', v') = n.f \int_y\int_x f(x, y)e^{i2\pi(u'x+v'y)}dxdy \quad ....(8)$$

and for N circular apertures [15].

$$\because x = x' + x_j,\ \ y = y' + y_j \quad ....(9)$$

$$F_1(u', v') = n.f \sum_{j=1}^{N}\int_y\int_x f(x, y)e^{i2\pi[u'(x'+x_j)+v'(y'+y_j)]}dxdy$$

....(10)

$$F_1(u', v') = n.f \sum_{j=1}^{N}\int_y\int_x f(x, y)e^{i2\pi(u'x'+v'y')}.e^{i2\pi(u'x_j+v'y_j)}dxdyy$$

....(11)

$$F_1(u', v') = n.f \int_y\int_x f(x, y)e^{i2\pi(u'x'+v'y')}dxdy.\sum_{j=1}^{N}e^{i2\pi(u'x_j+v'y_j)}$$

....(12)

By substituting the equation (7) in equation (10), we obtain:

$$F_1(u', v') = n.f \int_y\int_x \tau(x, y)e^{ikw(x,y)}.e^{i2\pi(u'x'+v'y')}dxdy.\sum_{j=1}^{N}e^{i2\pi(u'x_j+v'y_j)}$$

....(13)

Let $\tau(x, y) = 1$

$$F_1(u', v') = n.f \int_y\int_x e^{ikw(x,y)}.e^{i2\pi(u'x'+v'y')}dxdy.\sum_{j=1}^{N}e^{i2\pi(u'x_j+v'y_j)}$$

....(14)

Where $k = \frac{2\pi}{\lambda}$

$$F_1(u',v') = n.f \int_y \int_x e^{i2\pi[w(x,y)+(u'x'+v'y')]} dxdy . \sum_{j=1}^{N} e^{i2\pi(u'x_j+v'y_j)}$$

……………………………………………(15)

By substituting the integral boundary from equation (3) the area of synthetic aperture includes

$$F_1(u',v') = n.f \int_{-\frac{1}{\sqrt{N}}}^{\frac{1}{\sqrt{N}}} \int_{-\sqrt{\frac{1}{N}-y^2}}^{\sqrt{\frac{1}{N}-y^2}} e^{i2\pi[w(x,y)+(u'x'+v'y')]} dxdy . \sum_{j=1}^{N} e^{i2\pi(u'x_j+v'y_j)}$$

………………………………………..…(16)

Where $e^{i\theta} = \cos\theta + i\sin\theta$

$$F_1(u',v') = n.f \left[\int_{-\frac{1}{\sqrt{N}}}^{\frac{1}{\sqrt{N}}} \int_{-\sqrt{\frac{1}{N}-y^2}}^{\sqrt{\frac{1}{N}-y^2}} \begin{bmatrix} \cos\{2\pi[w(x,y)+u'x'+v'y']\} + \\ i\sin\{2\pi[w(x,y)+u'x'+v'y']\} \end{bmatrix} dxdy \right]$$
$$.\left[\sum_{j=1}^{N} \cos\{2\pi(u'x_j+v'y_j)\} + i\sin\{2\pi(u'x_j+v'y_j)\} \right]$$

……………………………………………(17)

Let $z' = 2\pi u'$ and $m' = 2\pi v'$. Then (17) becomes:

$$F_1(z',m') = n.f \left[\int_{-\frac{1}{\sqrt{N}}}^{\frac{1}{\sqrt{N}}} \int_{-\sqrt{\frac{1}{N}-y^2}}^{\sqrt{\frac{1}{N}-y^2}} \begin{bmatrix} \cos\{2\pi w(x,y)+z'x'+m'y'\} + \\ i\sin\{2\pi w(x,y)+z'x'+m'y'\} \end{bmatrix} dxdy \right]$$
$$.\left[\sum_{j=1}^{N} \cos\{z'x_j+m'y_j\} + i\sin\{z'x_j+m'y_j\} \right]$$

……………………………………………(18)

The intensity distribution on the two axes (z',m') is symmetric, so; we can reduce it to one axis only; let $(m'=0)$, then equation (18) will take a new form .

$$F_1(z') = n.f \left[\int_{-\frac{1}{\sqrt{N}}}^{\frac{1}{\sqrt{N}}} \int_{-\sqrt{\frac{1}{N}-y^2}}^{\sqrt{\frac{1}{N}-y^2}} [\cos\{2\pi w(x,y)+z'x'\} + i\sin\{2\pi w(x,y)+z'x'\}] dxdy \right]$$
$$.\left[\sum_{j=1}^{N} [\cos(z'x_j) + i\sin(z'x_j)] \right]$$

……………………………………………(19)

Equation (19) represents the complex amplitude for array of circular synthetic apertures.

The complex amplitude function for array of circular obscurations from equation (7) can be found as:

$$F_2(u',v') = n.f \int_y \int_x f(x',y') e^{i2\pi(u'x'+v'y')} dx'dy'$$

…. ……………………………….....…. (20)

By using the same steps and substituting the integral boundary from the equation (5) we obtain.

$$F_2(z') = n.f \left[\int_{-\frac{\varepsilon}{\sqrt{N}}}^{\frac{\varepsilon}{\sqrt{N}}} \int_{-\sqrt{\frac{\varepsilon^2}{N}-y^2}}^{\sqrt{\frac{\varepsilon^2}{N}-y^2}} [\cos\{2\pi w(x',y')+z'x'\} + i\sin\{2\pi w(x',y')+z'x'\}] dx'dy' \right]$$
$$.\left[\sum_{j=1}^{N} [\cos(z'x_j) + i\sin(z'x_j)] \right]$$

……………………………...………(21)

By using the physical conception of the equation (6), we can obtain the following:

$$PSF = n.f \left| \begin{matrix} \left[\int_{-\frac{1}{\sqrt{N}}}^{\frac{1}{\sqrt{N}}} \int_{-\sqrt{\frac{1}{N}-y^2}}^{\sqrt{\frac{1}{N}-y^2}} [\cos\{2\pi w(x,y)+z'x'\} + i\sin\{2\pi w(x,y)+z'x'\}] dxdy . \sum_{j=1}^{N} [\cos(z'x_j) + i\sin(z'x_j)] \right] \\ - \left[\int_{-\frac{\varepsilon}{\sqrt{N}}}^{\frac{\varepsilon}{\sqrt{N}}} \int_{-\sqrt{\frac{\varepsilon^2}{N}-y^2}}^{\sqrt{\frac{\varepsilon^2}{N}-y^2}} [\cos\{2\pi w(x',y')+z'x'\} + i\sin\{2w(x',y')+z'x'\}] dx'dy' . \sum_{j=1}^{N} [\cos(z'x_j) + i\sin(z'x_j)] \right] \end{matrix} \right|^2$$

……………………………………...…..(22)

From the relation $|x+iy|^2 = x^2 + y^2$ the equations above become:

$$PSF = n.f\left|\begin{array}{l} \displaystyle\int_{-\frac{1}{\sqrt{N}}}^{\frac{1}{\sqrt{N}}}\int_{-\sqrt{\frac{1}{N}-y^2}}^{\sqrt{\frac{1}{N}-y^2}}\cos\{2\pi w(x,y)+z'x'\}dxdy.\sum_{j=1}^{N}e^{iz'x_j} \\ \displaystyle -\int_{-\frac{\varepsilon}{\sqrt{N}}}^{\frac{\varepsilon}{\sqrt{N}}}\int_{-\sqrt{\frac{\varepsilon^2}{N}-y^2}}^{\sqrt{\frac{\varepsilon^2}{N}-y^2}}\cos\{2\pi w(x',y')+z'x'\}dx'dy'.\sum_{j=1}^{N}e^{iz'x_j} \\ \displaystyle +\int_{-\frac{1}{\sqrt{N}}}^{\frac{1}{\sqrt{N}}}\int_{-\sqrt{\frac{1}{N}-y^2}}^{\sqrt{\frac{1}{N}-y^2}}\sin\{2\pi w(x,y)+z'x'\}dxdy.\sum_{j=1}^{N}e^{iz'x_j} \\ \displaystyle -\int_{-\frac{\varepsilon}{\sqrt{N}}}^{\frac{\varepsilon}{\sqrt{N}}}\int_{-\sqrt{\frac{\varepsilon^2}{N}-y^2}}^{\sqrt{\frac{\varepsilon^2}{N}-y^2}}\sin\{2\pi w(x',y')+z'x'\}dx'dy'.\sum_{j=1}^{N}e^{iz'x_j} \end{array}\right|^2$$

……………………………………......……(23)

This equation represents the point spread function for an array of obscured circular synthetic apertures.

By substituting the value of normalizing factor ($\frac{1}{(\pi-\varepsilon^2\pi)^2}$) [15] and value of coma aberration from equation(1) , in equation (23), we obtain:

$$PSF = \frac{1}{(\pi-\varepsilon^2\pi)^2}\left|\begin{array}{l} \displaystyle\int_{-\frac{1}{\sqrt{N}}}^{\frac{1}{\sqrt{N}}}\int_{-\sqrt{\frac{1}{N}-y^2}}^{\sqrt{\frac{1}{N}-y^2}}\cos(2\pi w_{31}(x^2+y^2)(x\sin\Psi+y\cos\Psi)+z'x')dxdy \\ \displaystyle -\int_{-\frac{\varepsilon}{\sqrt{N}}}^{\frac{\varepsilon}{\sqrt{N}}}\int_{-\sqrt{\frac{\varepsilon^2}{N}-y^2}}^{\sqrt{\frac{\varepsilon^2}{N}-y^2}}\cos(2\pi w_{31}(x'^2+y'^2)(x'\sin\psi+y'\cos\psi)+z'x')dx'dy' \\ \displaystyle +\int_{-\frac{1}{\sqrt{N}}}^{\frac{1}{\sqrt{N}}}\int_{-\sqrt{\frac{1}{N}-y^2}}^{\sqrt{\frac{1}{N}-y^2}}\sin(2\pi w_{31}(x^2+y^2)(x\sin\psi+y\cos\psi)+z'x')dxdy \\ \displaystyle -\int_{-\frac{\varepsilon}{\sqrt{N}}}^{\frac{\varepsilon}{\sqrt{N}}}\int_{-\sqrt{\frac{\varepsilon^2}{N}-y^2}}^{\sqrt{\frac{\varepsilon^2}{N}-y^2}}\sin(2\pi w_{31}(x'^2+y'^2)(x'\sin\psi+y'\cos\psi)+z'x')dx'dy' \end{array}\right| \cdot\sum_{j=1}^{N}e^{iz'x_j} \quad \text{.....(24)}$$

Equation (24) represents the point spread function for array of obscured circular synthetic apertures with Coma aberration.

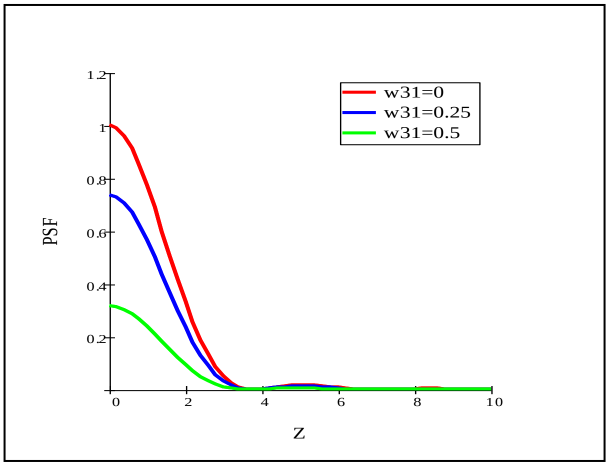


Figure (1)PSF for individual aperture with no obscuration and different values of coma aberration.

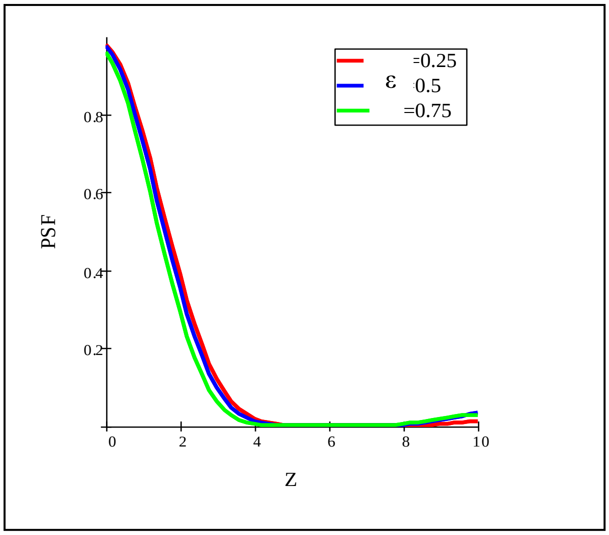


Figure (2)PSF for four apertures with coma aberration (0.5) and different values of obscuration ratio coefficient (R=1,ψ=90).

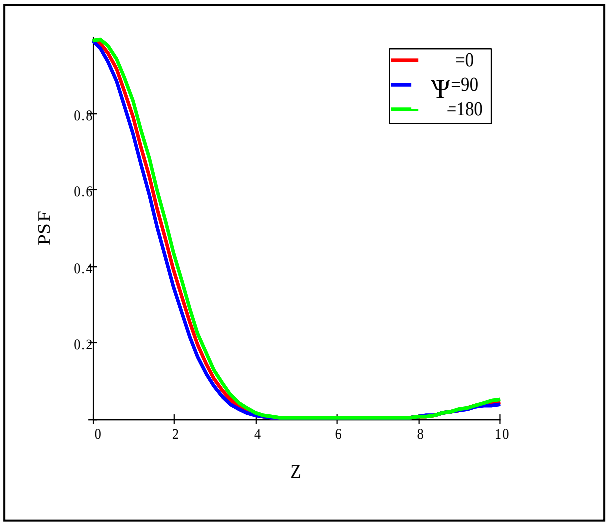


Figure (3)PSF for four apertures with coma aberration (0.25) and different values of rotation angle coefficient.(R=1,ε=0.75).

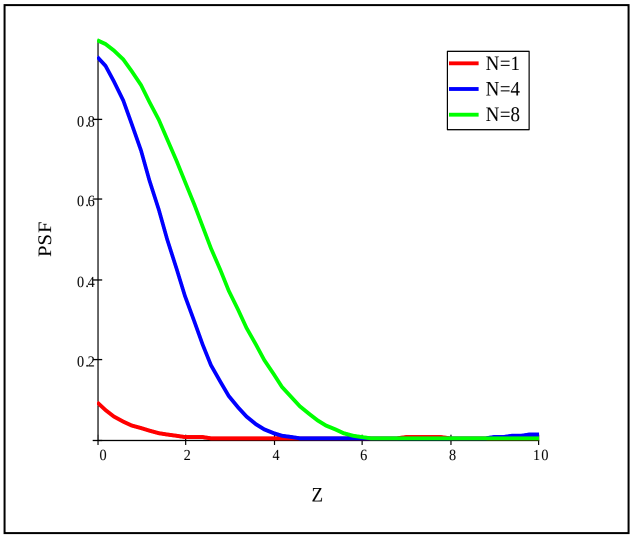


Figure (4)PSF for rotation angle (45) with obscuration ratio (0.25) and different values number of apertures.(R=1,w_{31}=0.75).

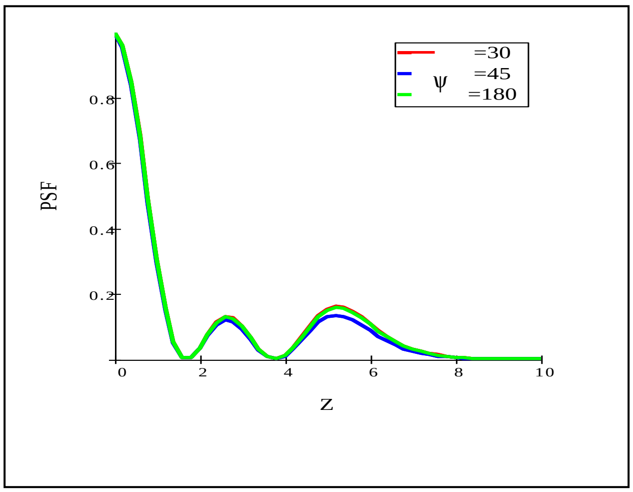
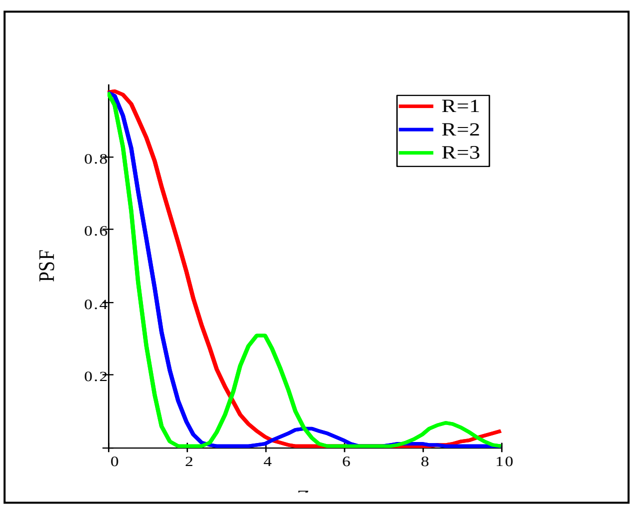


Figure (5)PSF for eight apertures with no obscuration and different values of rotation angle coefficient .(w_{31}=0.5,R=4).



Figure(6)PSF for four apertures with obscuration ratio (0.5) and different values of distance coefficient.(w_{31}=0.5,ψ=180).

3. Result and Discussion

The aim of using the obscured apertures is to make images better and reduce the effect of aberration. It also increases the resolving power of the optical system. We used MathCAD program to solve equation (24), which was derived in this research, to study the PSF for array synthetic circular obscured aperture with the presence of Coma aberration.

The results of PSF were drowning in figures (1-6), and we will discuss them as follows:

Figure (1) represents the intensity distribution curves for point spread function for values of Coma aberration (w_{31}=0, 0.25 and 0.5) with no obscuration (ε=0) for individual circular aperture without angle, we see the diffraction limited system in red curve normalizing to one and this proves the fact that the normalizing factor which we had derived in this research is true, where notes the aberration effect on individual aperture in two curves.

Figure (2) represents the intensity distribution curves for point spread function with different values of obscuration ratios (ε=0.25,0.5 and 0.75)when synthetic aperture are four and coma (w_{31}=0.5)for angle (ψ=90),where the resolving power for obscured aperture with obscuration ratio (ε=0.75),is better than resolving power with obscuration ratio(ε=0.5)and(ε=0.25) by using the obscured circular aperture system the one part transfers from central spot energy (Airy disk) to secondary peaks in diffraction pattern.

Figure(3) represents the intensity distribution curves for point spread function with different values of angles (ψ=0,90 and 180) when synthetic aperture are four with different values of focus error and obscuration ratio, we notes the state better when (ψ=90),but the effect an angle little.

Figure(4) represents the intensity distribution curves for point spread function of synthetic apertures (N=1,4 and 8) with angle (ψ=45) when different values from focus error and obscuration ratio, we notice that by increasing the number of apertures we see the positions of secondary peaks were moved inside (near the PSF), and not found secondary peaks, when increasing the number of apertures increases the higher peaks, where increasing strehl ratio.

(i.e. the resolving power becomes better with increasing the number of synthetic apertures by comparing with individual aperture).

Figure(5) represents the intensity distribution curves for point spread function which having different values of angles(ψ=30, 45,180) with no obscuration of synthetic aperture are eight (N=8) and distance (R=4), we notice the coma aberration leads to decreasing the secondary peaks for point spread function (Apodization).

Figure (6) represents the intensity distribution curves for point spread function which having different values of distance (R= 1,2 and 3) with angle (ψ=180) for synthetic aperture are four (N=4), we notice by increasing the distance between the synthetic apertures (R=1,2 and 3) (where R represents distance from the center of synthetic aperture to the center of Cartesian coordinates)we get secondary peaks with higher intensities, also to increasing the resolving power.

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