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Theoretical Study of Sensitivity of Slab-Sensor with Metamaterial

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Abstract

In this paper, a plane waveguide consisting of three layers was studied, where the metamaterial was placed in the middle region. The dispersion relation equations, field distribution, power flux, and sensitivity were derived, by Maxwell's equations. The results showed that the waveguide contains only the transverse modes TE and TM, while the fundamental mode is absent compared of normal materials. The study of dispersion curves showed that all modes contain a backward orientation in a certain range of V. This amount of backward orientation decreases with increasing the mode order. Also the study showed that the sensitivity increases by increasing the middle width of waveguide or mode order.

Keywords: Metamaterial, Waveguide, slab, Sensitivity, Sensor.

دراسة نظرية لحساسية حساس-شريحة بوجود المواد الفوقية

الخلاصة:

في هذا البحث، تمت دراسة دليل موجة مستوي مؤلف من ثلاثة طبقات، حيث وضعت المادة الفوقية وسط دليل الموجة. اشتقت علاقات التشتت باستخدام معادلات ماكس ويل، توزيع المجال، سيل القدرة والحساسية. اظهرت الدراسة وجود انماط مستعرضة فقط TE و TM حيث ان النمط الاساسي مفقود هنا مقارنة مع المواد الاعتيادية المواد الاعتيادية. اظهرت الدراسة ان جميع الانماط في منحنيات التشتت تعاني من وجود جزء توجه خلفي لمديات من التردد العياري V. هذا القدر من التوجه الخلفي يقل مع زيادة رتبة النمط. كما اظهرت الدراسة زيادة الحساسية مع زيادة عرض المنطقة الوسطية وزيادة مرتبة النمط.

الكلمات المفتاحية: المواد الفوقية، الدليل الموجي، لوح ، الحساسية ،الحساس.

1. Introduction

Metamaterials effectively can be used in the investigation of many physical phenomena. The optical waveguide sensors make use of guided modes in slab waveguides for chemical and biological applications [1]. The waveguide sensor consists of three layers: film, cladding (cover) and substrate. The sensing of such waveguides is accomplished by the evanescent tail in the cladding medium that causes an electromagnetic field into the substrate and cladding that senses an effective refractive index of the guided mode. This effective refractive index depends on the cladding thickness, dielectric permittivity (ε) and magnetic permeability (μ) [2].

The most important feature of designed material absorber (MA) structure is the resonant frequency that changes due to the environmental influences. The resonant frequency shift happens depending on the some physical parameters such as density, pressure and dielectric constant. This resonant frequency shift allowing us to precise detection of parameter change [3,4]. Different from the famous radar and remote sensing technology, microwave sensor can measure properties of materials based on microwave interaction with matter. There are two kinds of microwave sensor, i.e. nonresonant and resonant. Advantage of resonant sensor compared with the nonresonant sensor is that resonant sensor has higher sensitivity [5]. The sensor needs to have a sharp resonance in its frequency response and a high concentration of electric field to enable the detection of small changes in dielectric environment [5,6]. It is well known that sensing devices can detect a small change, depending on the following four criteria: 1) the sensor must have an operating frequency low enough to avoid to the background and substrate absorption. This poses a significant challenge as conventional sensing devices have a limited area, and such reduced space tens to increase the operating frequency of sensors [5], 2) the sensors must produce space a strong and measurable readout signal with a resonant behavior sharp enough to accurately track the shift in transmission spectra [6], 3) pertains to the linearity of sensing which is related to the quality factor of sensors, 4) the sensor sensitivity.

In this paper, the wave equations of $\vec{E}$ and $\vec{H}$ are presented for asymmetric slab waveguide with metamaterial. The filed distributions and pointing vectors are competed for each layer. In turn, the width of guiding layer that maximize the power are presented. Finally, the sensitivity of sensor is derived to construct the operation conditions of the sensor.

2. Slab Waveguide with Metamaterial

Planar waveguide characterizes by parallel boundaries with respect to one direction (x-axis), but is infinite in extent in the lateral directions (y-axis and z-axis). In this work, we investigate an asymmetric slab waveguide structure that sketched in Fig.(1). A metamaterial with (ε_g, μ_g) slab occupying the region $0 < x < h$ is sandwiched between a substrate with (ε_m, μ_m) occupying the region $-\infty < x < 0$ and a cover cladding with (ε_c, μ_c) occupying the region $h < x < \infty$. Their structure is uniform in the y and z directions. Only two-dimensional problem $(\partial / \partial y = 0)$ is considered because the index profile is independent of y-direction. For this waveguide type, the modes are either *TE* or *TM*. Electromagnetic waves can express as a linear combination of *TE* and *TM* modes.

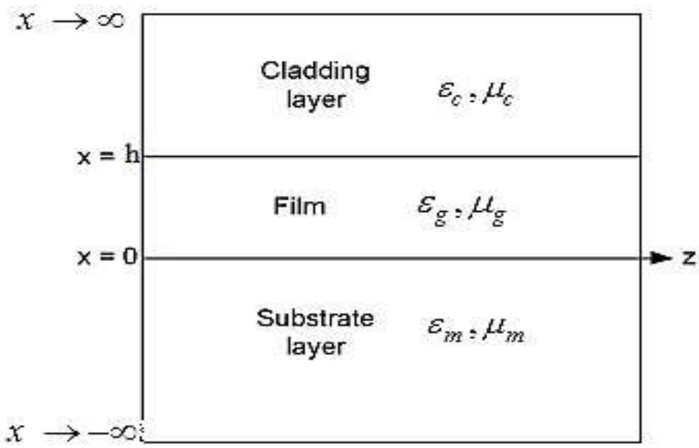


Fig. (1): an asymmetric metal-clad waveguide with metamaterial guiding layer.

For *TE* mode, the electric field $\vec{E}$ is parallel to the interface. It is pointing in the y-direction corresponds to the perpendicular to the plane of incidence (s-polarized). Therefore, the electric field is entirely in the y-plane direction but it has no component of the magnetic intensity $\vec{H}$ in this direction because the magnetic intensity is transverse. Thus $\vec{H}$ has components in x and z directions. For *TM* mode, the magnetic intensity vector is parallel to the interface, *TM* mode results a p-polarized wave. That is, the entire $\vec{H}$ vector is entirely in the y direction but not of the electric field. Thus $\vec{E}$ has components in x and z directions [3].

3. Maxwell's Equations

All the analyses of the electromagnetic waves propagation depend on Maxwell's equations that govern the time dependence of the intensity of the electric and magnetic field, respectively [4]

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \tag{1a}$$

$$\vec{\nabla} \times \vec{H} = \frac{\partial \vec{D}}{\partial t} \tag{1b}$$

where $\vec{D}$ is the electric displacement vector. We can also introduce the electric permittivity (ε) and the magnetic permeability (μ) to characterize the response of the materials to an external electric and magnetic fields as

$$\vec{D} = \varepsilon \vec{E} \tag{2a}$$

$$\vec{B} = \mu \vec{H} \tag{2b}$$

The electric and magnetic fields for the TE waves propagating along the z-axis with angular frequency ω and wavenumber β_z can express as[7]

$$\vec{E} = \begin{bmatrix} 0 \\ E_y \\ 0 \end{bmatrix} e^{i(\beta_z z - \omega t)}, \quad \vec{H} = \begin{bmatrix} H_x \\ 0 \\ H_z \end{bmatrix} e^{i(\beta_z z - \omega t)} \tag{3}$$

For similar parameters, the electric and magnetic fields of TM waves propagating along the z-axis will be

$$\vec{E} = \begin{bmatrix} E_x \\ 0 \\ E_z \end{bmatrix} e^{i(\beta_z z - \omega t)}, \quad \vec{H} = \begin{bmatrix} 0 \\ H_y \\ 0 \end{bmatrix} e^{i(\beta_z z - \omega t)} \tag{4}$$

A homogenous medium is one for which the quantities ε, μ, σ are constants through the medium, where σ is the conductivity of the metal.

The medium is isotropic if ε is a scalar constant so that the electric field and the displacement vectors have everywhere the same direction. Substituting Eqs.(2) into (1) and assuming that the dielectric waveguide is linear, isotropic and homogeneous, yields

$$\vec{\nabla}\times\vec{E}=-\mu\frac{\partial\vec{H}}{\partial t} \qquad (5b)$$

$$\vec{\nabla}\times\vec{H}=\varepsilon\frac{\partial\vec{E}}{\partial t} \qquad (5a)$$

Using Eqs.(3) into (5) and simplifying the result, yields

$$H_x=-\frac{\beta_z}{\mu\omega}E_y \qquad (6a)$$

$$H_z=\frac{i}{\mu\omega}\frac{\partial E_y}{\partial x} \qquad (6b)$$

$$E_y=\frac{i}{\varepsilon\omega}\left(\frac{\partial H_x}{\partial z}-\frac{\partial H_z}{\partial x}\right) \qquad (6c)$$

Substituting Eqs.(6a) and (6b) into (6c), simplifying the result using the fact $\partial E_y/\partial z=i\,\beta_z E_y$, and rearrange the final equation, we get

$$\frac{\partial^2 E_y}{\partial x^2}+\left(k_o^2\varepsilon_i\mu_i-\beta_z^2\right)E_y=0 \qquad (7)$$

where $\beta_z=k_oN$ is the propagation constant along the longitudinal direction and N is the modal index of the propagation mode , $\varepsilon=\varepsilon_r\varepsilon_o$ and $\mu=\mu_r\mu_o$

where ε_r is the relative permittivity, μ_r is the relative permeability, ε_i (where $i=c,g,m$) are the relative permittivity of cladding, metamaterial, and substrate, and μ_i (where $i=c,g,m$) are the relative permeability of cladding, metamaterial, and substrate. A similar manner for TM mode, one may be obtain

$$\frac{\partial^2 H_y}{\partial x^2}+\left(k_o^2\varepsilon_i\mu_i-\beta_z^2\right)H_y=0 \qquad (8)$$

4. Fields in the Waveguide Layers

In order to model the present waveguides, Eqs.(7) and (8) must be solved to construct the fields E_y and H_y in different layers. Consequently, Eqs.(6) will be used to explain the other components of the electric and magnetic fields. The solutions of Eq.(7) in the three-layers are given by [7]

$$E_{yc} = Ae^{-k_c(x-h)} \quad , \quad x \geq h \tag{9a}$$

$$E_{yg} = Be^{-ik_g x} + Ce^{ik_g x} \quad , \quad 0 \leq x \leq h \tag{9b}$$

$$E_{ym} = De^{k_m x} \quad , \quad 0 \geq x \tag{9c}$$

where

$$k_c = \sqrt{\beta_z^2 - k_o^2 \varepsilon_c \mu_c} \tag{10a}$$

$$k_m = \sqrt{\beta_z^2 - k_o^2 \varepsilon_m \mu_m} \tag{10b}$$

$$k_g = \sqrt{k_o^2 \varepsilon_g \mu_g - \beta_z^2} \tag{10c}$$

The parameters A, B, C, D represent the amplitudes of the waves in the different layers. The parameters $k_j, j = c, g, m$ represent the propagation constants in the x-direction for the three-layers.

The magnetic field component H_x can calculate using Eq.(6a) as follows

$$H_{xc} = -\frac{\beta_z}{\omega \mu_c} Ae^{-k_c(x-h)} \quad , \quad x \geq h \tag{11a}$$

$$H_{xg} = -\frac{\beta_z}{\omega \mu_g} (Be^{-ik_g x} + Ce^{ik_g x}) \quad , \quad 0 \leq x \leq h \tag{11b}$$

$$H_{xm} = -\frac{\beta_z}{\omega \mu_m} De^{k_m x} \quad , \quad 0 \geq x \tag{11c}$$

While the component H_z is calculated using Eq.(6b) as follows

$$H_{zc} = \frac{ik_c}{\omega\mu_c} A e^{-k_c(x-h)} \qquad , \quad x \geq h \qquad (12a)$$

$$H_{zg} = -\frac{k_g}{\omega\mu_g} (B e^{-ik_g x} - C e^{ik_g x}) \qquad , \quad 0 \leq x \leq h \qquad (12b)$$

$$H_{zm} = -\frac{ik_m}{\omega\mu_m} D e^{k_m x} \qquad , \quad 0 \geq x \qquad (12c)$$

Matching the components of the tangential and normal magnetic fields at $x = 0$ and $x = h$, yields

$$A = -i\eta_1 (B e^{-ik_g h} - C e^{ik_g h}) \qquad (13a)$$

$$A = B e^{-ik_g h} + C e^{ik_g h} \qquad (13b)$$

$$D = B + C \qquad (13c)$$

$$D = i\eta_2 (B - C) \qquad (13d)$$

Where

$$\eta_1 = \frac{k_g \mu_c}{k_c \mu_g} \qquad (14a)$$

$$\eta_2 = \frac{k_g \mu_m}{k_m \mu_g} \qquad (14b)$$

This determinant may be solved to obtain the form

$$\sin(k_g h) + \eta_1 \cos(k_g h) + \eta_2 \cos(k_g h) - \eta_1 \eta_2 \sin(k_g h) = 0 \qquad (15)$$

$$k_g h = \tan^{-1}\left(\frac{1/\eta_1 + 1/\eta_2}{1 - 1/\eta_1\eta_2} \right) + m\pi \qquad , \qquad m = 0,1,2,.. \qquad (16)$$

Using the definitions of η_1, η_2 in Eqs.(14), Eq.(16) will be

$$k_g h = \tan^{-1}\left(\frac{k_c \mu_g}{k_g \mu_c} \right) + \tan^{-1}\left(\frac{k_m \mu_g}{k_g \mu_m} \right) + m\pi \qquad (17)$$

This equation is called the characteristic equation of TE modes. Depending on the result in Eq.(15), the constants at the system in Eqs.(13) will take the following expressions and the characteristic equations of TM mode will be

$$k_g h = \tan^{-1}\left(\frac{k_c \varepsilon_g}{k_g \varepsilon_c}\right) + \tan^{-1}\left(\frac{k_m \varepsilon_g}{k_g \varepsilon_m}\right) + m\pi \tag{18}$$

5. The Sensitivity of Sensor

The sensitivity of sensor may be determined from Eq.(17) using the derivative $S = \partial N / \partial n_m$ [8]. Differentiating of Eq.(17) with respect to n_m yields

$$k'_g h = \frac{\mu_g}{\mu_c k_g^2} \frac{k_g k'_c - k'_g k_c}{a} + \frac{\mu_g}{\mu_m k_g^2} \frac{k_g k'_m - k'_g k_m}{b} \tag{19}$$

Where $a = 1 + \left(\frac{k_c \mu_g}{k_g \mu_c}\right)^2$, $b = 1 + \left(\frac{k_m \mu_g}{k_g \mu_m}\right)^2$

Eq.(19) may be reformed to explain

$$\left[\frac{\mu_m}{\mu_g} b k_g h + \frac{b\mu_m}{a\mu_c} \frac{k_c}{k_g} + \frac{k_m}{k_g}\right] k'_g = \frac{b\mu_m}{a\mu_c} k'_c + k'_m \tag{20}$$

S

The derivatives are explained as

$$k'_g = -\frac{k_o NN'}{\sqrt{n_g^2 - N^2}} = -\frac{k_o^2 NS}{k_g} \tag{21a}$$

$$k'_c = \frac{k_o NN'}{\sqrt{N^2 - n_c^2}} = \frac{k_o^2 NS}{k_c} \tag{21b}$$

$$k'_m = \frac{k_o (NN' - n_m)}{\sqrt{N^2 - n_m^2}} = \frac{k_o^2 (NS - n_m)}{k_m} \tag{21c}$$

Where S, the sensitivity of sensor. Using Eqs.(21) into (20) and simplifying the result, yields

$$-\left[\frac{\mu_m}{\mu_g} bk_g h + \frac{b\mu_m}{a\mu_c}\frac{k_c}{k_g} + \frac{k_m}{k_g}\right]\frac{NS}{k_g} = \frac{b\mu_m}{a\mu_c}\frac{NS}{k_c} + \frac{NS - n_m}{k_m} \tag{22}$$

The last equation may be rewritten as

$$NS\left[\frac{\mu_m}{\mu_g} bhk_m + \frac{b\mu_m}{a\mu_c}\frac{k_m k_c}{k_g^2} + \frac{k_m^2}{k_g^2} + \frac{b\mu_m}{a\mu_c}\frac{k_m}{k_c} + 1\right] = n_m \tag{23}$$

The sensitivity will be [2]

$$S = \frac{n_m}{N}\frac{1}{\left[\left(1 + \frac{k_m^2}{k_g^2}\right) + bhk_m \frac{\mu_m}{\mu_g} + \frac{b}{a}\frac{\mu_m}{\mu_c}\frac{k_m}{k_c}\left(1 + \frac{k_c^2}{k_g^2}\right)\right]} \tag{24}$$

Now, we introduce three normalized parameters

$$V = k_o h\sqrt{n_g^2 - n_m^2} \tag{25a}$$

$$p = \frac{N^2 - n_m^2}{n_g^2 - n_m^2} \tag{25b}$$

$$\Delta = \frac{n_m^2 - n_c^2}{n_g^2 - n_m^2} \tag{25c}$$

Using these parameters, we will explain

$$1+\frac{k_m^2}{k_g^2}=1+\frac{p}{1-p}=1+r_1, \qquad 1+\frac{k_c^2}{k_g^2}=1+\frac{p+\Delta}{1-p}=1+r_2$$

$$r_1=\frac{p}{1-p}, \quad r_2=\frac{p+\Delta}{1-p}, \quad a=1+\frac{\mu_g^2}{\mu_c^2}r_2, \quad b=1+\frac{\mu_g^2}{\mu_m^2}r_1$$

$$N=\sqrt{\frac{pV^2}{k_o^2h^2}+n_m^2}, \qquad k_m=\frac{V\sqrt{p}}{h}, \qquad \frac{k_m}{k_c}=\sqrt{\frac{p}{p+\Delta}}$$

Using the above results into Eq.(24), we obtain

$$S=\frac{1}{\sqrt{1+\frac{pV^2}{k_o^2h^2n_m^2}}}\frac{1}{\left[1+r_1+bV\sqrt{p}\frac{\mu_m}{\mu_g}+(1+r_2)\frac{b}{a}\frac{\mu_m}{\mu_c}\sqrt{\frac{p}{p+\Delta}}\right]} \tag{26}$$

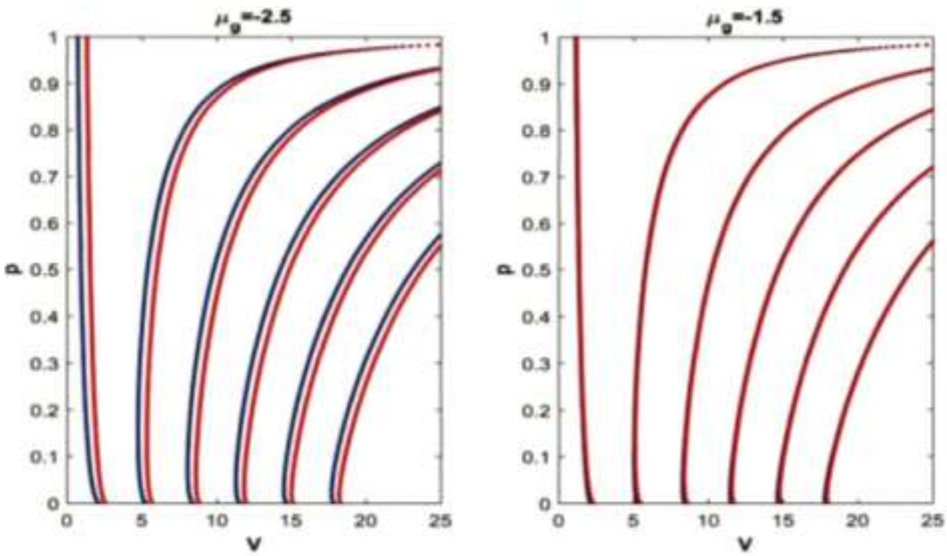
For symmetric metamaterial slab waveguide $\Delta=0$ and the sensitivity will be

$$S=\frac{1}{\sqrt{1+\frac{pV^2}{k_o^2h^2n_m^2}}}\frac{1}{\left[2+2r_1+bV\sqrt{p}\frac{\mu_m}{\mu_g}\right]} \tag{27}$$

Note that, in general, the sensitivity is restricted in the range $0\le S\le 1$ depending on the mode number, width of the metamaterial layer and the frequency. The main dependence is the metamaterial permeability, where the negative value of permeability μ will increase the sensitivity.

6. Results and Discussion

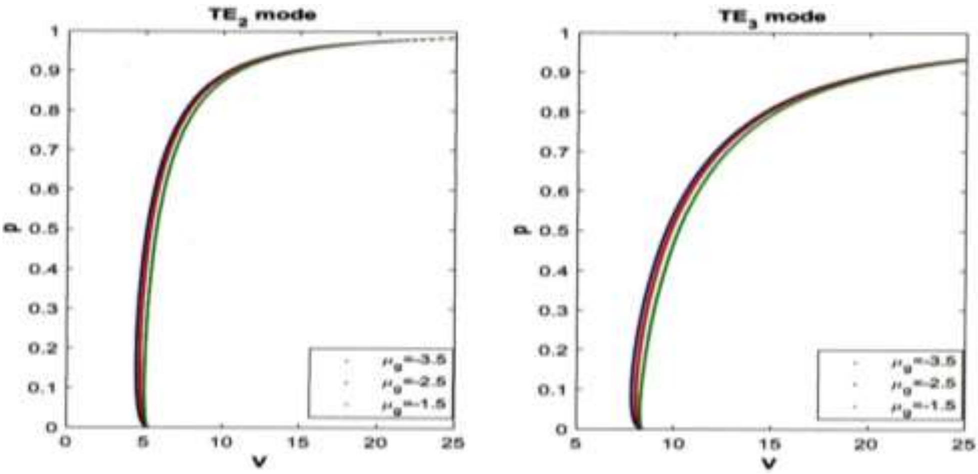
Fig.(2) explains the dispersion curves for many electric and magnetic modes for two values of μ_g where the blue and red lines indicate electric and magnetic modes, respectively. Note that, the fundamental modes TE_0 and TM_0 will note present in comparing with ordinary materials. It is clear that the increasing of $|\mu_g|$ leads to increase the separation between the $TE's$ and $TM's$ modes. The $TE's$ and $TM's$ will be degenerate by decreasing $|\mu_g|$ value. In general, TE_1 mode does not similar to the one in ordinary materials where the curvature dispersion curve of this mode is entirely to the left and not to the right. This behavior has a significant rule to the propagation. All other modes have a curvature to the left at a certain range of V and the other curvature is the right. There are many higher order modes that may be emergence by increasing V value. The physical modes properties (such as: distribution, velocity, ..etc.) vary with mode order.



Fig(2). The normalized dispersion curve of *TE* and *TM* guided mode with $\varepsilon_1 = 1, \mu_1 = 1, \varepsilon_2 = -2, \mu_2 = -2, \varepsilon_3 = 2, \mu_3 = 1$

Fig.(3) shows the dispersion relation of TE_2 and TE_3 modes with different values of $|\mu_g|$. The interest of this figure is to clarify the region that suffers from backwardness of the modes, which does not clear in Fig.(2). The value of $|\mu_g|$ (and perhaps the values of other) affects the shape of dispersion relation. In turn, the mode properties and propagation method will be changed. As $|\mu_g|$ increases, we will see a greater amount of dispersion relation that will be backward. In conclusion, the $|\mu_g|$ value is a fundamental indicator to the central power flux.

Fig.(4) represents a range V relationship that causes inversely propagation with the mode order of cases $\mu_g = -2.5, -2, -1.5$. We note that the quantity to which the cause inversely propagation are very large for mode TE_1 then decreased so much for following modes for all cases μ_g. Physically, this means that the mode TE_1 is suffering so much backward propagation and lead to less sensitivity for the lower order modes.



Fig(3). The normalized dispersion curve of TE_2 and TE_3 guided modes

with $\varepsilon_1 = 1, \mu_1 = 1, \varepsilon_2 = -2, \mu_2 = (-3.5, -2.5, -1.5), \varepsilon_3 = 1, \mu_3 = 1$

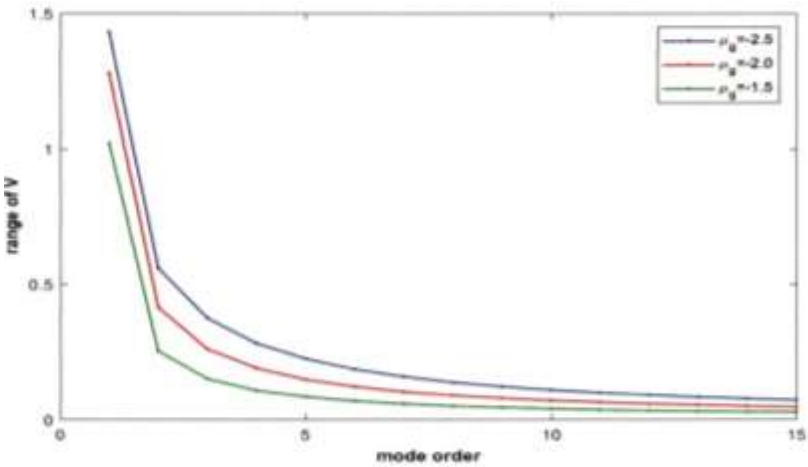
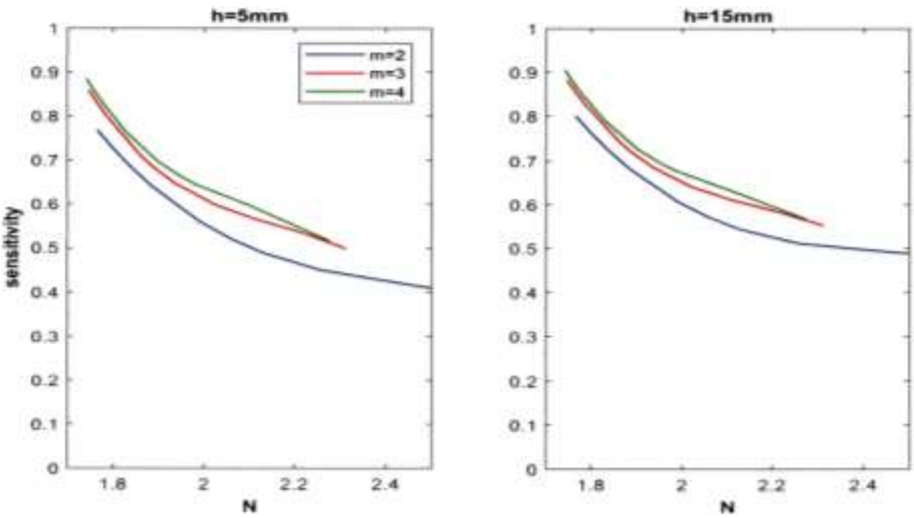


Fig.(4). Range of V relationship with mode order with
$\varepsilon_1 = 1, \mu_1 = 1, \varepsilon_2 = -2, \mu_2 = (-2.5, -2, -1), \varepsilon_3 = 2, \mu_3 = 1$.

Fig.(5) represents sensitivity as a function for effective refractive index of two values of the metamaterial layer thickness at the angular frequency $\omega = [4.1 - 4.6]GH$ of modes TE_1, TE_2 and TE_3. The sensitivity of sensor increases with the increasing mode order and so increase thickness of metamaterial layer and is near to 1. This figure explains that the range of sensitivity of the metamaterial is from [0-1], we note from figure the second mode is lessen sensitive of fourth and third modes when $1.2 < N < 2.5$, is less sensitive than the third and of the fourth mode when $0.8 < N < 2.3$, and the third mode is less sensitive than of the fourth mode when $0.7 < N < 2.25$ fourth mode is more sensitive. This means that the range of N is less with increasing the mode order, this reasonable physically because the inversely propagation is larger for mode is less order.



Fig(5). Sensitivity versus effective refractive index for three modes of two values of the metamaterial region thickness.

Fig.(6) represents the sensitivity as a function for the effective refractive index for three different values of the metamaterial layer thickness of three electric modes TE_1, TE_2 and TE_3. We note that of the figure the sensitivity of first mode increases by increasing the metamaterial layer, also we note that the first mode is less sensitivity.

The range of sensitivity in this mode $0.72 < S < 0.7$. The second electric mode is more sensitivity $0.37 < S < 0.8$ and the third mode is also more sensitivity when the range of sensitive $0.49 < S < 0.89$.

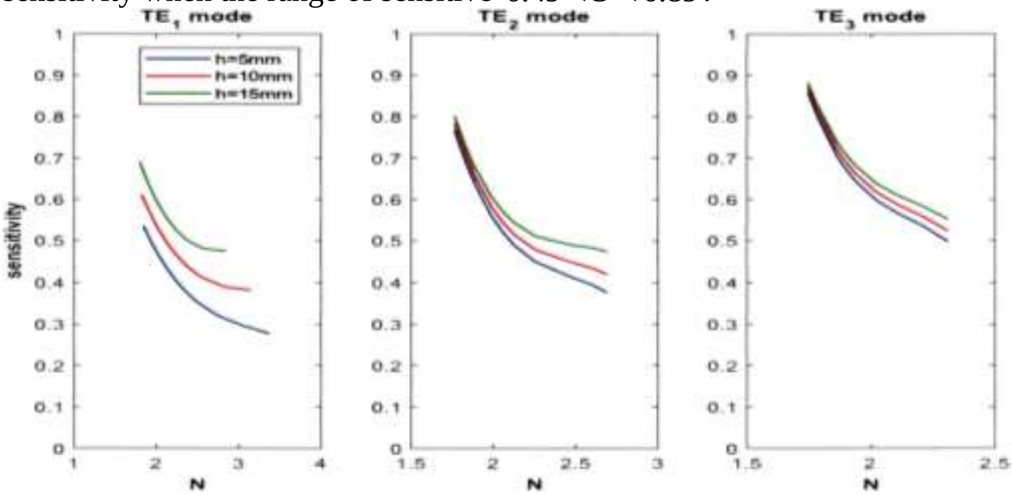


Fig.(6). Sensitivity versus effective refractive index for three modes of three values of the metamaterial region thickness.

Fig.(7) represents the sensitivity as a function of the thickness for the metamaterial modes TE_1 to TE_4. We note that from the figure the sensitivity increases with the increase of thickness and a great increasing with of modes order. With an increased mode order the relation tends to stability and be increased with thickness with a slow increase. We can get to know the thickness of the metamaterial that cannot be very large thickness without limitations. That is, it must be balanced to give appropriate sensitivity and be reasonable.

The results of this paragraph match the results in the reference [8] and somewhat inverse the results in the reference [1]. We note that the first opinion is nearest to the physical nature, where reference [8] indicates existence of the mode TE_0, this is contrary to most other references.

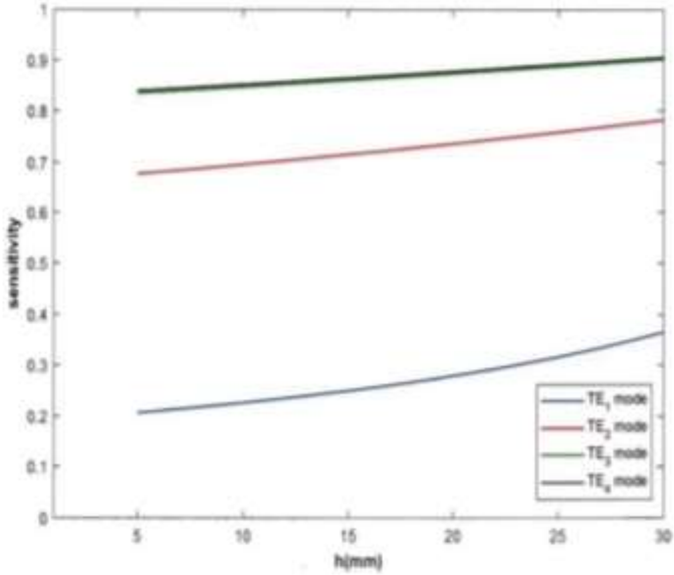


Fig.(7): Sensitivity as a function of the metamaterial thickness for four modes with

$$\omega_0 = 4GH\ , \omega_p = 10GH\ , \Gamma = 0.012 * \omega_p\ , F = 0.56\ ,$$
$$\varepsilon_1 = 1, \mu_1 = 1, \varepsilon_3 = 3, \mu_3 = 1.$$

7. Conclusions

There are many main conclusions that drawn from this paper as: there are many TE and TM modes that will show depending on the *V* value. The fundamental mode will not present as compared with the ordinary materials. All the higher modes contain a section of backward propagation that will reduce with increasing mode order. The sensitivity increasing mode order or increasing the action layer. The range of *V* that corresponds a backward propagation decreases rapidly by increasing mode order width.

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