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Other Classes of Generalized Compact Spaces Based on New Generalized ω- Open Sets

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ABSTRACT

The concept of generalized compactness is useful and essential not only in general topology but also in other advanced branches of pure and applied mathematics. The influence of topological spaces is evident in computer sciences, digital topology, computational topology for geometric and molecular design particle physics, high-energy physics, quantum physics, and superstring theory. Therefore, in this study, new concepts of generalized compact spaces, namely, $\omega_{\delta\text{-}ß}$-compact and nearly $\omega_{\delta\text{-}ß}$-compact spaces, based on new generalized ω-open sets are presented in topological spaces. Various essential characterizations related to these generalized compact spaces are investigated. Furthermore, the relationships among some kinds of generalized compact spaces are discussed. Some illustrative examples are also provided to highlight the realized improvements. The discoveries in this article are expected to aid scientists in conducting research on general topology to establish a general framework for their practical applications in all advanced branches of mathematics and other sciences.

1- Introduction

The notion of generalized compactness is useful and fundamental in general topological spaces and other advanced branches of mathematics and sciences. In previous years, various generalizations of open and closed sets, such as semi-open, α-open, pre-open, semi-pre-open, ƀ-open, δ-ß-open, and E-open sets, have been considered. These sets play a vital role in the generalization of continuity in topological spaces. The concepts of ω-open and ω-closed sets were studied by Hdeib [1]; he presented in [2] the notion of ω-continuous maps. Al-Zoubi and Al-Nashef [3] established the family of each ω-open set in $\mathcal{X}$ −form topology on $\mathcal{X}$. Novel ideas of extended closed sets called ω-closed sets and regular extended and ω-continuous maps were provided by Al-Omari [4].
A new concept of extended open sets called δ-ß-open was introduced by Hatir and Noiri [5], along with δ-ß continuity. Meanwhile, Aljarrah et al. [6]. presented an extended ω_{β}-closed set. Al Ghou [7] presented the concept of ω_s irresoluteness as a strong form of ω_s continuity and reported that ω_s irresoluteness is independent of continuity and irresoluteness. Recently, Waqas and Ali [8] investigated an extended form of continuous maps called contra ω_{pre}-continuous maps by utilizing the notion of ω_{pre}-open sets. Sasmaz and Ozkoc [9] introduced the notion of δ_{ω}-open sets and proved various types of continuity. Additionally, Abdulwahid and Al. Jumaili [10] studied new ideas of extended continuous maps by using a novel extended open set. Compactness is important in other advanced branches of mathematics. The theory of compact spaces was presented by Alexandroff and Urysohn [11]. Balachandran et al. [12] studied the GO compactness of topological spaces and verified some product theorems of compact spaces. In other literature, various types of generalized paracompactness, such as S [13] and P_3 paracompactness [14], have been investigated. Some classical results regarding compact

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and Lindelöf spaces were generalized to λ-compact spaces by Namdari and Siavoshi [15]. Meanehile, J. H. Park and J. K. Park [16] presented a new class of compact spaces related to some new generalized continuous functions. Later, Patil [17] introduced a new kind of compact space, namely, ω_α-compact space, and some characterizations were obtained. Evidently, many researchers [18–23] have investigated various fundamental properties of compactness in topological spaces.

The main objective of this study is to investigate new concepts of generalized compact spaces called $\omega_{\delta\text{-}ß}$-compact and nearly $\omega_{\delta\text{-}ß}$-compact spaces on the basis of new generalized ω-open sets in topological spaces. Some essential characterizations related to these types of generalized compact spaces are introduced. Vital properties related to *E*-Lindelöf spaces are also obtained.

2- Prerequisites

Throughout this manuscript, $(\mathcal{X},\mathcal{T})$ and $(\mathcal{Y}, \mathcal{T}^*)$ $(\mathcal{X}\&\mathcal{Y})$ are used. In this part, several definitions and essential results that play a major role in our work are presented.

Definition 2.1: [1] Assume that $\mathcal{X}$ is a topological-sp. with $\mathcal{K} \subseteq \mathcal{X}$. Thus,

(**a**) $\wp \in \mathcal{X}$ is called the condensation of $\mathcal{K}$ *if each* $\mathcal{N} \in \mathcal{T}\&\wp \in \mathcal{N}$,$\mathcal{N}\cap\mathcal{K}$ is uncountable.

(**b**) $\mathcal{K}$ is said to be ω-open *if and only if* $\forall \wp \in \mathcal{K}\ \exists\mathcal{N} \in \mathcal{T}$(s. t) $\wp \in \mathcal{N}$, with $\mathcal{N}-\mathcal{K}$ being countable.

(**c**) ω-closure and ω-interior, denoted by $Cl_\omega(\mathcal{K})$ and $Int_\omega(\mathcal{K})$, respectively, are described as

(**1**)-$Cl_\omega(\mathcal{K}) = \cap\{\mathcal{A} \subseteq \mathcal{X}: \mathcal{A}\ is\ \omega-\text{closed with}\ \mathcal{K} \subseteq \mathcal{A}\}$;

(**2**)-$\text{int}_\omega(\mathcal{K}) = \cup\{\mathcal{B} \subseteq \mathcal{X}: \mathcal{B}\ is\ \omega-\text{open with}\ \mathcal{B} \subseteq \mathcal{K}\}$.

Remark 2.2:"The family of all ω-open sets of $\mathcal{X}$ indicating $\mathcal{T}_\omega$ or $\omega\Sigma(\mathcal{X},\mathcal{T})$ forms a topology that is finer than $\mathcal{T}$".

Definition 2.3: [24] A subset $\mathcal{K} \subseteq \mathcal{X}$ is δ-open *if* for all $x \in \mathcal{K}\exists$ open $\mathcal{N}$(s. t),$x \in \mathcal{N} \subseteq int(Cl(\mathcal{N})) \subseteq \mathcal{K}$.

Definition 2.4: [5] A subset $\mathcal{K}$ of $\mathcal{X}$ is δ-*ß*-open if $\mathcal{K} \subseteq Cl\left(Int\left(Cl_\delta(\mathcal{K})\right)\right)$. The complement of an δ-*ß*-open set is δ-*ß*-closed, the intersection of each δ-*ß*-closed containing $\mathcal{K}$ is the δ-*ß*-closure of $\mathcal{K}$ and indicated by $Cl_{\delta-ß}(\mathcal{K})$, and the union of all δ-*ß*-open sets of $\mathcal{X}$ contained in $\mathcal{K}$ is the δ-*ß*-interior of $\mathcal{K}$ and indicated by $Int_{\delta-ß}(\mathcal{K})$.

Remark 2.5: The collection of each δ-*ß*-open(*resp.*δ-*ß*-closed) subset of $\mathcal{X}$ containing $\wp \in \mathcal{X}$ is defined by $\delta-ß\Sigma(\mathcal{X},\wp)$ $(resp.\delta-ßC(\mathcal{X},\wp)$. Each δ-*ß*-open-(*resp.*δ-*ß*-closed) subset of $\mathcal{X}$ is indicated by $\delta-ß\Sigma(\mathcal{X},\mathcal{T})(resp.\delta-ßC(\mathcal{X},\mathcal{T}))$.

Proposition 2.6: [5] The following statements hold for $\mathcal{X}$.

(***a***) The union of any collection of δ-*ß*-open sets in $\mathcal{X}$ is an δ-*ß*-open set.

(***b***) The intersection of the arbitrary family of δ-*ß*-closed in $\mathcal{X}$ is an δ-*ß*-closed set.

Lemma 2.7: [5] Assume that $\mathcal{K},\mathcal{D} \subseteq \mathcal{X}$. *If* $\mathcal{K}$ is open and $\mathcal{D}$ is δ-*ß-open*, then $\mathcal{K}\cap\mathcal{D}$ is δ-*ß-open*.

Definition 2.8: [25] $\mathcal{N} \subseteq \mathcal{X}$ is called the δ-*ß*-neighborhood of $\wp \in \mathcal{X}$ if $\exists$ δ-*ß*-open $\mathcal{K}$ of $\mathcal{X}$(s.t) $\wp \in \mathcal{K} \subseteq \mathcal{N}$.

Definition 2.9: $(\mathcal{X},\mathcal{T})$ is said to be

(***a***) Nearly compact [26] if each regular open cover of $\mathcal{X}$ has a finite subcover;

(***b***) Compact (comp for short) [27] if each open cover of $\mathcal{X}$ has finite subcover;

(***c***) δ-*ß*-compact [28] if every cover of $\mathcal{X}$ by δ-*ß*-open sets has a finite subcover;

(***d***) ω-compact [29] if each ω-open cover of $\mathcal{X}$ has a finite subcover.

Definition 2.10: [30] A map $\mathcal{F}: (D, \geq) \to \mathcal{X}$ from direct set $(D, \geq)$ to $\mathcal{X} \neq \emptyset$ is on $\mathcal{X}$ and indicated via $\{\eta_\lambda\}_{\lambda\in D}\forall\ \lambda \in D\ \exists\ \eta_\lambda \in \mathcal{X} \ni \mathcal{F}(\lambda) = \eta_\lambda$.

3 Several Characterizations and Essential Properties of *ωδ-ß*-Compact Spaces

In this section, various characterizations and fundamental properties related to $\omega_{\delta\text{-}ß}$-compact spaces based on $\omega_{\delta\text{-}ß}$-open sets are obtained in topological spaces.

Definition 3.1: A subset $\mathcal{K}$ of a space $(\mathcal{X},\mathcal{T})$ is said to be $\omega_{\delta\text{-}ß}$-open if $\forall \wp \in \mathcal{K}$. There exists an a$\delta$-*ß*-open subset $\mathcal{N}_\wp \subseteq \mathcal{X}$ containing $\wp$ (s.t), where $\mathcal{N}_\wp - \mathcal{K}$ is countable.

Remark 3.2: The complement of the $\omega_{\delta\text{-}ß}$-open subset is $\omega_{\delta\text{-}ß}$-closed, and the collection of each $\omega_{\delta\text{-}ß}$-open (*resp.*$\omega_{\delta\text{-}ß}$-closed) subset of $(\mathcal{X},\mathcal{T})$ is indicated by $\omega_{\delta-ß}\Sigma(\mathcal{X},\mathcal{T})\left(resp.\,\omega_{\delta-ß}C(\mathcal{X},\mathcal{T})\right)$.

Definition 3.3: $\mathcal{F}:(\mathcal{X},\mathcal{T}) \longrightarrow (\mathcal{Y},\mathcal{T}^*)$ is said to be
(***a***) $\omega_{\delta\text{-ß}}$-open if $\mathcal{F}(\mathcal{K})$ is a $\omega_{\delta\text{-ß}}$-open set in $(\mathcal{Y},\mathcal{T}^*)$ for all open subsets $\mathcal{K}$ of $(\mathcal{X},\mathcal{T})$;
(**b**) $\omega_{\delta\text{-ß}}$-closed if $\mathcal{F}(\mathcal{K})$ is a $\omega_{\delta\text{-ß}}$-closed set of $(\mathcal{Y},\mathcal{T}^*)$ for every closed subset $\mathcal{K}$ of $(\mathcal{X},\mathcal{T})$.

Definition 3.4: Let $(\mathcal{X},\mathcal{T})$ be a topological-sp and $\mathcal{K}$ be a subset of $\mathcal{X}$. Then,
the $\omega_{\delta\text{-ß}}$-interior and $\omega_{\delta\text{-ß}}$-closure of subset $\mathcal{K}$ are denoted by $Int_{\omega_{\delta-ß}}(\mathcal{K})$ and $Cl_{\omega_{\delta-ß}}(\mathcal{K})$, respectively, and described as follows:
(**1**) $Int_{\omega_{\delta-ß}}(\mathcal{K}) = \cup\{\mathcal{B} \subseteq \mathcal{X}: \mathcal{B} \text{ is } \omega_{\delta-ß} - \text{open} \,\&\, \mathcal{B} \subseteq \mathcal{K}\}$;
(**2**) $Cl_{\omega_{\delta-ß}}(\mathcal{K}) = \cap\{\mathcal{A} \subseteq \mathcal{X}: \mathcal{A} \text{ is } \omega_{\delta-ß} - \text{closed} \,\&\, \mathcal{K} \subseteq \mathcal{A}\}$.

Definition 3.5: Subset $\mathcal{K}$ of topological-sp $(\mathcal{X},\mathcal{T})$ is called
(***a***) Regular $\omega_{\delta\text{-ß}}$-open set if $\mathcal{K}=Int_{\omega_{\delta-ß}}\left(Cl_{\omega_{\delta-ß}}(\mathcal{K})\right)$;
(**b**) Regular $\omega_{\delta\text{-ß}}$-closed set if $\mathcal{K}=Cl_{\omega_{\delta-ß}}(Int_{\omega_{\delta-ß}}(\mathcal{K}))$.

Definition 3.6: $\mathcal{F}:(\mathcal{X},\mathcal{T}) \longrightarrow (\mathcal{Y},\mathcal{T}^*)$ is $\omega_{\delta\text{-ß}}$-continuous if $\mathcal{F}^{-1}(\mathcal{O})$ is $\omega_{\delta\text{-ß}}$-open in $\mathcal{X}$ for all open $\mathcal{O}$ in $\mathcal{Y}$.

Definition 3.7: A map $\mathcal{F}:(\mathcal{X},\mathcal{T}) \longrightarrow (\mathcal{Y},\mathcal{T}^*)$ is called $\omega_{\delta\text{-ß}}$-irresolute if $\mathcal{F}^{-1}(\mathcal{O})$ is $\omega_{\delta\text{-ß}}$-open in $\mathcal{X}$ for each $\omega_{\delta\text{-ß}}$-open $\mathcal{O}$ in $\mathcal{Y}$.

Definition 3.8: A space $(\mathcal{X},\mathcal{T})$ is called $\omega_{\delta\text{-ß}}$-$\mathcal{T}_2$-space if for each distinct point in $p, q \in \mathcal{X}, \exists$ disjoint is an $\omega_{\delta\text{-ß}}$-open set $\mathcal{U},\mathcal{V}$(s. t) $p \in \mathcal{U}$& $q \in \mathcal{V}$.

Definition 3.9: A space $\mathcal{X}$ is called $\omega_{\delta\text{-ß}}$-compact if each $\omega_{\delta\text{-ß}}$-open cover of $\mathcal{X}$ has a finite subcover.

Remark 3.10:
(**1**) Each $\omega_{\delta\text{-ß}}$-comp-sp is comp.
(**2**) Each ω-comp-sp is comp, but the opposite is not necessarily true in general, as shown in the following example.

Example 3.11: Assume $\mathcal{X} = \mathbb{R}$ with $\mathcal{T} = \{\emptyset, \mathcal{X}, \mathcal{Q}, \mathcal{Q}^c\}$. $\mathcal{X}$ is comp-sp, but it is not $\omega_{\delta\text{-ß}}$-comp because the collection $\{\mathcal{Q} \cup \mathcal{X} - p \notin \mathcal{Q}\}$ is a $\omega_{\delta\text{-ß}}$-open cover of $\mathbb{R}$. Consequently, $\mathcal{X} = \mathcal{Q} \cup \mathcal{Q}^c$, but it has no finite subcover.

Definition 3.12: $(\mathcal{X},\mathcal{T})$ is called nearly $\omega_{\delta\text{-ß}}$-comp if each $\omega_{\delta\text{-ß}}$-regular open cover of $\mathcal{X}$ has a finite subcover.

Remark 3.13: δ-ß-compact does not need to be ω-compact or $\omega_{\delta\text{-ß}}$-compact, as shown in the next example.

Example 3.14: Assume that $\mathcal{X} = \mathcal{Z}$ is an integer number with $\mathcal{T} = \{= \{\emptyset, \mathcal{X}, \mathcal{Z}^+, \mathcal{Z}^-\}$. Thus, $\delta - ß\Sigma(\mathcal{X},\mathcal{T}) = \{\mathcal{K} \subseteq \mathcal{X}: 0 \notin \mathcal{K}\} \cup \{\mathcal{X}\}$. $\mathcal{X}$ is δ-ß-comp because $\omega\Sigma(\mathcal{X},\mathcal{T}) = \omega_{\delta-ß}\Sigma(\mathcal{X},\mathcal{T}) = \{\mathcal{K}: \mathcal{K} \subseteq \mathcal{X}\}$, so $\mathcal{X}$ is neither ω-comp nor $\omega_{\delta\text{-ß}}$-comp.

Remark 3.15: ω-comp does not need to be δ-ß-comp nor $\omega_{\delta\text{-ß}}$-comp, as shown in the next example.

Example 3.16: Assume that $\mathcal{D}$ is uncountable and $\mathcal{X} = \mathcal{D} \cup \{r\}, r \notin \mathcal{D}$ with $\mathcal{T} = \{\emptyset, \mathcal{X}, \{r\}\}$. Hence, $\omega\Sigma(\mathcal{X},\mathcal{T}) = \{\emptyset, \mathcal{X}, \{r\}\} \cup \{G \subseteq \mathcal{X}: G^c \text{finite}\}$. $\mathcal{X}$ is ω-comp because
$\delta - ß\Sigma(\mathcal{X},\mathcal{T}) = \omega_{\delta-ß}\Sigma(\mathcal{X},\mathcal{T}) = \{\{r, s\}: s \in \mathcal{D}\}$, so $\mathcal{X}$ is neither δ-ß-compact nor $\omega_{\delta\text{-ß}}$-compact.

Remark 3.17: From the definitions and remarks above, we derive the following implications.

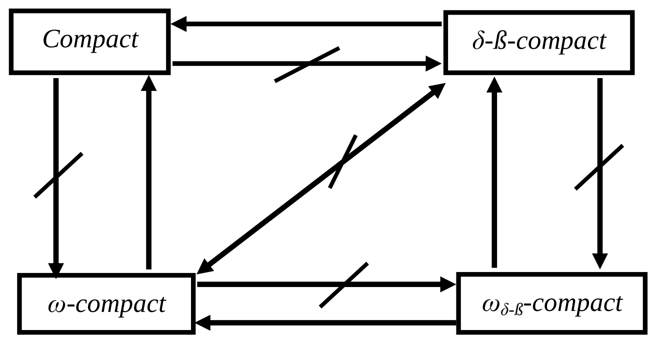

Diagram (1): Relationships among diverse kinds of compact spaces

Theorem 3.18: Let $\mathcal{F}:(\mathcal{X},\mathcal{T}) \to (\mathcal{Y},\mathcal{T}^*)$ be on a $\omega_{\delta\text{-ß}}$-continuous map. If $(\mathcal{X},\mathcal{T})$is $\omega_{\delta\text{-ß}}$-compact, then $\mathcal{Y}$ is compact.

Proof: Assume that $\{G_\alpha: \alpha \in \Delta\}$ is an open cover of $\mathcal{Y}$, so $\{\mathcal{F}^{-1}(G_\alpha): \alpha \in \Delta\}$ is an $\omega_{\delta\text{-ß}}$-open cover of $\mathcal{X}$. Given that $\mathcal{X}$ is $\omega_{\delta\text{-ß}}$-comp, $\mathcal{X}$ has a finite subcover $\{\mathcal{F}^{-1}(G_{\alpha_i}): i = 1,2,\ldots,n\}$, and $G_{\alpha_i} \in \{G_\alpha: \alpha \in \Delta\}$. Therefore, $\{G_{\alpha_i}: i = 1,2,\ldots,n\}$ is the finite subcover of $\mathcal{Y}$, and $\mathcal{Y}$ is comp.

Proposition 3.19: The following statements are equivalent for topological-sp $\mathcal{X}$.
(***a***) $\mathcal{X}$ is ωs −comp.
(***b***) For each collection of $\omega_{\delta\text{-ß}}$-closed sets $\{\mathcal{O}_\lambda: \lambda \in \mho\}$ of $\mathcal{X}$, (s. t) $\cap_{\lambda \in \mho} \mathcal{O}_\lambda = \emptyset$, there exists finite subset $\mho_\circ \subseteq \mho$ (s. t) $\cap_{\lambda \in \mho_\circ} \mathcal{O}_\lambda = \emptyset$.

Proof: (***a***) $\Longrightarrow$ (***b***): Presume that $\mathcal{X}$ is $\omega_{\delta\text{-ß}}$-comp, and $\{\mathcal{O}_\lambda: \lambda \in \mho\}$ is a collection of $\omega_{\delta\text{-ß}}$-closed subsets of $\mathcal{X}$, (s. t) $\cap_{\lambda \in \mho} \mathcal{O}_\lambda = \emptyset$. Then, the collection $\{\mathcal{X} - \mathcal{O}_\lambda: \lambda \in \mho\}$ is the $\omega_{\delta\text{-ß}}$-open cover of $\omega_{\delta\text{-ß}}$-comp $(\mathcal{X},\mathcal{T})$, and there exists a finite subset $\mho_\circ$ of $\mho$. Hence, $\mathcal{X} = \cup\{\mathcal{X} -$

$\mathcal{O}_\lambda: \lambda \in \mho_\circ\}$ and $\emptyset = \mathcal{X} - \bigcup\{\mathcal{X} - \mathcal{O}_\lambda: \lambda \in \mho_\circ\} = \bigcap\{\mathcal{X} - (\mathcal{X} - \mathcal{O}_\lambda): \lambda \in \mho_\circ\} = \bigcap\{\mathcal{O}_\lambda: \lambda \in \mho_\circ\}$.

$(\boldsymbol{b}) \Longrightarrow (\boldsymbol{a})$: Suppose that $\mathcal{N} = \{\mathcal{N}_\lambda: \lambda \in \mho\}$ is a $\omega_{\delta\text{-}\beta}$-open cover of $(\mathcal{X}, \mathcal{T})$. Thus, $\mathcal{X} - \{\mathcal{N}_\lambda: \lambda \in \mho\}$ is a collection of $\omega_{\delta\text{-}\beta}$-closed subsets of $(\mathcal{X}, \mathcal{T})$ and $\bigcap\{\mathcal{X} - \mathcal{N}_\lambda: \lambda \in \mho\} = \emptyset$ via supposition. $\exists$ is a finite subset $\mho_\circ$ of $\mho$. $\bigcap\{\mathcal{X} - \mathcal{N}_\lambda: \lambda \in \mho_\circ\} = \emptyset$, so $\mathcal{X} = \mathcal{X} - \bigcap\{\mathcal{X} - \mathcal{N}_\lambda: \lambda \in \mho_\circ\} = \bigcup\{\mathcal{N}_\lambda: \lambda \in \mho_\circ\}$, and $\mathcal{X}$ is $\omega_{\delta\text{-}\beta}$-comp.

Proposition 3.20: Let $\mathcal{F}: (\mathcal{X}, \mathcal{T}) \to (\mathcal{Y}, \mathcal{T}^*)$ be a $\omega_{\delta\text{-}\beta}$-irresolute map. $\mathcal{X}$ is $\omega_{\delta\text{-}\beta}$-comp-sp, so $\mathcal{F}(\mathcal{X})$ is $\omega_{\delta\text{-}\beta}$-comp.

Proof: Suppose that $\{\mathcal{D}_\alpha: \alpha \in \Delta\}$ is a $\omega_{\delta\text{-}\beta}$-open cover of $\mathcal{F}(\mathcal{X})$, and $\mathcal{F}(\mathcal{X}) \subseteq \bigcup_{\alpha\in\Delta} \mathcal{D}_\alpha$ (s. t) $\mathcal{F}^{-1}(\mathcal{F}(\mathcal{X})) \subseteq \mathcal{F}^{-1}(\bigcup_{\alpha\in\Delta} \mathcal{D}_\alpha) = \bigcup_{\alpha\in\Delta} \mathcal{F}^{-1}(\mathcal{D}_\alpha) \subseteq \mathcal{X}$. Thus, $\mathcal{X} = \bigcup_{\alpha\in\Delta} \mathcal{F}^{-1}(\mathcal{D}_\alpha)$ because $\mathcal{D}_\alpha$ is a $\omega_{\delta\text{-}\beta}$-open set in $\mathcal{Y}, \forall \alpha \in \Delta$. Given that $\mathcal{F}$ is $\omega_{\delta\text{-}\beta}$-irresolute, $\mathcal{F}^{-1}(\mathcal{D}_\alpha)$ is a $\omega_{\delta\text{-}\beta}$-open set in $\mathcal{X}, \forall \alpha \in \Delta$. $\{\mathcal{F}^{-1}(\mathcal{D}_\alpha): \alpha \in \Delta\}$ is the $\omega_{\delta\text{-}\beta}$-open cover of $\mathcal{X}$ because $\mathcal{X}$ is a $\omega_{\delta\text{-}\beta}$-comp space $\exists\, \alpha_1, \alpha_2, \dots, \alpha_n \in \Delta$ (s. t) $\mathcal{X} = \bigcup_{i=1}^n \mathcal{F}^{-1}(\mathcal{D}_{\alpha_i})$, $\mathcal{F}(\mathcal{X}) = \bigcup_{i=1}^n \mathcal{F}(\mathcal{F}^{-1}(\mathcal{D}_{\alpha_i})) \subseteq \bigcup_{i=1}^n \mathcal{D}_{\alpha_i}$. Consequently, $\mathcal{F}(\mathcal{X})$ is $\omega_{\delta\text{-}\beta}$-comp.

Definition 3.21: A subset $\mathcal{D}$ of $\mathcal{X}$ is called $\omega_{\delta\text{-}\beta}$-comp relative to $\mathcal{X}$ if every cover of $\mathcal{D}$ via $\omega_{\delta\text{-}\beta}$-open sets has a finite subcover of $\mathcal{D}$. The subset $\mathcal{D}$ is $\omega_{\delta\text{-}\beta}$-comp if it is $\omega_{\delta\text{-}\beta}$-comp as a subspace.

Theorem 3.22: The following statements are equivalent for $\mathcal{X}$.

$(\boldsymbol{a})$ $\mathcal{X}$ is $\omega_{\delta\text{-}\beta}$-comp.

$(\boldsymbol{b})$ For any collection $\mathcal{J}$ of $\omega_{\delta\text{-}\beta}$-open sets, if no finite subcollection of $\mathcal{J}$ covers $\mathcal{X}$, $\mathcal{J}$ does not cover $\mathcal{X}$.

$(\boldsymbol{c})$ For any collection $\mathcal{J}$ of $\omega_{\delta\text{-}\beta}$-closed sets, if $\mathcal{J}$ satisfies the finite intersection condition, then $\bigcap\{\mathcal{K}: \mathcal{K} \in \mathcal{J}\} \neq \emptyset$.

$(\boldsymbol{d})$ For any collection $\mathcal{J}$ of subsets of $\mathcal{X}$, if $\mathcal{J}$ satisfies the finite intersection condition, then $\bigcap\{Cl_{\omega_{\delta-\beta}}(\mathcal{K}): \mathcal{K} \in \mathcal{J}\} \neq \emptyset$.

Proof: $(\boldsymbol{a}) \Longrightarrow (\boldsymbol{b})$ and $(\boldsymbol{b}) \Longrightarrow (\boldsymbol{a})$ are apparent. Now, $(\boldsymbol{c}) \Longrightarrow (\boldsymbol{d})$. If $\mathcal{J} \subset P(\mathcal{X})$ satisfies the finite intersection condition, then $\bigcap\{Cl_{\omega_{\delta-\beta}}(\mathcal{K}): \mathcal{K} \in \mathcal{J}\}$ is a collection of $\omega_{\delta\text{-}\beta}$-closed sets that perceptibly satisfies the finite intersection condition.

$(\boldsymbol{d}) \Longrightarrow (\boldsymbol{c})$: $\mathcal{K} = Cl_{\omega_{\delta-\beta}}(\mathcal{K})$ for every $\omega_{\delta\text{-}\beta}$-closed set $\mathcal{K}$.

Proposition 3.23: If $(\mathcal{Y}, \mathcal{T}^*)$ is a $\omega_{\delta\text{-}\beta}$-open subspace of $\mathcal{X}$ and $\mathcal{D} \subseteq \mathcal{Y}$, then $\mathcal{D}$ is a $\omega_{\delta\text{-}\beta}$-comp set in $\mathcal{Y}$ iff $\mathcal{D}$ is $\omega_{\delta\text{-}\beta}$-comp in $\mathcal{X}$.

Proof: Assume that $\mathcal{D}$ is a $\omega_{\delta\text{-}\beta}$-comp set in $\mathcal{Y}$, with $\{\mathcal{O}_\lambda: \lambda \in \mho\}$ being a $\omega_{\delta\text{-}\beta}$-open cover of $\mathcal{D}$ in $\mathcal{X}$. Then, $\mathcal{D} \subseteq \bigcup_{\lambda\in\mho} \mathcal{O}_\lambda$ because $\mathcal{D} \subseteq \mathcal{Y}, \mathcal{D} \subseteq \bigcup\{\mathcal{Y} \cap \mathcal{O}_\lambda: \lambda \in \mho\}$. Given that $\mathcal{Y} \cap \mathcal{O}_\lambda$ is $\omega_{\delta\text{-}\beta}$-open relative to $\mathcal{Y}$, $\{\mathcal{Y} \cap \mathcal{O}_\lambda: \lambda \in \mho\}$ is a $\omega_{\delta\text{-}\beta}$-open cover of $\mathcal{D}$ relative to $\mathcal{Y}$. We have $\mathcal{D} \subseteq (\mathcal{Y} \cap \mathcal{O}_\lambda) \cup \dots \cup (\mathcal{Y} \cap \mathcal{O}_{\lambda_n})$, so $\mathcal{D}$ is $\omega_{\delta\text{-}\beta}$-comp in $\mathcal{X}$.

Conversely: Presume that $\mathcal{D}$ is a $\omega_{\delta\text{-}\beta}$-comp set in $\mathcal{X}$, with $\{\mathcal{N}_\lambda: \lambda \in \mho\}$ being a $\omega_{\delta\text{-}\beta}$-open cover of $\mathcal{D}$ in $\mathcal{Y}$. Then, $\mathcal{D} \subseteq \bigcup_{\lambda\in\mho} \mathcal{N}_\lambda$ as a result $\exists\, \mathcal{O}_\lambda$ being $\omega_{\delta\text{-}\beta}$-open relative to $\mathcal{X}$, (s. t) $\mathcal{N}_\lambda = \mathcal{Y} \cap \mathcal{O}_\lambda, \forall \lambda \in \mho$. Thus, $\mathcal{D} \subseteq \bigcup_{\lambda\in\mho} \mathcal{O}_\lambda$, where $\{\mathcal{O}_\lambda: \lambda \in \mho\}$ is a $\omega_{\delta\text{-}\beta}$-open-cover of $\mathcal{D}$ relative to $\mathcal{X}$ because $\mathcal{D}$ is $\omega_{\delta\text{-}\beta}$-comp in $\mathcal{X}, \exists \lambda_1, \lambda_2, \dots, \lambda_n \in \mho$ (s. t) $\exists \mathcal{D} \subseteq \bigcup_{i=1}^n \mathcal{O}_{\lambda_i}$. Given that $\mathcal{D} \subseteq \mathcal{Y}, \mathcal{D} \subseteq \mathcal{Y} \cap \{\mathcal{O}_{\lambda_1} \cup \mathcal{O}_{\lambda_2}, \dots, \cup \mathcal{O}_{\lambda_n}\} = (\mathcal{Y} \cap \mathcal{O}_{\lambda_1}) \cup, \dots, \cup (\mathcal{Y} \cap \mathcal{O}_{\lambda_n})$ and $\mathcal{Y} \cap \mathcal{O}_{\lambda_i} = \mathcal{N}_i$, $\mathcal{D}$ is $\omega_{\delta\text{-}\beta}$-comp in $\mathcal{Y}$.

Theorem 3.24: The following statements are equivalent for $\mathcal{X}$.

$(\boldsymbol{a})$ $\mathcal{X}$ is nearly $\omega_{\delta\text{-}\beta}$-comp-sp.

$(\boldsymbol{b})$ Each $\omega_{\delta\text{-}\beta}$-open cover $\mu = \{\mathcal{O}_\lambda: \lambda \in \mho\}$ of $\mathcal{X}, \exists$ finite subset $\mho_\circ \subseteq \mho$ (s. t) $\mathcal{X} = \bigcup_{\lambda\in\mho_\circ} Int_{\omega_{\delta-\beta}}(Cl_{\omega_{\delta-\beta}}(\mathcal{N}_\lambda))$.

Proof: $(\boldsymbol{a}) \Longrightarrow (\boldsymbol{b})$ Let $\mu = \{\mathcal{O}_\lambda: \lambda \in \mho\}$ be a $\omega_{\delta\text{-}\beta}$-open cover of $\mathcal{X}$. Hence, $\{Int_{\omega_{\delta-\beta}}(Cl_{\omega_{\delta-\beta}}(\mathcal{O}_\lambda)): \lambda \in \mho\}$ is the $\omega_{\delta\text{-}\beta}$-regular open cover of nearly $\omega_{\delta\text{-}\beta}$-comp-sp $\mathcal{X}$, and there exists a finite subset $\mho_\circ \subseteq \mho$ (s. t) $\mathcal{X} = \bigcup_{\lambda\in\mho_\circ} Int_{\omega_{\delta-\beta}}(Cl_{\omega_{\delta-\beta}}(\mathcal{O}_\lambda))$.

$(\boldsymbol{b}) \Longrightarrow (\boldsymbol{a})$: $\omega_{\delta\text{-}\beta}$-regular open is $\omega_{\delta\text{-}\beta}$-open.

Definition 3.25: A point $\wp \in \mathcal{X}$ is called the $\omega_{\delta\text{-}\beta}$-cluster point of a net $\{\eta_\lambda\}_{\lambda\in\Delta}$ and frequently exists in each $\omega_{\delta\text{-}\beta}$-open set containing $\wp$. We indicate via $\omega_{\delta\text{-}\beta}$-CP$\{\eta_\lambda\}_{\lambda\in\Delta}$ the set of each $\omega_{\delta\text{-}\beta}$-cluster point of $\{\eta_\lambda\}_{\lambda\in\Delta}$.

Theorem 3.26: A space $\mathcal{X}$ is $\omega_{\delta\text{-}\beta}$-comp if and only if every $\{\eta_\lambda\}_{\lambda\in\Delta}$ in $\mathcal{X}$ has at least one $\omega_{\delta\text{-}\beta}$-cluster point.

Proof: Assume that $\mathcal{X}$ is $\omega_{\delta\text{-}\beta}$-comp-sp and $\exists$ some net $\{\eta_\lambda\}_{\lambda\in\Delta}$, (s. t) $\omega_{\delta\text{-}\beta}$-CP$\{\eta_\lambda\}_{\lambda\in\Delta}$ is empty. Let $\wp \in \mathcal{X}$, so $\exists\, G(\wp) \in \omega_{\delta-\beta}\Sigma(\mathcal{X}, \wp)$ is not frequent. As a result, $\exists\, \lambda(\wp) \in \Delta$, (s. t) $\wp_\delta \notin G(\wp)$. When $\delta \geq \lambda(\wp), \delta \in \Delta$, the collection $\{G(\wp): \wp \in \mathcal{X}\}$ is a cover of $\mathcal{X}$ via $\omega_{\delta\text{-}\beta}$-open sets and has a finite subcover. $\{G_k: k = 1,2, \dots, n\}$ (s. t), $G_k = G(\wp_k)$ for $k = 1,2, \dots, n, \{\wp_k: k =$

$1,2,\ldots,n$ and $\gamma \in \Delta$. Thus, $\lambda \geq \lambda \wp k, \forall k = 1,2,\ldots,\gamma$, for each $\delta \in \Delta$ (s. t) $\delta \geq \gamma$. We have $\wp\delta \notin G\wp, k = 1,2,\ldots,n$, so $\wp_\delta \notin \mathcal{X}$ is a contradiction.

Conversely: Suppose that $\mathcal{X}$ is not $\omega_{\delta\text{-}\beta}$-comp. Then, $\exists \{G_i : i \in \Delta\}$ is the cover of $\mathcal{X}$ via $\omega_{\delta\text{-}\beta}$-open sets and has no finite subcover. Assume that $P(\Delta)$, the collection of every finite subset of Δ with obvious $(P(\Delta), \subseteq)$, is a directed set $\forall j \in$ I. We select $\wp_j \in \mathcal{X} - \cup\{G_i : i \in \mathrm{I}\}$ and consider $\{\eta_j\}_{j \in P(\Delta)}$ via supposition of $\omega_{\delta\text{-}\beta}$-CP$\{\eta_j\}_{j \in P(\Delta)}$ to be nonempty. We presume $\wp \in \omega_{\delta\text{-}\beta}$-CP$\{\eta_j\}_{j \in P(\Delta)}$ and let $i_\circ \in \Delta$. Thus, $\wp \in G_{i_\circ}$ via the definition of the $\omega_{\delta\text{-}\beta}$-cluster point $\forall j \in P(\Delta)$. $\exists j^* \in P(\Delta)$ (s.t) $j \subseteq j^*$ and $\wp_{j^*} \in G_{i_\circ}$ for $j = \{i_\circ\}, \exists j^* \in P(\Delta)$ (s. t) $i_\circ \in j^*$ with $\wp_{j^*} \in G_{i_0}$, but $\wp_{j^*} \in \mathcal{X} - \cup\{G_i : i \in j^*\} \subset \mathcal{X} - G_{i_0}$ is a contradiction. Therefore, $\mathcal{X}$ is $\omega_{\delta\text{-}\beta}$-comp.

Next, we provide characterizations of $\omega_{\delta\text{-}\beta}$-comp by means of filter bases. We recall a nonempty collection $\mathfrak{F}$ of subsets of $\mathcal{X}$ (called a filter base on $\mathcal{X}$) if $\emptyset \notin \mathfrak{F}$ and each intersection of two members of $\mathfrak{F}$ contains a third member of $\mathfrak{F}$. Each chain in the family of every filter base on $\mathcal{X}$ has an upper bound. The union of all members of the chain is denoted by Zorn's lemma, and the collection of filter bases on $\mathcal{X}$ has at least one maximal element. Likewise, the collection of filter bases on $\mathcal{X}$ containing a given filter base $\mathfrak{F}$ has at least one maximal element.

Definition 3.27: A filter base $\mathfrak{F}$ on $(\mathcal{X}, \mathcal{T})$ is called

(***a***) $\omega_{\delta\text{-}\beta}$-converge to $\wp \in \mathcal{X}$ iff every $\omega_{\delta\text{-}\beta}$-open $\mathcal{N}$ contains $\wp, \exists \mathcal{D} \in \mathfrak{F}$ (s. t) $\mathcal{D} \subset \mathcal{N}$.

(***b***) $\omega_{\delta\text{-}\beta}$-accumulate at $\wp \in \mathcal{X}$ if $\mathcal{N} \cap \mathcal{D} \neq \emptyset, \forall$ $\omega_{\delta\text{-}\beta}$-open set $\mathcal{N}$ contains $\wp$ with $\forall \mathcal{D} \in \mathfrak{F}$.

Proposition 3.28: Let maximal filter base $\mathfrak{F}$ be $\omega_{\delta\text{-}\beta}$-accumulate at $\wp \in \mathcal{X}$. Then, $\mathfrak{F}$ $\omega_{\delta\text{-}\beta}$-converge to $\wp$.

Proof: Assume that $\mathfrak{F}$ is a maximal filter base with $\omega_{\delta\text{-}\beta}$-accumulate at $\wp \in \mathcal{X}$. If $\mathfrak{F}$ is not $\omega_{\delta\text{-}\beta}$-converge to $\wp$, there exists $\omega_{\delta\text{-}\beta}$-open set $\mathcal{N}_\circ$ containing $\wp$, (s. t) $\mathcal{N}_\circ \cap \mathcal{D} \neq \emptyset$ & $(\mathcal{X} - \mathcal{N}_\circ) \cap \mathcal{D} \neq \emptyset, \forall \mathcal{D} \in \mathfrak{F}$. Thus, $\mathfrak{F} \cup \{\mathcal{N}_\circ \cap \mathcal{D} : \mathcal{D} \in \mathfrak{F}\}$ is a filter base that contains $\mathfrak{F}$, which is a contradiction.

Theorem 3.29: The following statements are equivalent for $\mathcal{X}$.

(***a***) $\mathcal{X}$ is $\omega_{\delta\text{-}\beta}$-comp.

(***b***) Each maximal filter base is $\omega_{\delta\text{-}\beta}$-converge to some points of $\mathcal{X}$.

(***c***) Each filter base is $\omega_{\delta\text{-}\beta}$-accumulate at some points of $\mathcal{X}$.

Proof: (***a***) $\Longrightarrow$ (***b***) Assume that $\mathfrak{F}_\circ$ is a maximal filter base on $\mathcal{X}$ and $\mathfrak{F}_\circ$ is not $\omega_{\delta\text{-}\beta}$-converge to an arbitrary point of $\mathcal{X}$. Then, on the basis of Proposition 3.28, $\mathfrak{F}_\circ$ is not $\omega_{\delta\text{-}\beta}$-accumulate at an arbitrary point of $\mathcal{X}, \forall$ $\wp \in \mathcal{X}$, so $\exists$ is $\omega_{\delta\text{-}\beta}$-open set $\mathcal{N}_\wp$ containing $\wp$ with $\mathcal{D}_\wp \in \mathfrak{F}_\circ$. Thus, $\mathcal{N}_\wp \cap \mathcal{D}_\wp = \emptyset$, and the collection $\{\mathcal{N}_\wp : \wp \in \mathcal{X}\}$ is the cover of $\mathcal{X}$ via $\omega_{\delta\text{-}\beta}$-open and via (***a***) $\exists$ finite subset $\{\wp_1, \wp_2, \ldots, \wp_n\}$ of $\mathcal{X}$. Therefore, $\mathcal{X} = \cup\{\mathcal{N}_{\wp_k} : k = 1,2,\ldots\}$. Given that $\mathfrak{F}_\circ$ is a filter base, $\exists \mathcal{D}_\circ \subset \mathfrak{F}_\circ$ (s. t) $\mathcal{D}_\circ \subset \cap\{\mathcal{D}_{\wp_k} : k = 1,2,\ldots,n\} = \mathcal{X} - \cup\{\mathcal{N}_{\wp_k} : k = 1,2,\ldots\}$, so $\mathcal{D}_\circ = \emptyset$ is a contradiction.

(***b***) $\Longrightarrow$ (***c***) Assume that $\mathfrak{F}$ is a filter base on $\mathcal{X}$, and there exists a maximal filter base $\mathfrak{F}_\circ$. Thus, $\mathfrak{F} \subset \mathfrak{F}_\circ$ via (b). $\mathfrak{F}_\circ$ is $\omega_{\delta\text{-}\beta}$-converge to some point $\wp_\circ \in \mathcal{X}$. Assume that $\mathcal{D} \in \mathfrak{F}\, \forall \mathcal{N} \in \delta - \beta\Sigma(\mathcal{X}, \wp)$. As a result, $\exists \mathcal{D}_\mathcal{N} \in \mathfrak{F}_\circ$ (s. t) $\mathcal{D}_\mathcal{N} \subset \mathcal{N}$, so $\mathcal{N} \cap \mathcal{D} \neq \emptyset$. It contains the member $\mathcal{D}_\mathcal{N} \cap \mathcal{D}$ of $\mathfrak{F}_\circ$, so $\mathfrak{F}$ $\omega_{\delta\text{-}\beta}$-accumulates at $\wp_\circ$.

(***c***) $\Longrightarrow$ (***a***) Suppose that $\{\mathcal{O}_i : i \in \Delta\} = \emptyset$ is an arbitrary collection of $\omega_{\delta\text{-}\beta}$-closed sets (s. t) $\cap\{\mathcal{O}_i : i \in \Delta\} = \emptyset$. We establish that there exists a finite subset Δ_0 of Δ, so $\cap\{\mathcal{O}_i : i \in \Delta\}$ via Theorem (3.22) (***a***). Assume that $P(\Delta)$ is the collection of finite subsets of Δ and presume that $\cap\{\mathcal{O}_i : i \in \mathcal{J}\} = \emptyset$ for each $\mathcal{J} \in P(\Delta)$….**. Hence, $\mathfrak{F} = \{\cap\{\mathcal{O}_i : i \in \mathcal{J}\} : \mathcal{J} \in P(\Delta)\}$ is a filter base on $\mathcal{X}$ via (***c***), and $\mathfrak{F}$ is $\omega_{\delta\text{-}\beta}$-accumulate to some point $\wp_\circ \in \mathcal{X}$. Given that $\{\mathcal{X} - \mathcal{O}_i : i \in \Delta\}$ is a cover of $\mathcal{X}, \exists i_\circ \in \Delta$, then $\wp_\circ \in \mathcal{X} - \mathcal{O}_{i_\circ}$, (s. t) $\mathcal{X} - \mathcal{O}_{i_\circ}$ is $\omega_{\delta\text{-}\beta}$-open and contains $\wp_\circ, \mathcal{O}_{i_\circ} \in \mathfrak{F}$ with $(\mathcal{X} - \mathcal{O}_{i_\circ}) \cap \mathcal{O}_{i_\circ} = \emptyset$, which is in contradiction with the truth. $\mathfrak{F}$ being $\omega_{\delta\text{-}\beta}$-accumulate at $\wp_\circ$ indicates that (**) is untrue.

Conclusion

The concept of compactness is of paramount importance in mathematics and other sciences. Therefore, new classes of generalized compact spaces, namely, $\omega_{\delta\text{-}\beta}$-compact and nearly $\omega_{\delta\text{-}\beta}$-compact spaces, based on other generalized ω-open sets are investigated in this study. Several essential characterizations related to these kinds of generalized compact spaces are discussed. We refer readers to [31, 32].

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أصناف أخرى من الفضاءات المتراصة المعممة بناءً على مجموعات مفتوحة أوميكا معممة جديدة

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الخلاصة:

يعد مفهوم التراص المعمم مفهومًا مهما وأساسيًا ليس فقط في التبولوجيا العامة ولكن أيضًا في فروع الرياضيات البحته والتطبيقية المتقدمة الأخرى. كذلك يمكن للمرء ملاحظة تأثير الفضاءات التبولوجية العامة أيضًا في علوم الكمبيوتر والتبولوجيا الرقمية، التبولوجيا الحسابية لفيزياء الجسيمات ذات التصميم الهندسي والجزيئي، وفيزياء الطاقة العالية، وفيزياء الكم، ونظرية الأوتار الفائقة. لذلك، تم تقديم مفاهيم جديدة للفضاءات المتراصة المعممة، وهي الفضاء المتراصة-$\omega_{\delta\text{-}\beta}$ والفضاء المتراصة تقريبًا-$\omega_{\delta\text{-}\beta}$ بناءً على مجموعات ω-مفتوحة معممة جديدة في الفضاءات التبولوجية. في هذه الورقة، تم دراسة الخصائص الأساسية المختلفة المتعلقة بهذه الأنواع من الفضاءات المتراصة المعممة في الفضاءات التبولوجية. علاوة على ذلك، فقد تمت مناقشة العلاقات بين بعض أنواع الفضاءات المتراصة المعممة في الفضاءات التبولوجية. كما تم تقديم بعض الأمثلة التوضيحية لتسليط الضوء على النتائج المقدمة في هذه الورقة. ونتوقع أن تساعد الاكتشافات الواردة في هذا المقال العلماء في تطوير الأبحاث المتعلقة بالتبولوجيا العامة من أجل الارتقاء بإطار عام لتطبيقاتها العملية في جميع الفروع المتقدمة للرياضيات والعلوم الأخرى.

الكلمات المفتاحية: مجموعات مفتوحة-$\delta\text{-}\beta$ ، مجموعات مفتوحة-ω، متراصة-$\omega_{\delta\text{-}\beta}$، الفضاءات المتراصة- $\omega_{\delta\text{-}\beta}$-تقريبا.