

# PERFORMANCE EVALUATION OF ORTHOGONAL FREQUENCY DIVISION MULTIPLEXING OVER FREQUENCY SELECTIVE RAYLEIGH FADING HF CHANNEL

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## Abstract

The effectiveness of Orthogonal Frequency Division Multiplexing (OFDM) with  $\pi/4$ -DQPSK modulation is investigated, for communication over HF channel. The Fast Fourier Transform (FFT) is used to satisfy the orthogonality between subcarriers and reduce the implementation complexity. The system performance is investigated for various delay spreads, coherence time and number of propagation paths. A guard time with cyclic extension is used to reduce channel dispersion, intersymbol and intercarrier interference. Clipping is used to minimize the high peak-to-average power (PAP) ratio.

## الخلاصة

في هذا البحث تم التحقق من فعالية منظومة التعدد التقسيمي الترددي المتعامد (OFDM) مع استخدام تقنية التضمين ( $\pi/4 - \text{DQPSK}$ ) للاتصال عبر قناة متغيرة مع الزمن مثل قناة الراديوية عالية التردد (HF Channel). استخدم تحويل فوريير السريع (FFT) لتحقيق التعمد بين الحاملات الثانوية وتقليل التعقيد في بناء المنظومة. تم التحقق من أداء المنظومة بقيم مختلفة من تأخير الانتشار (delay spread)، زمن التشاكة (Coherence time) وعدد المسارات. تم استخدام الفترة الحارسة (guard time) مع الامتداد الدوري (cyclic extension) لتقليل التداخل بين الرموز والحاملات. تم استخدام طريقة القطع (clipping method) لتقليل نسبة القدرة للقيمة - إلى - المعدل (PAP) العالية.

## 1 Introduction

Orthogonal Frequency Division Multiplexing (OFDM) splits a high-rate data stream into a number of lower rate streams that are transmitted simultaneously over a number of overlapped subcarriers. These subcarriers are modulated with subcarriers spacing, which is selected such that modulated subcarriers are orthogonal over symbol duration. Increasing symbol duration will result in a lower rate parallel subcarriers. This decreases the relative amount of dispersion in time caused by multipath delay spread. Intersymbol Interference (ISI) is eliminated almost completely by introducing a guard time in every

OFDM symbol. In guard time, the OFDM symbol is cyclically extended to combat the frequency-selective of the channel and to avoid Intercarrier Interference (ICI).

## 2 Orthogonal Frequency Division Multiplexing (OFDM)

Orthogonal Frequency Division Multiplexing (OFDM) system may be viewed as a conventional FDM system, as shown in Fig.(1-a). In this arrangement, the spectra of different subchannels do not overlap. In such a system, there is a sufficient guard space between adjacent subchannels to isolate them at the receiver using the

conventional filters. This arrangement does not achieve effective use of bandwidth. A much more efficient use of bandwidth can be obtained with a parallel system if the spectra of the individual subchannels are permitted to overlap, as shown in Fig.(1-b). With the addition of coherent detection and the use of subcarrier separated by the reciprocal of the signal element duration, independent separation of the multiplexed subcarriers is possible. If this condition is satisfied then this OFDM system will achieve orthogonality among its consistent subcarriers<sup>[1,2]</sup>.

The transmitted OFDM symbol waveform can be represented as:

$$S(t) = \text{Re} \left\{ \sum_{k=0}^{N-1} d(k) \exp(j2\pi f_k t) \right\}$$

(1)

where

$d(k)$  is the modulated data symbol  
 $f_k$  is subcarrier frequency of  $k^{\text{th}}$  subcarrier which is equal to  $(f_c + k\Delta f)$ .

$f_c$  is the carrier frequency

$\Delta f$  is subcarrier spacing (bandwidth) equal to  $(1/NT)$ .

$T$  is the symbol time duration.

$N$  is the subcarriers number.

This expression represents the passband OFDM signal. The equivalent complex baseband notation is given by :

$$S(t) = \sum_{k=0}^{N-1} d(k) \exp(j2\pi k\Delta f t)$$

(2)

Equation (2) represents the general form of complex baseband OFDM signal.

### **3 Implementation of OFDM Using Fast Fourier Transform(FFT)**

The main objections to the use of parallel systems are the complexity of the equipment required to implement the parallel system and the possibility of severe mutual interference among subchannels when the transmission medium distorts the signal. System design is greatly reduced by eliminating any pulse shaping and demodulated by using the Discrete Fourier Transform (DFT) to implement the modulation processes,<sup>[1,2,3,4]</sup>.

If the signal is sampled at a rate of  $(T)$ , then  $(t=nT)$ , and for orthogonality  $\Delta f = 1/NT$ , then Eq.(2) can be rewritten as

$$S(n) = \sum_{k=0}^{N-1} d(k) \exp(j2\pi kn/N)$$

(3)

Equation.(3) is exactly the Inverse Discrete Fourier Transform (IDFT) of the data sequence  $d(k)$ . All operations that occur in the transmitter are reversed in the receiver<sup>[1,2]</sup>. Further reductions in complexity are possible by using the Fast Fourier Transform (FFT) algorithm to implement the DFT. To eliminate the Intersymbol Interference (ISI) almost completely, a guard time is introduced for each OFDM symbol. The guard time is chosen larger than the expected delay spread such that multipath components from one symbol cannot interfere with the next symbol. The guard time could consist of no signal at all<sup>[3,5]</sup>. However, the problem of Intercarrier Interference (ICI) would arise. ICI is a crosstalk between different subcarriers, which means that they are no longer orthogonal. To eliminate ICI, the OFDM symbol is cyclically extended in the guard time, which is done by taking symbol period samples from the end of OFDM symbol and appending them to the start of

OFDM symbol [2,4]. Figure.(2) shows the OFDM system model used in this paper.

#### **4 Peak to Average Power (PAP) Ratio Problem**

The PAP ratio, is defined as the maximum signal value divided by r.m.s Signal value [2,6]. Considering the OFDM baseband signal  $S(t)$ , if the peak and rms values of this signal are denoted by  $S_{max}$  and  $S_{r.m.s}$  respectively, then:

$$PAP = \frac{S_{max}}{S_{r.m.s}} \quad (4)$$

An OFDM signal consists of a number of independently modulated subcarriers, which give a large peak-to-average (PAP) ratio when added up coherently. When  $N$  signals are added with the same phase, they produce a peak power that is  $N$  times the average power [2]. This effect is illustrated in Fig.(3). A large PAP ratio brings disadvantages like an increased complexity of the analog-to-digital and digital-to-analog converters and a reduced efficiency of the RF power amplifier [2,6].

To reduce the PAP ratio is to clip the signal, such that the peak amplitude becomes limited to some desired maximum level. This technique is performed by passing the OFDM modems output baseband signal,  $S(t)$ , through a digital limiting device prior to transmitter stage [6,7], as shown in Fig.(4).

There are a few problems associated with it. First by distorting the OFDM signal amplitude, a kind of self-interference is introduced that degrades the BER (some information is lost in the OFDM signal). Second, the nonlinear distortion of the OFDM signal

significantly increases the level of the out-of-band radiation [2].

The first problem can be solved by optimum choice of clipping level for each designed system. This is made depending on the analysis of system performance at different clipping levels, such that the minimum PAP ratio with good BER performance can be achieved.

To solve the out-of-band problem of clipping, large signal peaks are multiplied with a certain nonrectangular window. To minimize the out-of-band interference, ideally the window should be as narrowband as possible [2]. On the other hand, the window should not be too long in time domain, because using long time window will affect many signal samples, which increase the BER. Examples of suitable window functions are the cosine, Kaiser, and Hamming windows [2].

#### **5 Performance Analysis Of The OFDM System:**

Figure.(5) illustrates the bit error rate (BER) versus signal to noise ratio for OFDM signal having bit rate  $R=4800$  bit/sec and subcarriers number  $N=128$ , transmitted in AWGN channel, number of transmission bits is about 50000 bits. The first curve is for OFDM system with differential detection by using  $(\pi/4)$ -DQPSK modulation scheme, while the second curve is for OFDM system with coherent detection by using (QPSK) modulation scheme. In this figure, the amount of SNR required to obtain BER of  $10^{-4}$  for  $(\pi/4)$ -DQPSK is larger than that of QPSK by about 2 dB. The noise performance and the SNR loss of  $\pi/4$ -DQPSK are not important factors compared with receiver complexity introduced by coherent detection

requirement of the QPSK system. The OFDM/QPSK system without using channel estimation does not give a useful channel (BER is closed to 0.5), but the performance of OFDM/ ( $\pi/4$ -DQPSK) system is improved when SNR is increased. In consequence, the performance of ( $\pi/4$ -DQPSK) modulation scheme is much better than the performance of QPSK modulation scheme without using channel estimation in HF channel.

A comparison between the (BER) performance of OFDM system and the performance of single carrier system is illustrated in Fig.(6). The ( $\pi/4$ -DQPSK) modulation scheme is used in these two systems. Both systems operate at bit rate  $R=1200$  bit/sec, transmitted in HF channel having channel coherence time  $(\Delta t)_c=1$  sec and channel delay spread  $T_d=1$  msec; Number of transmission bits is 50000 bits. At all values of SNR (0-20) dB we observe that the performance of OFDM system is better than the performance of single carrier system. In fact, the OFDM system splits the serial data stream into  $N$  ( $N=128$ ) parallel lower data streams, then the symbol duration is increased by a factor of  $N$ . The ISI is eliminated completely since the delay spread is now less than the symbol duration. In other words, the frequency selective fading of the channel will be converted to flat fading, where each symbol is only affected by fluctuation of amplitude and phase.

The effect of the guard time with cyclic extension is tested on the OFDM system performance obtained in the HF channel with channel coherence time  $(\Delta t)_c=5$  sec and delay spread  $T_d=3$  msec (Moderate conditions). Figure.(7) illustrates the BER versus SNR for

different values of guard time represented by four cases. Case-1 represents the no guard time case ( $T_g=0$ ), case-2 represents the case when the guard time ( $T_g=1$  msec) is less than the delay spread ( $T_d=3$  msec), case-3 represents the case when  $T_g = T_d = 3$  msec, and finally case-4 represents the case when the guard time is greater than the delay spread ( $T_g=5$  msec). At all values of SNR from (0-20) dB, we observe the OFDM signal without any guard time has higher BER compared with OFDM signal with guard time, and the performance is improved when guard time is increased. The best performance is in case-4 when the guard time is greater than the delay spread of the channel because the effect of channel dispersion is reduced by increasing the guard time, so that ISI is eliminated almost completely.

To have a better understanding on the minimum requirement of the guard time based on the maximum delay spread expected on the HF channel, Figure.(8) shows the bit error rate (BER) performance versus the ratio ( $T_g/T_d$ ) at SNR=15 dB, bit rate  $R=4800$  bits/sec, coherence time  $(\Delta t)_c=5$  sec and delay spread  $T_d=3$  msec. From this figure we observe that as this ratio is increased, the effect of delay spread decreases and the BER decreases quite significantly. However, further addition of the guard time will no longer decrease the BER so that, the best performance is achieved when the guard time is almost twice the delay spread ( $T_g \approx 2T_d$ ). Any further increase of guard time will decrease the throughput of the system without any further improvement to the BER.

To illustrate the effect of increasing subcarriers number ( $N$ ) on the OFDM system performance, the BER for the

( $\pi/4$ -DQPSK-OFDM) system operates at bit rate  $R=4800$  bit/sec in the HF channel having channel coherence time  $(\Delta t)_c=5$  sec and channel delay spread  $T_d=3$ msec (Moderate conditions) are described for various number of subcarriers  $N$ . Number of transmitted bits is 100000 bits. Figure.(9) shows simulation results for various values of  $N$  where two extreme cases are considered. When no guard time is added ( $T_g=0$ ) and when the guard time with cyclic extension eliminates the ISI perfectly ( $T_g=5$  msec). From this figure, we observe that, without guard time, the BER performance of OFDM system is not further improved when  $N$  is increased. As  $N$  increases, the frequency separation between the adjacent subcarriers decreases, so that the OFDM signal becomes more sensitive to frequency offset and small frequency shift will cause ICI. Hence, when  $N$  increases the effect of ISI is decreased, but the effect of ICI is increased. With guard time and cyclic extension ( $T_g>T_d$ ), the BER performance is further improved as  $N$  increased, because the ISI is eliminated completely and ICI is further reduced by using cyclic extension.

To determine an optimum number of subcarriers that gives minimum ICI with good BER performance and low equipment complexity, Fig(10) illustrates BER versus SNR for various number of subcarriers of OFDM system which operates at bit rate  $R=4800$  bit/sec and has guard time with cyclic extension that eliminates ISI perfectly ( $T_g=5$  msec). Testing is in HF channel having channel coherence time  $(\Delta t)_c=5$  sec and channel delay spread  $T_d=3$  msec (Moderate conditions). From this figure, we observe that when subcarriers number increases, the BER performance

is improved. But, by increasing the number of subcarriers the frequency separation between adjacent subcarriers would be small; therefore, any small frequency offset causes ICI between adjacent subcarriers and the peak-to-average power (PAP) ratio is increased when subcarriers number is increased. Another effect is FFT implementation complexity as subcarriers number is increased. From the above discussion and by looking to Fig.(10), then for  $BER=10^{-3}$ ,  $SNR=19$  dB if  $N=32$ ,  $SNR=13$  dB if  $N=128$  and  $SNR=11$  dB if  $N=512$ . In terms of frequency separation between adjacent subcarriers, at  $N=32$  subcarriers, frequency separation= $75$  Hz, at  $N=128$  subcarriers, frequency separation= $18.75$  Hz, and the frequency separation= $4.6$  Hz for  $N=512$  subcarriers. The PAP ratio for  $N=32$  subcarriers is about 4dB, compared with 5dB at  $N=128$  and 6dB for  $N=512$  subcarriers. The FFT implementation complexity can be measured by a number of complex multiplications required to implement the FFT. Hence, the complexity of FFT is given by  $(N/2*\log_2N)$  then the complexity is increased when the subcarrier increased. Hence, in this study, the optimum number of subcarriers is 128 and the OFDM system at this value gave good BER performance with suitable complexity, frequency separation and PAP ratio.

## **6 Effect of Clipping Method on the OFDM System Performance:**

To illustrate the effect of clipping method on the performance of OFDM system, Fig.(11-a) shows the BER performance of OFDM system in HF channel for various clipping voltage levels (CL) at  $SNR=20$  dB. Figure.(11-b) shows the PAP ratio for various clipping voltage levels. From these two

figures, we observe that, when clipping voltage level is increased, the BER performance is better, but, the PAP ratio is high. When the clipping voltage level is low, the amount of peaks removed from OFDM signal is high, then the signal is distorted very much and the performance degrades (higher BER and less PAP ratio).

The clipping method is associated with some difficulties, concerning the choice of the appropriate clipping voltage level for a given OFDM system configuration. Hence, the BER performance of the designed OFDM system function of the clipping voltage level is calculated, for different values of SNR in order to indicate the suitable clipping voltage level for the designed system. The results obtained are shown in Fig.(12). The OFDM system operates at bit rate  $R=4800$  bit/sec,  $N=128$  subcarriers, and guard time  $T_g=5$  msec, HF channel having channel coherence time  $(\Delta t)_c=5$  sec and delay spread  $T_d=3$  msec (Moderate conditions). Number of transmission bits is 50000 bits. From this figure, we can see that, at clipping voltage level =12 v, the degradation in SNR=2 dB at BER  $10^{-3}$  compared with SNR for OFDM system without clipping and about 50% reduction in PAP ratio before the clipping (PAP ratio about 5 dB without clipping and about 2.5 dB at clipping voltage level =12 v). From these results, one can conclude that the clipping voltage level is a tradeoff between PAP ratio reduction and degradation in BER.

## **7 Conclusions**

The behavior of the OFDM system is studied. The study shows that the OFDM system is the efficient way to deal with frequency selective multipath fading channel

in comparison with single carrier system, which provides large improvement in BER performance in HF channel environment. OFDM system is capable of supporting high data rate transmission in comparison with single carrier system. The OFDM system is designed to operate with bit rate 4800 bit/sec, without extra-added equipment complexity, which is an important achievement in HF channel system design field. By the insertion of the guard time with cyclic extension in each OFDM symbol, the ISI can be eliminated completely and ICI is reduced. The guard time must be much greater than the delay spread of the channel to eliminate ISI perfectly. The simulation shows that the best performance of the guard time has to be about twice the delay spread. In this work, optimum guard time used is 5 msec. The simulation shows that when the number of subcarriers is increased, the BER performance is improved, but, the PAP ratio is increased, frequency separation is decreased, and implementation complexity is increased. Therefore, the simulation shows that the optimum number of subcarriers is chosen to be 128 subcarriers, which offers good BER performance, suitable complexity, frequency separation and PAP ratio.

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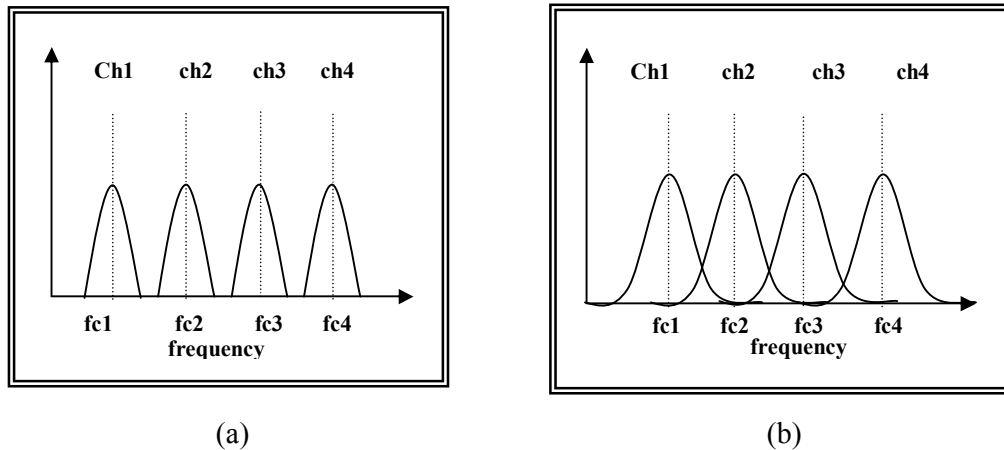


Fig.(1) Transmitted signal spectrum of FDM system

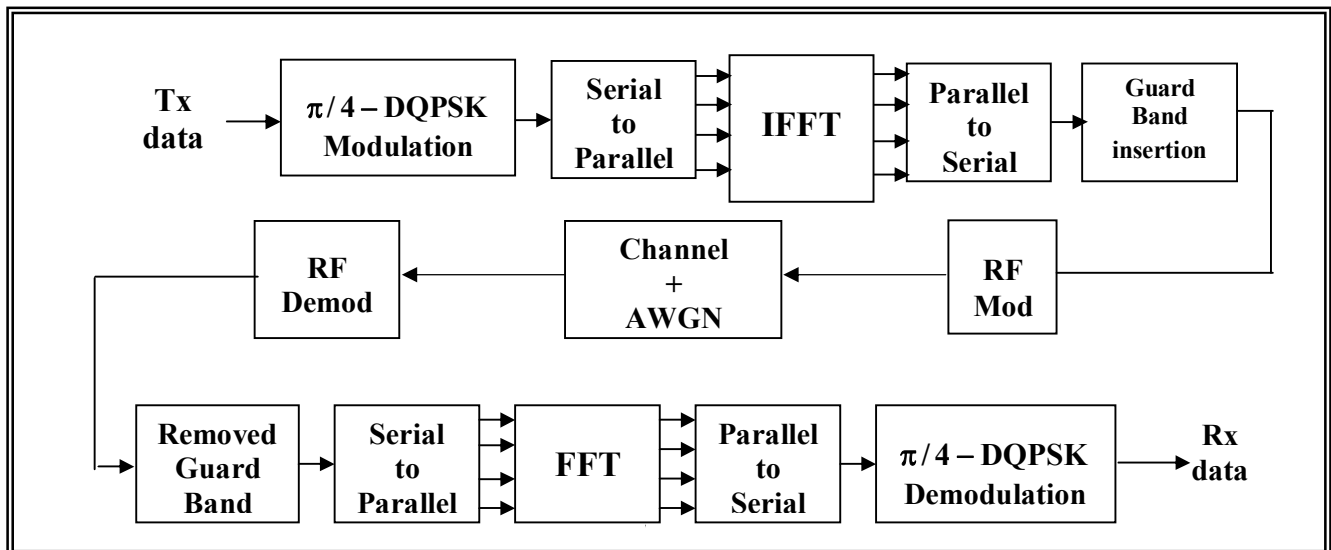


Fig. (2) Block diagram of the OFDM system model

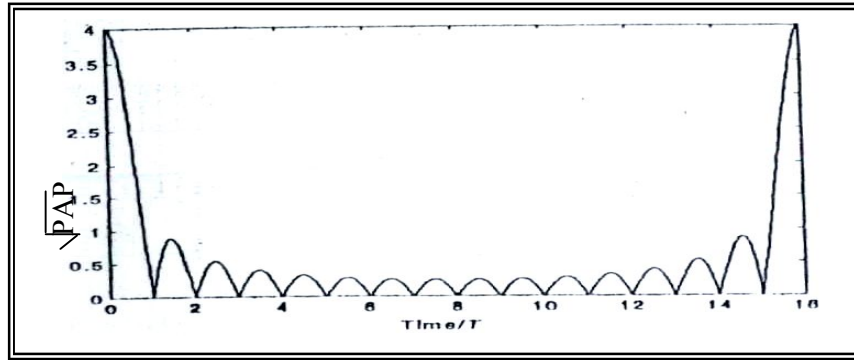


Fig.(3) square root of peak-to-average power ratio for a 16 subcarriers OFDM signal, modulated with the same initial phase for all subcarriers

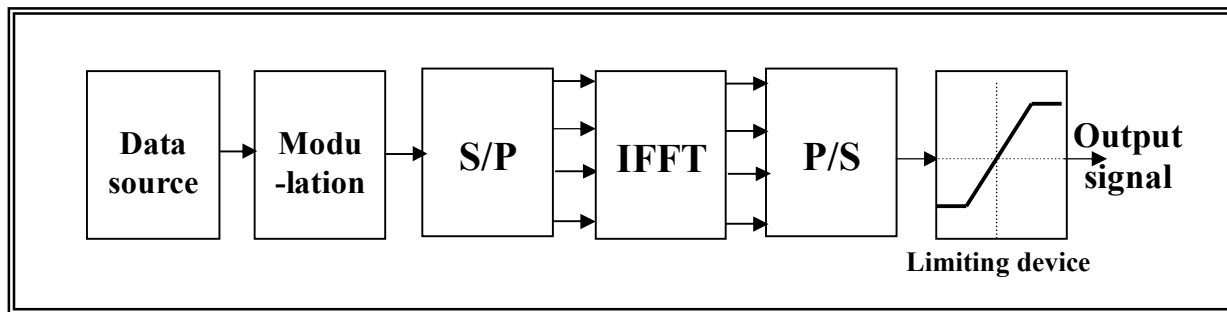
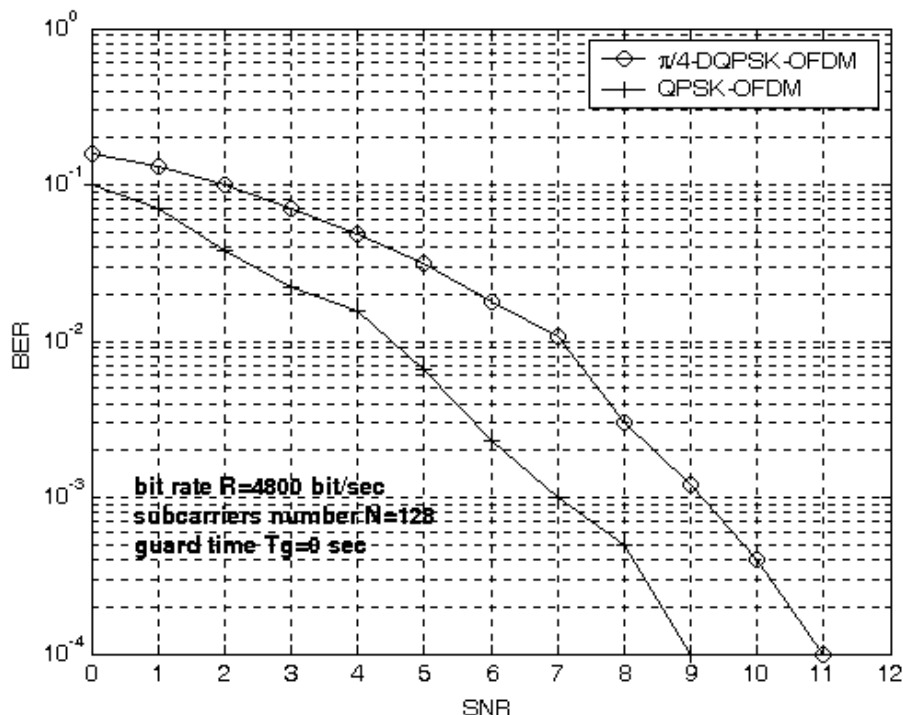


Fig.(4) Block diagram to show limiting of a composite OFDM waveform



Fig(5) comparison between  $\pi/4$ -DQPSK and QPSK modulation scheme of OFDM system in AWGN channel



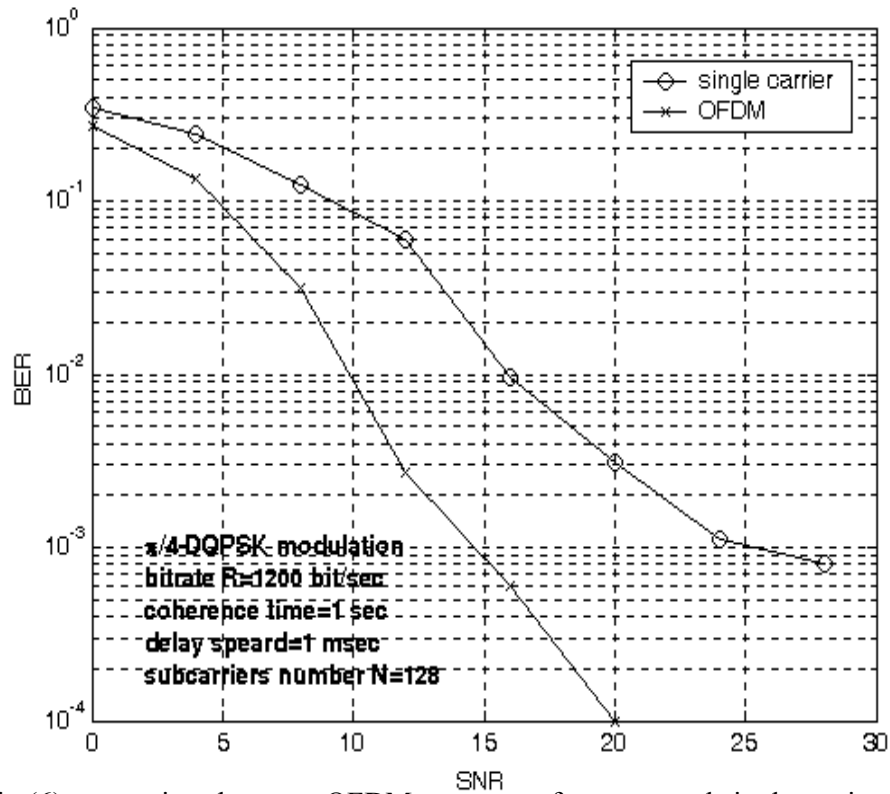


Fig.(6) comparison between OFDM system performance and single carrier system performance in HF channel

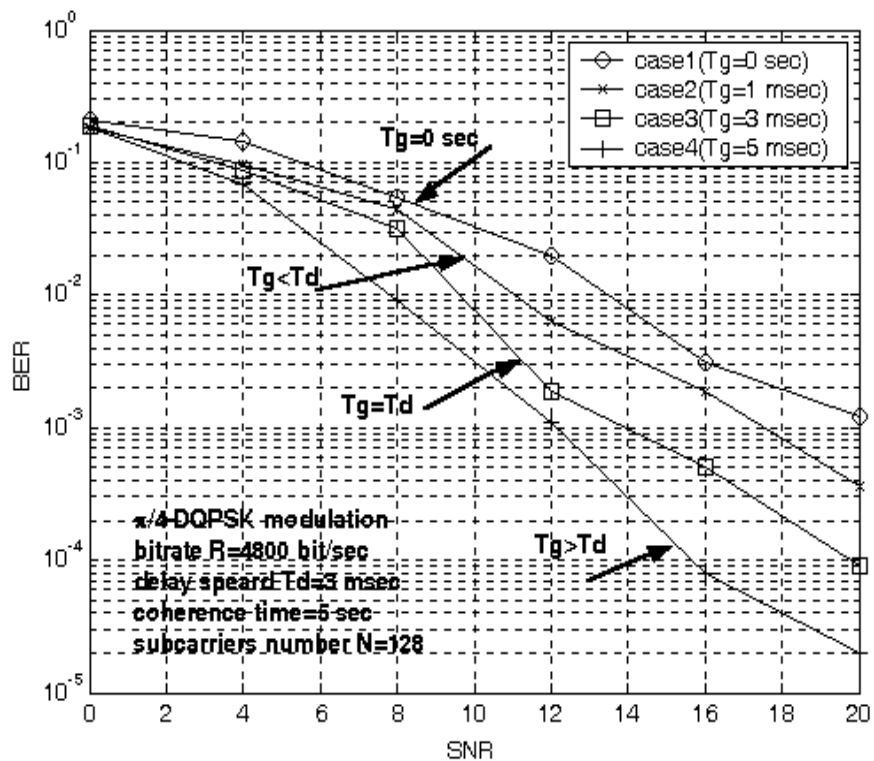


Fig.(7) BER performance of OFDM system, at different cases of time guard band use

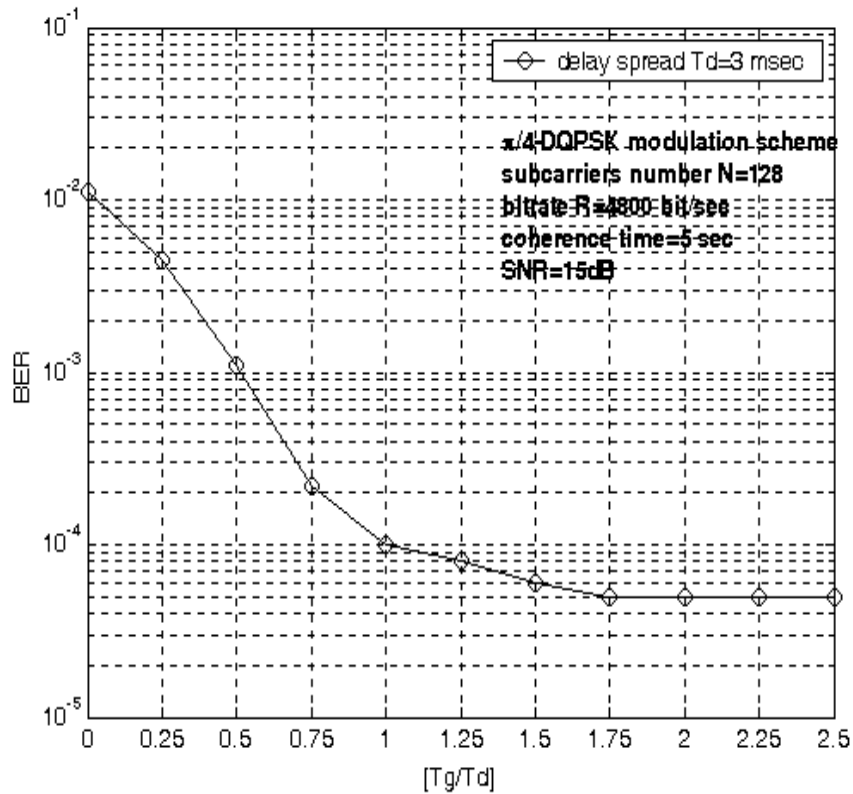
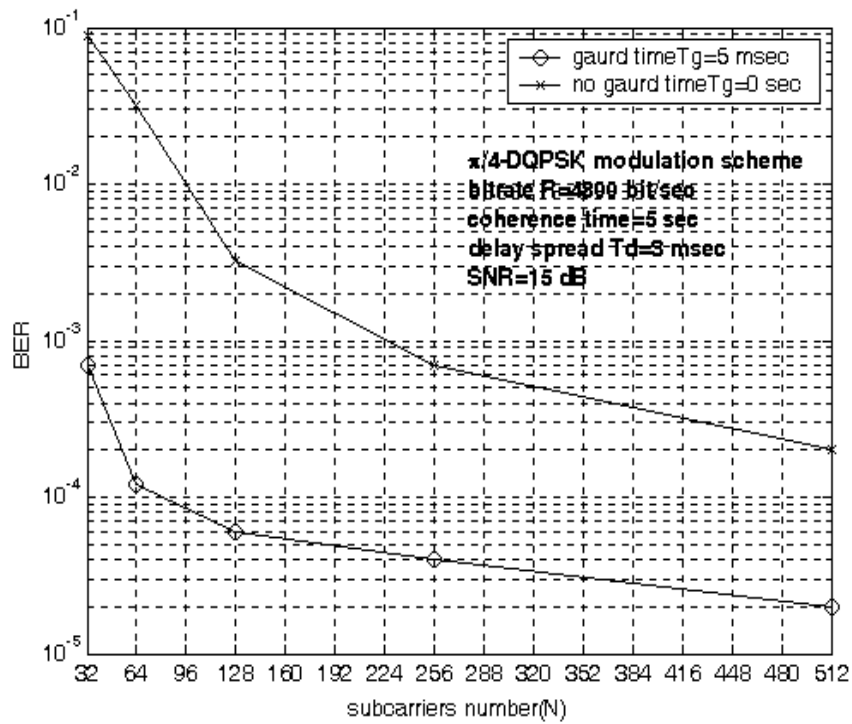


Fig.(8) BER versus normalized ratio ( $T_g / T_d$ ) for OFDM system in HF channel



Fig(9) BER versus number of subcarriers for OFDM system performance with and without guard time

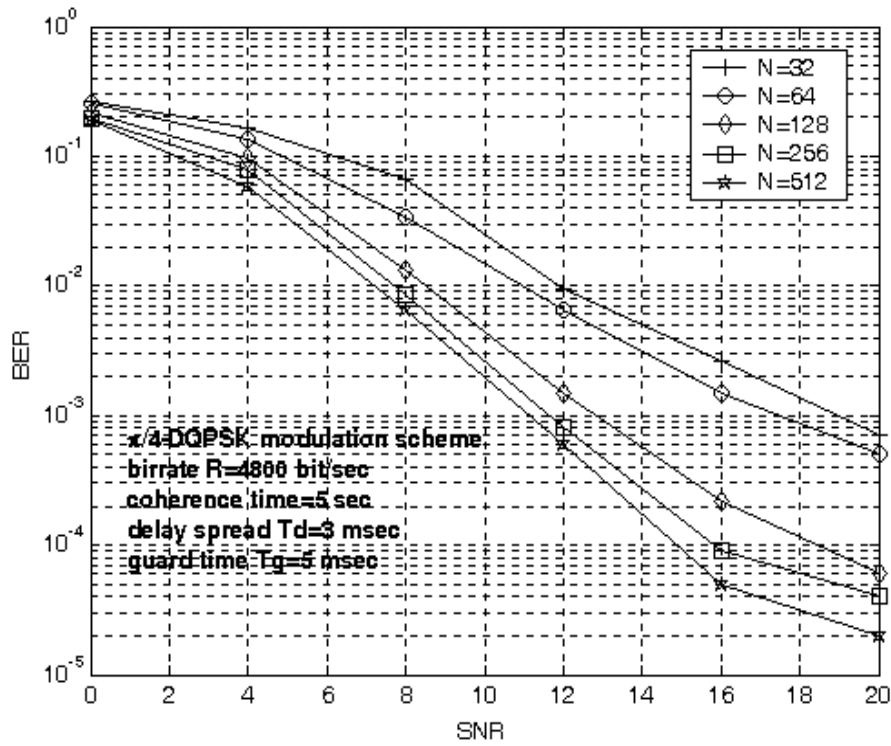
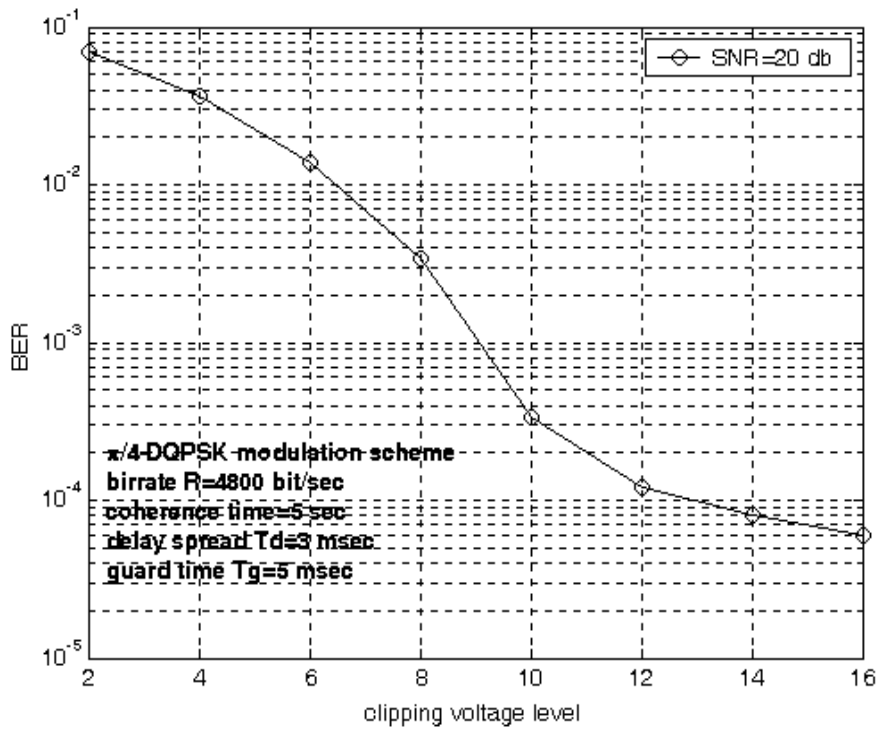
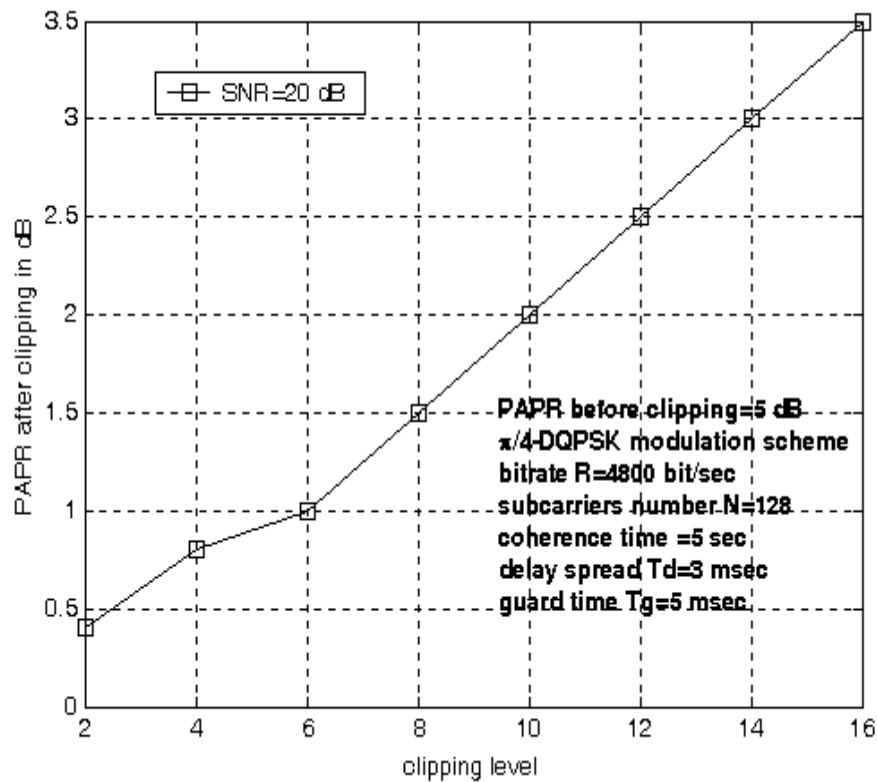


Fig.(10) BER performance of OFDM system for various number of subcarriers



Fig(11-a) BER performance of OFDM system for various clipping voltage level



Fig(11-b) PAP ratio versus clipping voltage level

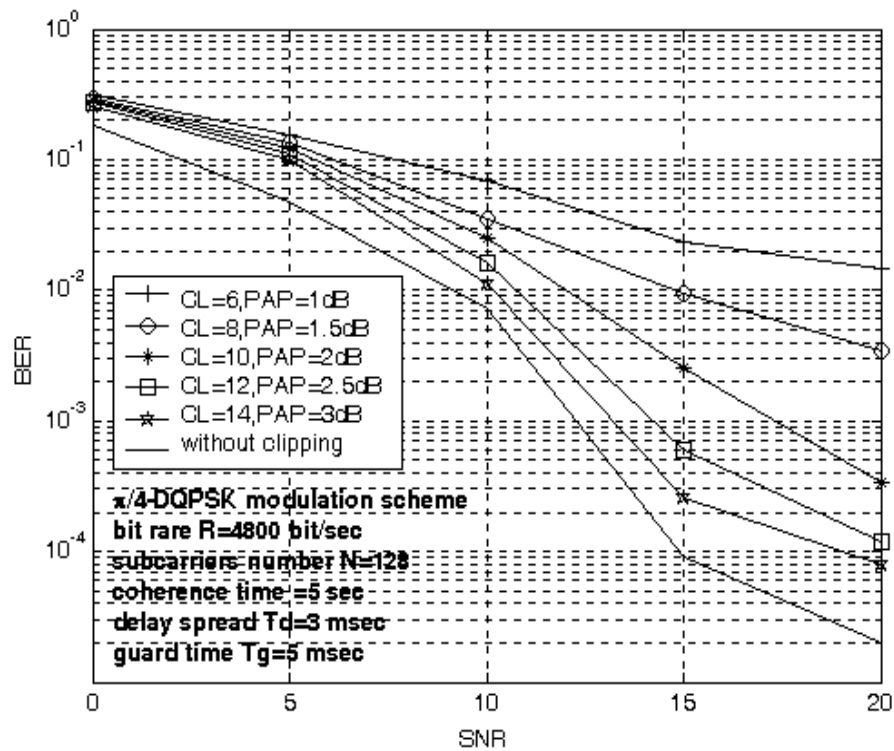


Fig.(12) BER performance of OFDM system for different clipping voltage level