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# Evaluation the Initial Values for Eccentric Anomaly for an Ellipse Orbit: Article Review

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**ABSTRACT**

The equation of Kepler is used to solve different problems associated with celestial mechanics and the dynamics of the orbit. It is an exact explanation for the movement of any two bodies in space under the effect of gravity. This equation represents the body in space in terms of polar coordinates; thus, it can also specify the time required for the body to complete its period along the orbit around another body. This paper is a review for previously published papers related to solve Kepler's equation and eccentric anomaly. It aims to collect and assess changed iterative initial values for eccentric anomaly for forty previous years. Those initial values are tested to select the finest one based on the number of iterations, as well as the run time for each starting initial value that is required for completing the solution. The method of Newton–Raphson is employed to acquire a final value for an eccentric anomaly; this method considers a typical method for a solution with less divergence as compared with an ideal solution, and the best initial value is chosen. The applicable selection of the initial value of the eccentric anomaly will decrease the calculation time and confirm the convergence of the curves of the eccentric anomaly with ideal curves.

## Introduction

The systematic study of celestial objects and space actions is identified as astronomy. This science clarifies the development, source, growth, and astronomical actions in space via physics and mathematics [1]. From 1650 to the present, numerous studies have been employed to find the value of an eccentric anomaly. Many methods have been described to calculate this value. Essentially, its calculation depends on the motivation of the solver and the mathematical procedure or tools that are obtainable according to the requirements of calculation. The era of calculators and computers has facilitated rapid and precise calculations of the value of eccentric anomaly. In particular, Matlab is a very influential program that is simple and easy to use. Moreover, a standard programming language, like Fortran, is used for numerical calculations [2, 3, 4]. The Kepler equation deals with dissimilar difficulties related to celestial mechanics. It is a description of the motion of two bodies in space under the impact of gravitational forces on each other. It represents the body in terms of polar coordinates, so it can also determine the required time for the body to complete its period along the orbit [5, 6, 7]. This equation requires three parameters to work: eccentricity (e), mean anomaly ($M_e$), and initial value of eccentric anomaly ($E_i$).

Eccentricity defines the shape of the orbit; mean anomaly describes the motion of a body along untrue orbit; and the last is considered a starter value for a solution to find a final value for eccentric anomaly [8, 9, 10]. In general, this is solved by three methods: classical methods [11], iterative methods, and non-iterative methods. The classical methods use power series (E) for solution, so it is considered a direct solution to obtain the value of the eccentric anomaly for one period (0°–360°) and does not need a tolerance to complete the solution [12, 13, 14]. It is divided into two types: one based on the Bessel function, and the other is based on the Lagrange series. An iterative of the Newton–Raphson method is employed to calculate the final value of eccentric anomaly using different initial values for eccentric anomaly. The non-iterative method is similar to the classical method; it provides a direct solution for the equation of Kepler and also requires a tolerance [15, 16]. In 1987, Mikkola used a non-iterative initial value, which yields a rough calculation for the eccentric anomaly value [17].

## Theory

- **The Formula of Kepler's Equation**

It is a non-linear equation and seems simple; it is illustrated below [5]:

$$E_{i+1} = Me + e_i \sin(E_i) \qquad (1)$$

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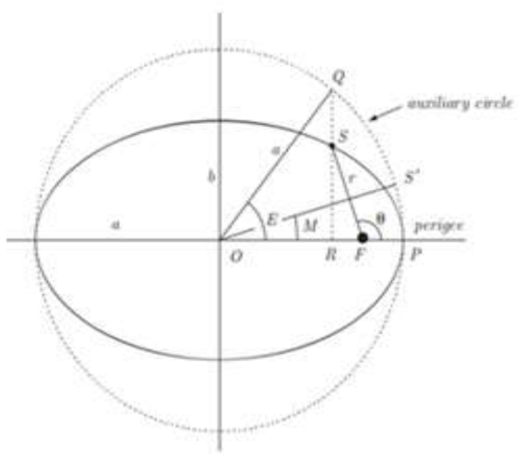
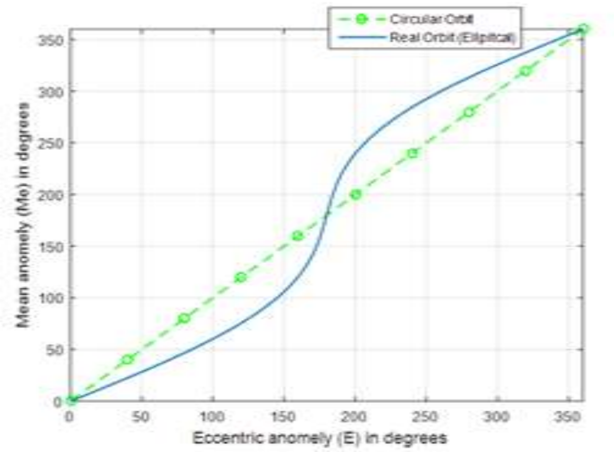
Here, $e_i$ is designated to eccentricity and defined by (e × 57.3248 °), which means that it is measured in degrees. Its range for an elliptical orbit is from 0 to 1. $M_e$ designates the motion of an object along a circle orbit. It is an angle measured in degrees from the center of the circle and has a range from 0° to 360° [4, 5, 7]. $E_i$ represents the angle from the closest point to the object to the position of an extended object along an auxiliary circle, measured in degrees from 0° to 360°, as shown in Figures 1a and 1b [5]. This parameter is considered a starter value for a solution, and it has different values to test by the Newton–Raphson method. $E_{i+1}$ is the final value for an eccentric anomaly. The general form of the equation of Kepler is Equation (1), which reflects a direct solution to obtain the value of the eccentric anomaly. In some references, this equation is measured in radians. Eccentricity and mean anomaly must be identified to find the eccentric anomaly value, and eccentric anomaly can then be calculated directly [4, 5]. The Newton–Raphson method is one of the best standard methods for determining the root of a well-behaved function, as presented in Figure 2. The reiteration using this method will continue until the next value ($E_{i+2}$) is estimated from the previous value of the eccentric anomaly ($E_{i+1}$), ceasing only when the user-defined degree of precision is achieved. The Newton–Raphson method is expressed as follows [4, 14]:

$$E_{i+1}=E_i+ [\, E_i - e \times \sin(E_i) - M_e \,] / [1 - e \times \cos(E_i)] \quad (2)$$




(a)




(b)

**Figure 1.** (a) Representation of the eccentric and mean anomalies in their orbits [5], and (b) the result of mean anomaly as a function of eccentric anomaly for the initial value $E_i=M_e$ and eccentricity = 0.8 by the Newton–Raphson method [5].

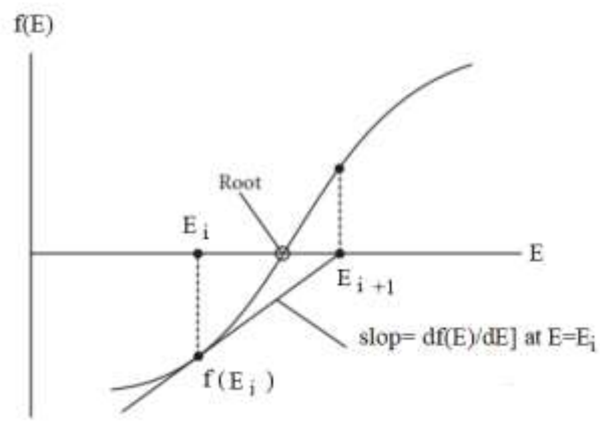



**Figure 2.** Representation of the Newton–Raphson method [5].

Once the user uses the above method, he will face two problems that must be considered. First, the required property for the algorithm offers the degree of convergence. By contrast, if the starting initial value is not correctly near to the solution, the curve's behavior for an eccentric anomaly will deviate. Second, the derivative of the function has a zero value. Thus, this method begins to miss the required property for the algorithm that makes the degree of convergence (quadratic) available. This comes about when the solution has a slope with a zero value. Thus, at each stage of iteration, the slope of the function must not be zero. Halley used the second derivative, which results from Tylor series expansion with the Newton–Raphson method [15]:

$$E_{i+1} = E_i - 2\, f(E_i) \times [{}^{\prime}f(E_i)/2[{}^{\prime}f(E_i)]^2 - f(E_i) \times {}^{\prime\prime}f(E_i)] \quad (3)$$

where $f(E_i), {}^{\prime}f(E_i)$, and ${}^{\prime\prime}f(E_i)$ are the first, second, and third derivatives, respectively. Extra terms of derivatives for the Halley method are considered to determine the convergence behavior at each stage. This method is created by the initial value of an eccentric anomaly with a high convergence property [5].

- **Initial Values of Eccentric Anomaly ($E_i$)**

Regardless of using the non-iterative method and classical methods to determine eccentric anomaly, numerous researchers and scholars have proposed that, unlike initial values, the majority are derived from eccentricity and mean anomaly parameters, or one of them. Specific initial values incorporate an additional formula that is based on the sine or cos function. Furthermore, the formula encompasses eccentricity, mean anomaly, or both. Changed initial values are used in Equation 1, especially instead of the (Ei) parameter, to obtain a rough final value for the eccentric anomaly ($E_{i+1}$). The reiteration by the Newton–Raphson method will carry on until the next value, which is ($E_{i+2}$) from the previous value of the eccentric anomaly ($E_i$+1). At that point, the values of ($E_{i+3}$), ($E_{i+4}$), and ($E_{i+5}$) are calculated based on the previous values [4, 5, 7]. In 1972, complex adaptable analysis was conducted to develop an ideal solution to find the value of eccentric anomaly for both types of orbits, which are elliptical and hyperbolic. Riemann problems were appropriately displayed; they were formed by the simple properties of canonical solutions. Thus, the final results will be obtained by elementary quadrature formulas [18]. In 1978, Smith used the root of the Kepler equation and considered the eccentric anomaly instead of $M_e$ and $M_e$+ e to provide an initial value of the eccentric anomaly as below [19]:

$$E_i = M_e - e \times [\sin(M_e)/1 - \sin(M_e + e) + \sin(M_e)] \quad (4)$$

The root of Kepler's equation between $M_e$ and $M_e$ + e above reflected a linear approximation. A comparison was made by Smith between the initial value and the Newton–Raphson method within two sections with respect to another value. These sections are imaginary: section 1 has a limit of $0.05 \le M_e \le \pi$ and $0.01 \le e \le 0.9$, and section 2 has a limit of $0.005 \le M_e \le 0.4$ and $0.95 \le e \le 0.99$. The average number of repetitions was determined by Smith's measurement to obtain a good initial value for eccentric anomaly via the Newton–Raphson method to each section. The tolerance to discontinue the program was $5\times10^{-8}$. After obtaining the initial values, an assessment was made between them to choose the finest, which is given by [19]:

$$E_i = M_e + \alpha\,(-\frac{\alpha^2}{2}) \quad (5)$$

At this point, α = e sin ($M_e$)/1− e cos ($M_e$). In section 2, the variation between $M_e$ + e and the initial value for Smith was not remarkable. As the solver uses a suitable initial value, a correction factor does not need to be added by using the Newton–Raphson method. Two manners must be taken into account for calculation: good convergence and required number of iterations. Smith used another initial value [19]:

$$E_i = M_e \quad (6)$$
$$E_i = M_e + e \quad (7)$$
$$E_i = M_e + e \sin(M_e) \quad (8)$$
$$E_i = M_e - e \times [\sin(M_e) / 1 - \sin(M_e + e) + \sin(M_e)] \quad (9)$$
$$E_i = M_e + e \sin(M_e) + e^2 \sin(M_e) \cos(M_e) \quad (10)$$
$$E_i = M_e + \alpha\,(-\frac{\alpha^2}{2}) \quad (11)$$

In 1979, Edward used the same method that Halley followed. He created separate regions in space, which were $M_e$ and e. The first three regions had topical designs. He found that the equation of Kepler was treated as a third-degree function with nearly $M_e$ = 0 and e = 1, and he used a third-degree root for this region [20]. In 1981, Alefeld used Halley's method with the below initial value [21]:

$$E_{i+1} = E_i - f(E_i)/[\square f(E_i) - 0.5\square\square f(E_i)[f(E_i)/\square f(E_i)]] \quad (12)$$

At this point, i must be greater than or equal to 0. The above equation was known as the tangent to the hyperbola orbit [21]. 1983, Danby and Burkardt suggested another iterative initial value, which was $E_i$=$M_e$. He considered the behavior of function directly toward the upper. The rate of the deviation was reduced as follows [22]:

$$E_{i+1} = E_i + \delta_1 \quad (13)$$

where $\delta_1 = -f(E_i)/\square f(E_i)$, which was the Newton–Raphson method. Further derivatives were used $\delta_2 = -f(E_i)/\square f(E_i) - [0.5 \times \delta_1 \square\square f(E_i)]$ and $\delta_3 = -f(E_i)/\square f(E_i) - [0.5 \times \delta_2 \square\square f(E_i)] + [0.166\times\delta_2^2 \square\square\square f(E_i)]$. The use of $\delta_1 = -f(E_i)/\square f(E_i)$ signified Halley's method and the fourth derivative of convergence or meeting, respectively [22, 23]. In 1985, Ioakimidis and Papadakis suggested a new simple method for transcendental equations and the non-linear algebraic by an integral formulation of the closed procedure.

Furthermore, they used Gauss category quadrature procedures to develop accurate results [24]. In 1986, Serafin employed the property of the sin function to describe the intervals that contained the root of the equation of Kepler. He identified a good initial value for eccentric anomaly, but he needed to find the root of eccentric anomaly [25]. In the same year, Gooding and Odell calculated twelve different initial values. They considered the rapid convergence in large eccentricity and small mean anomaly to be reasonable when the initial values showed a good state for eccentric anomaly [26]. Conway used Leguerr's method to obtain the root of a polynomial [27]:

$$E_{i+1} = E_i - \left[k\,f(E_i)/{\prime}f(E_i) \pm \sqrt{(k-1)^2({\prime}f(E_i))^2 - k(k-1){\prime}{\prime}f(E_i)}\right] \quad (14)$$

Where k is a parameter, and the selected value was 5. The convergence by Equation 14 is assured, regardless of the used initial value [27], as shown in Table 1. After 4 years, Danby separated the regions into two parts, as shown in Table 2 [28]:

**Table 1.** Clarify the intervals of initial value for eccentric anomaly [27].

| **Mean anomaly ($M_e^{\circ}$)** | **Initial value for eccentric anomaly ($E_i^{\circ}$)** |
|---|---|
| [0, 1– eα] | $\frac{M_e}{1-2\frac{e}{\pi}} \le E \le \frac{M_e}{1-e}$ |
| [0, 1– $e\alpha_o$ , (π/2) – e] | $\frac{M_e}{1-2\frac{e}{\pi}} \le E \le M_e + e$ |
| [(π/2) – e, π- (1– $e\alpha_o$)] | $\frac{M_e+2e}{1+2\frac{e}{\pi}} \le E \le M_e + e$ |
| [π – (1– $e\alpha_o$), π] | $\frac{M_e+2e}{1+2\frac{e}{\pi}} \le E \le \frac{M_e+e\pi}{1+e}$ |

**Table 2.** Illustrate the initial values for eccentric anomaly and their intervals [28].

| **Used Initial value of eccentric anomaly ($E_i^{\circ}$)** | **Interval** |
|---|---|
| $M_e+((6M_e)^{1/3}-M_e)\,e^2$ | $0 \le M_e < 0.1$ |
| $M_e+0.85\,e$ | $0.1 \le M_e \le \pi$ |

Taff evaluated 13 changed initial values in 1989, and the finest one was $E_i=M_e+e$ by using Wegstein's method [29]. In 1991, Nijenhuis also separated $M_e$ and e into four sections, using dissimilar initial values for each sector. His work was similar to that of Edward and Danby but included slight modification. The initial values were as follows [30]:

Section 1: It has a large mean anomaly as

$$E_i= M_e+ e\pi / 1+ e \quad (15)$$

Section 2: It has a middle mean anomaly as

$$E_i= M_e / 1- e \quad (16)$$

Section 3: It has a small mean anomaly as

$$E_i= M_e / 1- e \quad (17)$$

Section 4: It contains a space near the mean anomaly = 0 and eccentricity = 1, which used Mikkola's method as follows:

$$E_i= M_e +e\,(3R- 4R^3) \quad (18)$$

Here, R is a parameter necessary for the solution [17]. In 1995, Markly suggested a final value for eccentric anomaly, which was built on Pede calculations for the sine function; he reduced the trigonometric function using [31]. Besides, Shiming and Desmond also used Halley's method, but their calculations were based on the Kantorovich theorem. The purpose of using this theorem was to decrease the region conditions and give an operator equation for the Kantorovich theorem [32]. In the same year, Chobtov compared Conway's with the Newton–Raphson method. He showed that, despite the assured convergence behavior of Conway's method, the Newton–Raphson method for calculating the completing time was preferred [33]. In 1996, Toshio determined an approximate solution, and he considered the starter value of (y) to be insignificant or a trivial solution, and (x) represents (y) and (j) is the solution index. The Newton–Raphson method included an additional approximation, which was the corrected value for (y). It was an iterative initial value that did not require the evolution of a transcendental function [34]. In 1997, Toshio solved the Kepler equation for all types of orbits, namely, elliptic, parabolic, and hyperbolic. He studied two kinds of orbits: elliptic and hyperbola; two different initial values were used as follows [35]:

$$E_i= M_e +e \quad (19)$$

$$E_i= M_e + e \sin M_e + e \times \sin 2M_e /2 \quad (20)$$

He concluded that the initial value in Equation 20 that was solved by the Newton–Raphson method, as compared with Equations 5 and 19, had a convergence performance with a lower number of iterations. He recognized that the starting value in Equation 19 was 15% faster than the initial value in Equation 5, whereas the initial value in Equation 19 was 2% faster. He selected the initial value in Equation 19 for the hyperbola and elliptic orbits based on the total calculating time to skip the deviation by Newton–Raphson and reduce the time of calculation [35]. In 1998, Charles and Tatum indicated that the Newton–Raphson method analyzes the initial values $E_i = M_e$ and $E_i = \pi$. He reported that the Newton–Raphson method was convergent for $E_i = \pi$, but there was a risk for divergence when $E_i = M_e$. The following equation was offered to obtain an enhanced beginning value [36]:

$$E_i = M_e + e\,[(\pi^2 M_e)^{1/3} - \pi \sin Me/15 - M_e] \qquad (21)$$

In 2006, Feinstein proved that a non-iterative method was better than using dynamical discretization methods in conjunction with a dynamic program to all earlier published procedures [37]. Mortari and Clocchiatti offered a non-iterative solution in 2007 to calculate the eccentric anomaly by using Bézier curves. In contrast to the method of dynamic discretization, this method did not require any initial calculated data [38]. In 2010, Curtis used the following initial values for the solution [39]:

$$E_i = \begin{cases} M_e + (e/2) & M_e < \pi \\ M_e + (e/2) & M_e > \pi \end{cases} \qquad (22)$$

In the same year, Mohammed used the root based on an iterative solution by enhancing the method of convergence. An entirely new enhancement to Aitken's method was used to quickly arrive at a numerical solution for the Kepler equation [40]. Boubaker proposed analytical initial values [41]. An algorithm was employed by Calvo et al. in 2013 for the iterative solution of an elliptic orbit. A new global effectiveness was calculated to compare the quality of initial values, and certain well-known initial values with low computing costs were assessed. Considering the measurements, an optimized starting value was obtained [42]. A number of researchers in 2014 [43–45] attempted to resolve several iterative approaches to determine the value of an eccentric anomaly. In 2017, the Adomian decomposition method was used by Aisha and Abdelhalim to obtain a periodic, analytical, and accurate solution [46]. In the same year, an effective program was generated by Raposo-Pulido and Pelaez to determine the eccentricity of an elliptical orbit [47]. In 2019, Rasha and Abdul-Rahman used different initial values based on the Newton–Raphson method to calculate state vectors for the satellites at different orbits [48]. In 2020, Mohammed et al. refined the Halley method to provide a superior approximation of the ideal solution. They used the third order to find a value for eccentric anomaly [49]. Fouad and Abdulrahman investigated state vectors and predicted the directional and dimensional elements for the Spot-6 satellite by using the value of the eccentric anomaly derived from the Newton–Raphson method [50]. In 2021, Dike and Isaac determined the eccentric anomaly for a satellite with perturbations. Using hypothetical numerical examples with various mean anomaly and eccentricity values, the perturbation based on seeded secant iterations was demonstrated. The convergence cycle extended as eccentricity grew, according to the results of the eccentric anomaly with a mean anomaly of 30° and eccentricity limits between 0.001 and 1. Specifically, at eccentricity of 1, the cycle extended the design from 2 at eccentricity of 0.01 to 8 at eccentricity of 1. These results implied that additional iterations were required to determine the value [51]. In 2023, Selim used the homotopic continuation method and suggested convergent order using an effective iterative method that was designed to solve Kepler's equation. This formulation has a dynamic component, moving from one iterative model to another with supplementary guidance. This method does not need any prior knowledge of initial assumptions and avoids crucial situations arising from deviations associated with numerical methods reliant on an initial estimation. As a result, the computed algorithm and a numerical demonstration of the method were provided [52]. In the same year, Doaa et al. improved the precision of prayer times and then calculated the change with geographical latitudes [53]. Duaa and Abdul-rahman calculated the optimal orbit for a satellite that rotates around Earth before being directed to another orbit [54].

**Method**

In this article, two major parameters were taken into consideration to compare and select the best initial values of the eccentric anomaly. Those parameters were as follows:

1. The time required to run the initial value by the Newton–Raphson method for each step of iteration in the program.
2. The number of iterations.

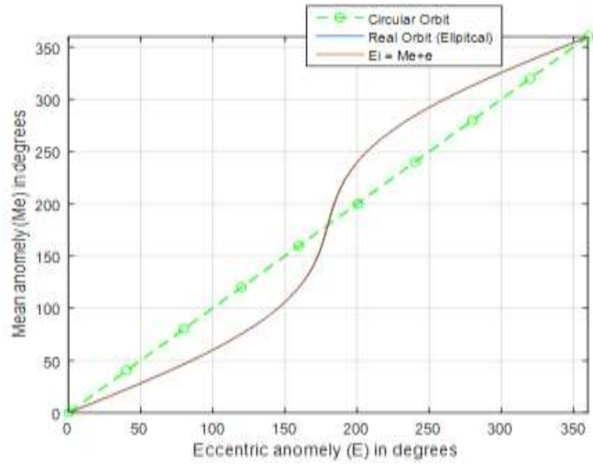
The ideal solution depended on the above two parameters. The Newton–Raphson method was proposed as a standard method for finding the final value of eccentric anomaly. Thus, this method was selected to reduce the iteration numbers by choosing the best initial value. The only difficulty with the Newton–Raphson method was the probability of discrepancy in some areas according to the initial values cited previously in Equations 2–23 and Table 2 [55, 56, 57]. The original value of eccentric anomaly derived from Equation 1 will be improved, and this enhanced value will be used to obtain a further refined value for eccentric anomaly. A tolerance of $10^{-10}$ was used to terminate the method [58, 59, 60, 61]. The derived eccentric anomaly was utilized to compute the distance and velocity components of satellites, with and without perturbations, and study the impact of these perturbations on the satellites [62, 63, 64, 65, 66].

## Results and Discussion

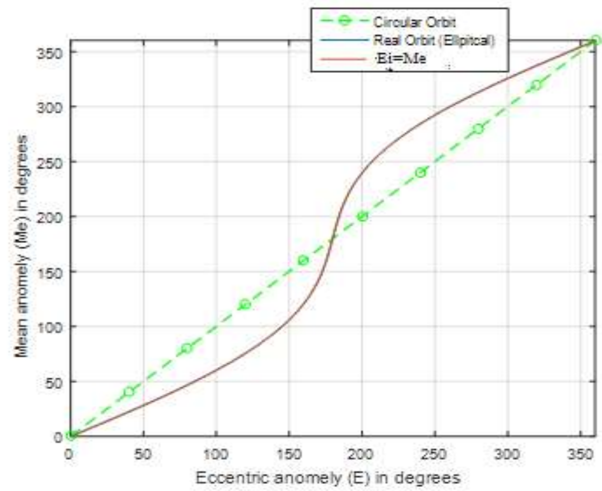
A Matlab program was designed to test iterative initial values for eccentric anomaly to obtain the best final value for eccentric anomaly. The selection for the finest final value of eccentric anomaly depends on the number of iterations and the run time required for the initial value to complete one period (0°–360°). The Newton–Raphson method was used to apply the results at eccentricity of 0.8, mean anomaly of 0°–360°, and tolerance of $10^{-10}$, with eccentric initial values in Table 3. The relation between the eccentric and mean anomalies was plotted and applied for one period. All the initial values that were mentioned before were used to find a new value of eccentric anomaly and display identical behavior, as clarified in Figure 3a. The object in an elliptical orbit started to move from perigee, which was the closest point of focus. The curve of the initial value for eccentric anomaly and the real Kepler's orbit matched each other regardless of the initial value that starts the solution. This initial value was considered a basic value to start the solution. The new initial value was used another time to find a new value for the eccentric anomaly until it reached the desirable value. If the solver tried to use another initial value from Table 3, the same behavior was obtained, as illustrated in Figures 3b–3f. The same performance would appear for all ranges of eccentricity in elliptical orbit (0–1). All the figures below represent the values of eccentric anomaly at perigee $M_e$ = 0°, apogee $M_e$ = 180°, and perigee $M_e$ = 360°. Additionally, the run time for some initial values was calculated and compared with other methods, as shown in Figure 4. The results for the run time were between 175.09 and 195 ms, and this range was close to those of other methods. The initial values ($M_e - e$), ($M_e + e$), and $M_e + e\sin(M_e) / \sqrt{1 - e\cos(M_e) + e^2}$ were faster than others. The selection of those initial values was based on the total calculation time to shrink the calculation time [26, 59].

**Table 3.** Initial values of eccentric anomaly ($E_i^{\circ}$) that were tested in the program by the Newton–Raphson method [15].

| | |
|---|---|
| | **$E_i^{\circ}$** |
| **1** | **$M_e + e$** |
| **2** | **$M_e$** |
| **3** | **$M_e + (e/2)$** |
| **4** | **$M_e - e$** |
| **5** | **$M_e + e\sin(M_e)$** |




**(a)**

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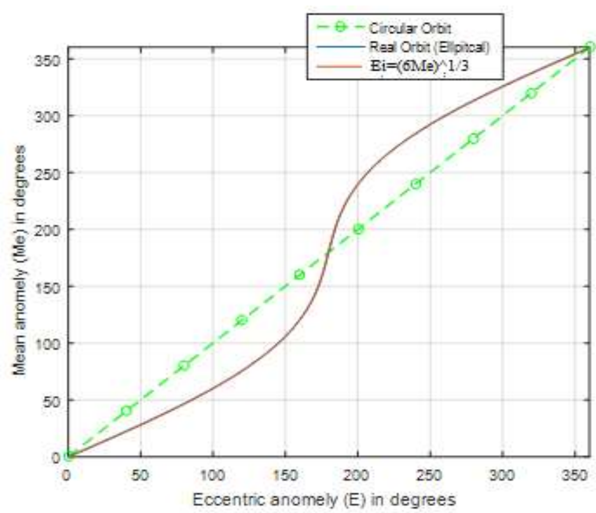
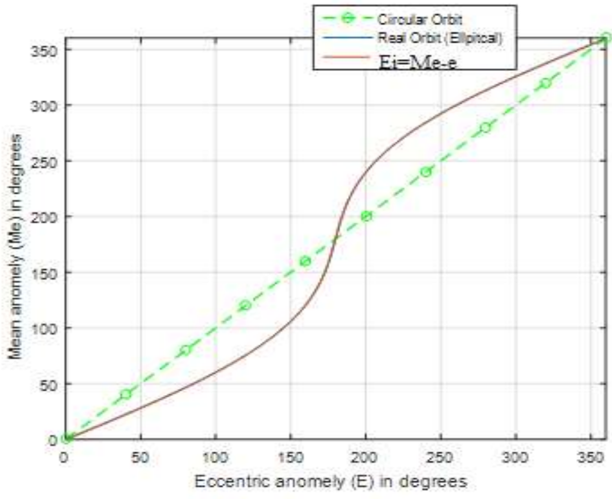
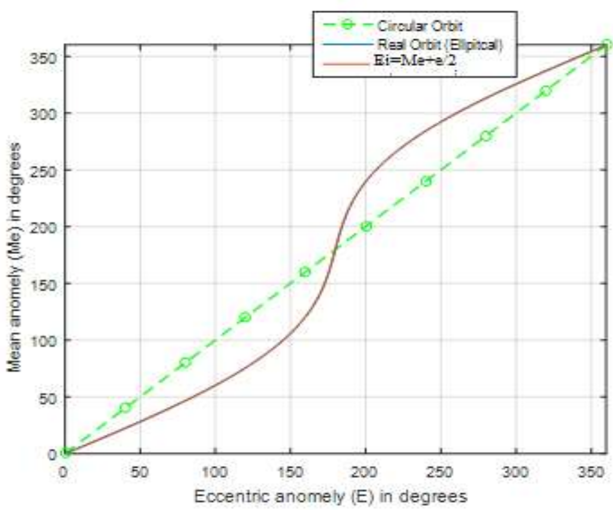


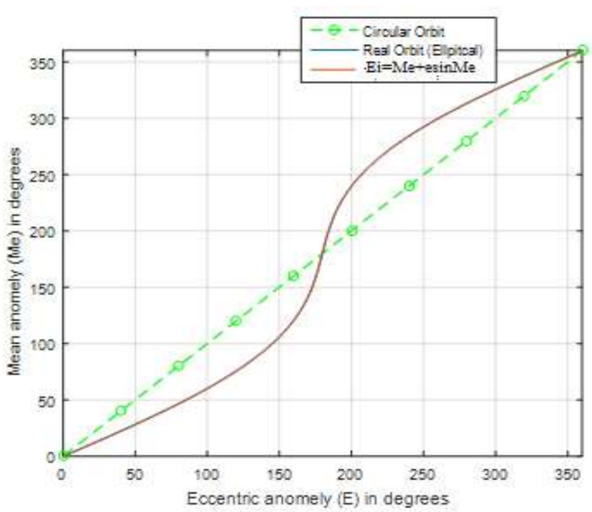

**(c)**




**(d)**




**(e)**




**(f)**

**Figure 3.** Relation between $M_e$ as a function of $E_{i+1}$ for e = 0.8 [15, 28, 48].

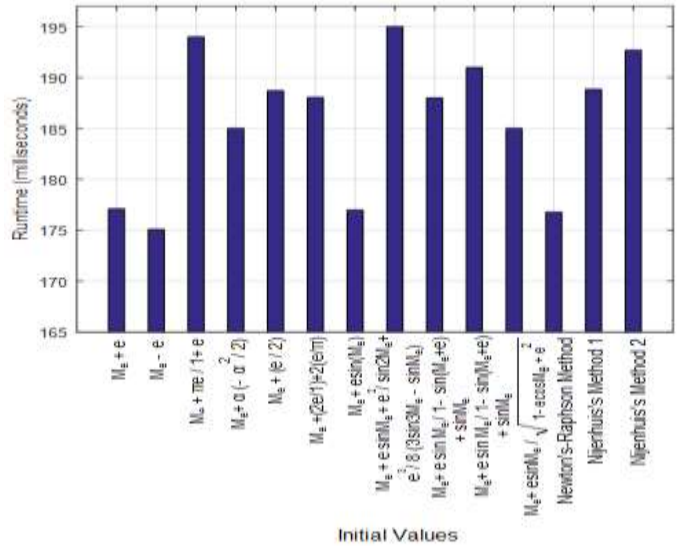



**Figure 4.** Run time for each initial value and the comparison with other methods at e = 0.8 and $M_e$ = (0 - 360)° [15, 28].

## Conclusions

The conclusions of this article review showed several important points, which are as follows:

1. The value of an eccentric anomaly converged with a low amount of iteration. The rate of convergence varied, based on the initial value formula for eccentric anomaly.
2. The precise selection of the starting value and the quick property were both considered when choosing the initial value. By observing the time and iteration numbers required for running the Newton–Raphson method, a quick and precise beginning value was found.
3. Each initial value has a certain limit of eccentricity and mean anomaly, but all of the initial values of eccentric anomaly were studied, and they all exhibited optimal presentation for the application of the solution. However, those initial values were considered.
4. When near the starting value ($M_e$ + e), the runtime for each initial value was brief.
5. The only difficulty with the Newton–Raphson method was the potential for disagreement in some sections according to the initial values mentioned previously.
6. Nijenhui's method will produce convergence behavior (coming close) for any starting value selected, preventing divergence in the final values of eccentric anomaly. As a result, Mikkola's initial value was not considered as a solution.

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**Conflict of Interest**

The authors declare that they have no competing interests.

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## تقييم القيم الابتدائية للانحراف الشاذ في مدار القطع الناقص: مقال مراجعة

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**الخلاصة:**

تستخدم معادلة كبلر لحل العديد من المواضيع المتعلقة بالميكانيك السماوي وديناميكية المدارات. تعتبر هذه المعادلة توضيح لحركة الجسمين في الفضاء تحت تأثير قوة الجاذبية وتمثل بشكل الاحاثيات القطبية وعلى هذا الاساس يمكن تحديد الوقت المستغرق للجسم حتى يكمل دورة حول جسم اخر. هذا البحث هو مقال مراجعة للبحوث المنشورة السابقة لحل معادلة كبلر في مدار اهليجي وايجاد قيمة الانحراف الشاذ، يهدف البحث لجمع وتقييم عدة قييم ابتدائية تكرارية مختلفه للانحراف الشاذ خلال الاربعين سنة الماضية. يتم اختبار هذه القيم الابتدائية لاختيار أفضلها بالاعتماد على عدد التكرارات والوقت المستغرق لكل قيمية ابتدائية في البرنامج. استخدمت طريقة نيوتن-رافسن للحل للحصول على القيمة النهائية للانحراف الشاذ، تعتبر هذه الطريقة أفضل طريقة للحل وبتباعد قليل عن الحل المثالي، تم ايضا اختيار أفضل قيمة ابتدائية للانحراف الشاذ. يقلل الاختيار المناسب للقيمة الابتدائية من زمن التنفيذ في البرنامج ويضمن سلوك التقارب للمنحني الخاص بقييم الانحراف الشاذ مع منحني الحل المثالي.

***الكلمات المفتاحية:*** *ديناميكية المدار، معادلة كبلر، طريقة نيوتن-رافسن، الانحراف الشاذ، معدل الانحراف.*