ESTIMATION OF THE NUMBER OF SOURCES BY APPLYING MDL TO THE OUTPUT OF QUADRATURE MIRROR FILTER BANK

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Abstract

Estimating the number of sources impinging on an array of sensors is a well known and well investigated problem. A common approach for solving this problem is to use an information theoretic criterion, such as Minimum Description Length (MDL). MDL technique is very important in many applications, but its response degrades under low signal-to-noise ratio (SNR) conditions. This paper proposes a new system to estimate the number of sources by applying MDL to the outputs of filter bank consisting quadrature mirror filters (QMF). Some numerical experiments show that the proposed method can estimate the number of sources under low signal-to-noise ratio (SNR).

Key Words: MDL, QMF, Wavelet Transform

الخلاصة:

سوف نتحرى في هذا البحث عن مشكلة تخمين عدد المصادر الساقطة على نظام المتحسسات. الطريقة الشائعة لحل هذه المشكلة هو أستعمال معيار نظرية المعلومات, مثل أستعمال وصف الحد الأدنى للطول (MDL) . تقنية MDL جدا مهمة في الكثير من التطبيقات, لكن أستجابتها تسؤ عندما تكون نسبة الأشارة الى الضوضاء (SNR) واطئة. في هذا البحث نقترح نظام جديد لتخمين عدد المصادر بواسطة تسليط MDL على نتاج حزمة المرشح المحتوية على مرشحات مربعة المرأة. بعض التجارب العددية تبين ان الطريقة المقترحة في هذا البحث تستطيع تخمين عدد المصادربشكل صحيح عندما تكون نسبة الاشارة الى الضوضاء(SNR) واطئة .

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1. Introduction

The wavelet transform has diffused into most signal processing applications. It plays a very important role in the denoising of signals since it gives an effective, informative and compact description of the analyzed signals. In 1988, Mallat produced a fast wavelet transform. The Mallat algorithm for discrete wavelet transform (DWT) known as quadrature mirror filters (QMF)[1].This property also endows wavelets with a remarkable aptitude for denoising by means of a simple nonlinear thresholding filter. Mallat and Hwang [2] first showed that effective noise suppression may be achieved by transforming the noisy signal into the wavelet domain by QMF, and preserving only the local maxima of the transform. A wavelet reconstruction that uses only large-magnitude coefficients has also been shown to approximate well the uncorrupted signal; in other words, noise suppression is achieved by thresholding the wavelet transform of the contaminated signal.

The problem of estimating the number of sources impinging on a passive array of sensors has received a considerable amount of attention during the last two decades. The first to address this problem were Wax and Kailath, [3]. In their seminal work [3] it is assumed that the additive noise process is a spatially and temporally white Gaussian random process. Given this assumption, the number of sources can be deduced from the multiplicity of the received signal correlation matrix's smallest eigenvalue [4]-[5]. In order to avoid the use of subjective thresholds required by multiple hypothesis testing detectors, Wax and Kailath suggested the use of the Minimum Description Length (MDL) criterion for estimating the number of sources. The MDL estimator can be interpreted as a test for determining the multiplicity of the smallest eigenvalue [5], but its response degrades under low Signal-to-Noise Ratio (SNR) condition due to errors in estimating the data covariance matrix from finite data. In this paper, based on wavelet transform, a new system is proposed to determine the number of sources using MDL and QMF bank analysis. The method can estimate the number of sources under low SNR environment.

2. Wavelet Transform

The wavelet transform (WT) is a time-scale representation technique, which describes a signal by using the correlation with translation and dilatation of a function called "mother wavelet". Wavelet shape can be selected to match the shapes of components embedded in the signal to be analyzed. Such wavelets are excellent templates to separate those components and events from the analyzed signal waveform. The Mallat algorithm for discrete wavelet transform (DWT)[6] known as quadrature mirror filters (QMF). The QMF analyses a finite-length time-domain signal at different frequency bands with different resolutions by successive decomposition into coarse approximation and detail information. Approximations represent the slowly changing features of the signal and conversely details represent the rapidly changing features of the signal.

The wavelet de-noising approach is based on the assumption that random errors in a signal are present over all the coefficients, while deterministic changes get captured in a small number of relatively large coefficients. As a result, a nonlinear thresholding (shrinking) function in the wavelet domain will tend to keep a few larger coefficients representing the underlying signal, while the noise coefficients will tend to reduce to zero.

Practically, the wavelet denoising method consists in applying the QMF bank to the original noisy data, thresholding the wavelet coefficients, and then inverse transforming the thresholded coefficients to obtain the time-domain de-noised data [7]. It should be noted that the performance of the wavelet de-noising depends to the choice of the thresholding rule, the type of wavelet, the maximum depth of wavelet decomposition and the initial SNR.

In this paper, soft thresholding method is used to eliminate noise from noisy data. According to soft thresholding method, the wavelet coefficient between $-\delta$ and δ is set zero [8], while the other are shrunk in absolute value. The threshold δ proposed by Donoho is

$$\delta = \sigma\sqrt{2\log(N)} \qquad (1)$$

where N is the total number of samples and σ is the standard deviation of noise. In order to apply this method in practice, one usually needs to estimate σ. Donoho & Johnstone suggested using as an estimator the median of the coefficients on the finest level divided by 0.6745 which usually works well as long as the signal is contained mainly in the low frequency coefficients [8].

2.1. Quadrature Mirror Filters

Figure 1 shows the structure of the QMF bank. The input signal is processed by the high-pass and the low-pass FIR filters having their impulse responses g(n) and h(n) , respectively. The output of each filter is down sampled. The following relation is held between g(n) and h(n) in the QMF[9].

$$g(n) = (-1)^{1-n} h(1-n) \qquad (2)$$

$$\text{for} \quad n = 0,1,2,\ldots\ldots, N_{tap} - 1$$

where N_{tap} is the number of taps for both FIR filters. The decomposition level of QMF bank is denoted by M (select a suitable number of levels based on the nature of the signals, or on a suitable criteria such as entropy)

3. MDL Principle

Consider an uniform linear array of p sensors and denote by Y(t) the received, p-dimensional, signal vector at time instant t. Denote by $q < p$ the number of signals impinging on the array. A common model for the received signal vector is [10,11]:

$$Y(t) = AS(t) + n(t), for \quad t = 1,2,\ldots\ldots, N \quad (3)$$

where $A = [a(\theta_1), a(\theta_2), \ldots\ldots\ldots, a(\theta_q)]$ is a p×q matrix composed of p-dimensional vectors, and $a(\theta_i)$ = a px1 complex vector, parameterized by an unknown parameter vector (θ_i) associated with *ith* signal. $a(\theta_i)$ is called array response vector or the steering vector and A is referred to as the steering matrix. $S(t) = [s_1(t), s_2(t), \ldots\ldots\ldots, s_q(t)]^T$ is the qx1 vector, and $s_i(t)$ = scalar complex waveform referred to as the *ith* signal. *n(t)* = a px1 complex vector referred to as the additive noise.

The signal parameters which are of interest, are of spatial nature, and thus require the cross-covariance information among the various sensors, i.e., the spatial covariance matrix

$$\begin{aligned} R &= E[Y(t)Y(t)^H] \\ &= AE\{S(t)S(t)^H\}A^H + E\{n(t)n(t)^H\} \end{aligned} \qquad (4)$$

where $E\{.\}$ denotes statistical expectation, and "H" the Hermitian (i.e., transpose conjugate),

$$E\{S(t)S(t)^H\} = R_S \qquad (5)$$

is the source covariance matrix and

$$E\{n(t)n(t)^H\} = \sigma^2 I \qquad (6)$$

is the noise covariance matrix and I denotes the identity matrix , the latter covariance structure is a reflection of the noise having a common variance σ^2 at all sensors and being uncorrelated along all sensors.

The spectral factorization of R will be of central importance, and it is positively guarantees the following representation.

$$R = AR_S A + \sigma^2 I = U \Lambda U^H , \quad (7)$$

with U unitary and $\Lambda = diag\{\lambda_1, \lambda_2, ..., \lambda_P\}$ a diagonal matrix of real eigenvalues ordered such that $\lambda_1 \geq \lambda_2 \geq .... \geq \lambda_P \geq 0$. Observe that any vector orthogonal to A is an eigenvector of R with the eigenvalue σ^2. There is p-q linear independent such vectors. Since the remaining eigenvalues are all larger than σ^2, the eigenvalue/eigenvector pairs can be partitioned into noise eigenvectors (corresponding to eigenvalues $\lambda_{q+1} = \lambda_{q+2} = ... = \lambda_p = \sigma^2$) and signal eigenvectors (corresponding to eigenvalues $\lambda_1 \geq \lambda_2 \geq ... \geq \lambda_q \geq \sigma^2$). Hence, it can be written as

$$R = U_S \Lambda_S U_S^H + U_n \Lambda_n U_n^H , \quad (8)$$

where $\Lambda_n = \sigma^2 I$. Since all noise eigenvectors are orthogonal to A, the columns of U_S must span the range space of A whereas those of U_n span its orthogonal component (the null space of A^H).

As stated earlier that in practice only sample estimates which is denoted by a hat "^" are available. A natural estimates of R is the sample covariance matrix [12]

$$\hat{R} = \frac{1}{N} \sum_{t=1}^{N} Y(t) Y(t)^H \quad (9)$$

for which spectral representation similar to that of R is defined as

$$\hat{R} = \hat{U}_S \hat{\Lambda}_S \hat{U}_S^H + \hat{U}_n \hat{\Lambda}_n \hat{U}_n^H . \quad (10)$$

The problem of is to determine the number of the signals. In theory, the number of the signals can be determined from the multiplicity of the smallest eigenvalue of R. However, in practice , we do not have access to the true data covariance matrix but to its finite sample estimate $\hat{R}$. The smallest eigenvalues of $\hat{R}$ are all different with probability 1, complicating thus the determination of the number of signals. In this case, one may estimate the number of signals by using the MDL criterion [2]

$$MDL(K) = -\log \left(\frac{\prod_{i=1+K}^{P} l_i^{1/(P-K)}}{\frac{1}{P-K} \sum_{i=K+1}^{P} l_i} \right) + \frac{1}{2} K(2P-K) \log(N) \quad (11)$$

where l_i denotes the eigenvalues of $\hat{R}$. The number of signals is taken to be the value of $K \in \{0,1,......,P-1\}$ for which MDL is minimized.

4. Proposed Model

Figure 2 shows the block diagram of the proposed system to estimate the number of sources. The main procedure of the system is described as follows. First, we receive the signals by antennas. Next, these signals are processed by the QMF bank. Then, in every bank, the channel that exceeds threshold level (δ) as compared with other channels, will be chosen by the selection unit. The selected channel's output data is applied to IDWT (inverse of discrete wavelet transform). The output of IDWT is used to compute its covariance matrix. Finally MDL is calculated to estimate the numbers of sources.

4.1. Selection Unit

Figure 3 shows the structure of selection unit. The role of the unit is to select the channel that exceeds threshold level (δ) as compared with other channels. In every bank, the soft threshold is applied to the output of QMF bank. The gate switches are simultaneously close when the output of QMF banks exceeds threshold level (δ).

4.2 MDL Using Wavelet Coefficient

The output of Selection Unit is sent to IDWT, then the output of IDWT is applied to covariance matrix computation unit. Later, estimation the number of sourcescan be by applying the output of the covariance matrix computation unit to MDL.

5. Simulation and Results

In this subsection simulation results with synthetic data are presented. We consider a uniform linear array with 7 elements, and assume three equal-power and independent sources having signal-to-noise ratio (SNR) per element of 10 dB. The sources' directions of arrival (DOA's) are taken to be [10^o,20^o,25^o]. We consider two cases: the first corresponds to classical system (without applying proposed model for MDL); and the second corresponds to modified system (with applying proposed model for MDL). The parameters that used in second case include Daubechies wavelet (db18) and one level decomposition. In first case, the eigenvalues of the sample – covariance matrix are 15.5889, 8.1894, 1.4713, 1.1601, 1.0169, 0.9209, and 0.6529, and the response of MDL method is shown in table (1), the minimum value of MDL is obtained incorrectly for K = 2. In second case, these eigenvalues of the sample – covariance matrix are 14.5598, 6.7785, 0.3785, 0.1369, 0.1110, 0.0948, and 0.0688, and the response of MDL method is shown in table (1), the minimum value of MDL is obtained correctly for K = 3.

We present some numerical experiments for classical system and modified system with two SNR conditions (as shown in figure 4). Figure 4 illustrates that modified system has the higher performance as compared to classical system and the modified system has less number of snapshots than classical system. Moreover, the estimates are consistent as the number of snapshots becomes large and the superiority of the proposed system remains intact despite degradation in SNR.

6. Conclusion

In this paper, a new system has been introduced to determine the number of sources by applying MDL to the outputs of filter banks consisting QMF. The proposed method can give precise estimation for the number of the sources even in low SNR environments. It is based on the idea that wavelet denoising improves the SNR of a noisy signal. We have preceded to perform wavelet denoising of the signal from each sensor of the array independently, prior to estimating the MDL. The utilization of wavelet denoising has reduced the error of the array data covariance estimated from finite data and thus reduces the error of the MDL estimation. In addition, the proposed method can decrease the number of snapshots used for MDL than the classical system.

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	K	0	1	2	3	4	5	6
Classical System	MDL(K)	505.5	297.9	72.5*	85.1	97.1	106.3	110.2
Modified System	MDL(K)	1475.2	985.9	160.4	90.2*	99.3	104.6	115.6

** refers to the minimum value*

Table 1: The Response of the MDL Method

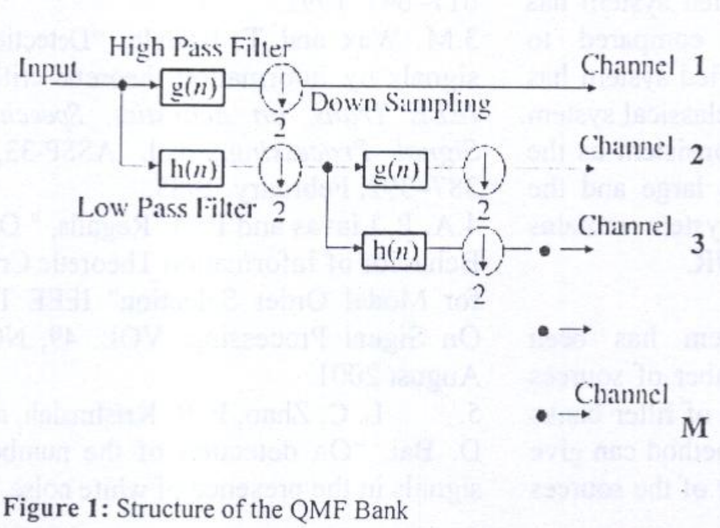

Figure 1: Structure of the QMF Bank

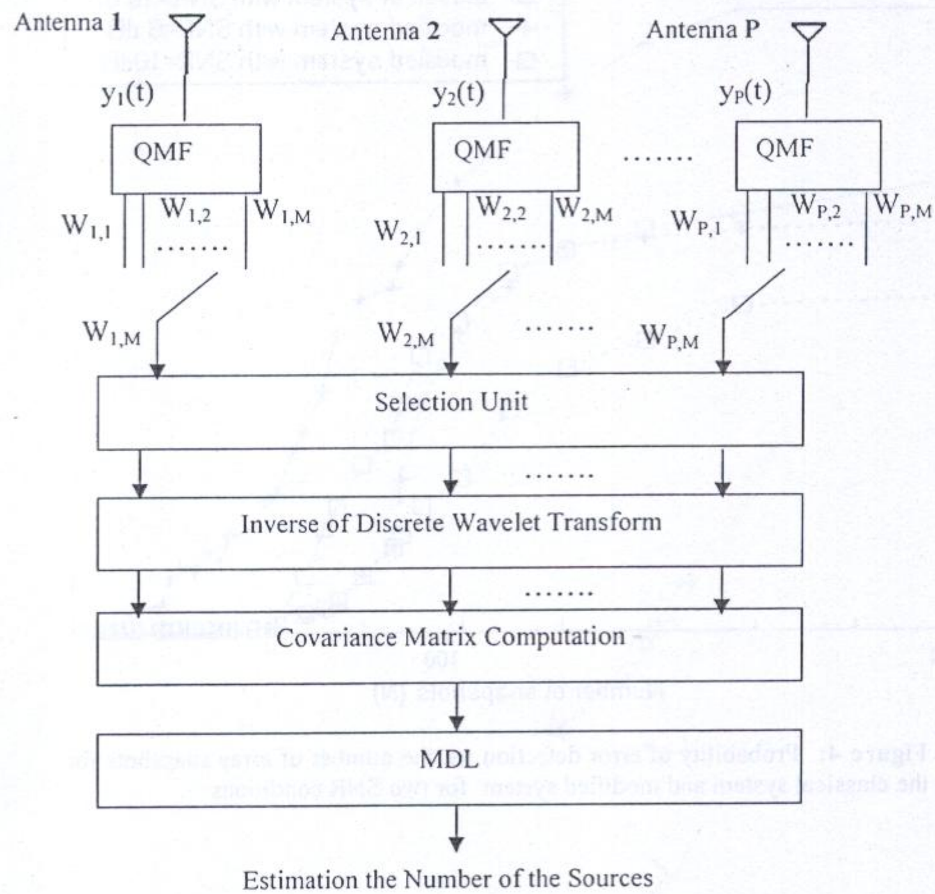

Figure 2: Block Diagram of the Proposed System

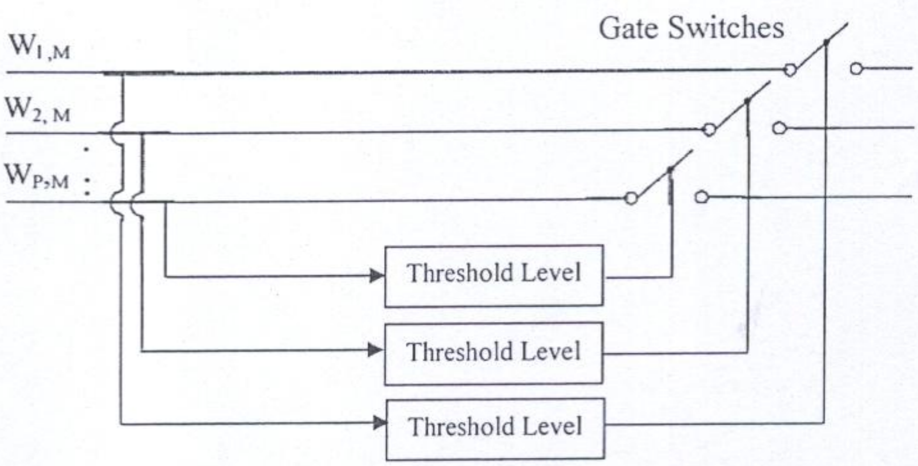

Figure 3: Structure of the Selection Unit

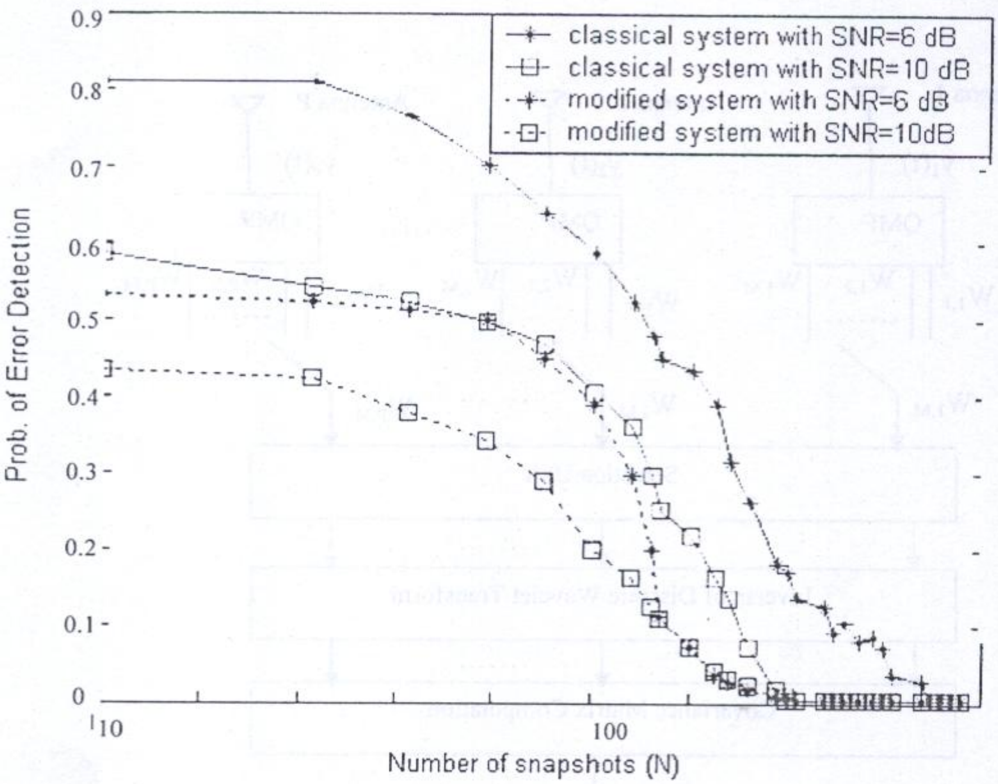

Figure 4: Probability of error detection vs. the number of array snapshots for the classical system and modified system for two SNR conditions