Parallel Kalman Filtering for Real-Time Signal Identification

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Abstract

The applicability of Kalman filter (KF) to real-time signal processing problems is limited by the relatively complex mathematical operations necessary in computing Kalman filtering algorithms. However, with the rapid development of fast processing/memory devices that has offered a new research direction in high-speed real-time, systematic implementation on KF. Presently, the research trend is to achieve a major improvement in computational speed that will come from the concurrent use of many processor cells.Parallel processing usually makes a major impact in real-time signal identification. These require high-speed computations, which must be performed on continuous data streams. This result here in the stimulation of novel architectures for parallel Kalman filter (PKF). The implementation of the PKF is achieved on a simulated radar signal that works in real-time by using the simulink package.

Key words: Signal identification; Real-time signal processing; Parallel processing; Parallel Kalman filter; Simulink package.

الخلاصة

إن تطبيق مرشح كالمن لمعالجة المسائل في الزمن الحقيقي محدود بسبب العمليات الرياضية المعقدة نسبيا التي من الضروري استخدامها في حساب الخوارزمية. لذلك وبسبب التطور في سرعة المعالجات و التي بدورها هيأة اتجاه بحثي جديد في الزمن الحقيقي لتطبيق مرشح كالمن، فان هدف البحث هو تسخير هذه التطورات في سرعة الحساب الناتجة من الاستخدام التوافقي لعدد من خلايا المعالج.

المعالجات المتوازية لها التأثير الفاعل في تعريف الإشارة بالزمن الحقيقي و هذا يتطلب سرعة الحسابات التي تجري علي البيانات المستمرة، مما أثمر عن هيكلية حديثة العهد لتمثيل مرشح كالمن المتوازي. محاكاة هذه الهيكلية تمت علي إشارة رادارية مولدة باستخدام حقيبة السميلنك

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Introduction

The Kalman filter is a linear estimator, which is an optimal recursive data processing algorithm. It is optimal with respect to virtually any criterion that makes sense. One aspect of this optimality is that the KF incorporates all information that can be provided to it. It processes all available measurements, regardless of their precision, to estimate the current value of the variables of interest, with use of

a) Knowledge of the system and measurement device dynamics.
b) The statistical description of the system noises, measurement errors, and uncertainty in the dynamics models.
c) Any available information about initial conditions of the variables of interest.

Conceptually, what any type of filter tries to do is obtain an optimal estimate of desired quantities from data provided by a noisy environment. Here optimal meaning that it minimizes errors in some respect.

The word recursive in the previous description means that, the KF does not require all previous data to be kept in storage and reprocessed every time a new measurement is taken. However, KF is a set of mathematical equations that provides an efficient computational (recursive) means to estimate the state of a process, in a way that minimizes the mean of the squared error. The filter is very powerful in several aspects: it supports estimations of past, present, and even future states. It can do so even when the precise nature of the modeled system is unknown.

This paper is organized as follows: the basic concept of KF is presented in the next section. Hence an illustration to the Estimated process is given in section 2. Sections 3 give the computational and the probabilistic origins. The algorithm that is used to develop the PKF is given in section 4. The development of PKF is given in section 5, which describes the decoupling procedure to obtain the was implemented in the simulink to identify the signal in real time as described in section 7. Conclusions are given in section 8.

In 1960, R.E. Kalman published his famous paper describing a recursive solution to the discrete data linear filtering problem [1]. Since that time, due in large part to advances in digital computing, the KF has been the subject of extensive research and application, particularly in the area of autonomous or assisted navigation. An introduction to the general idea of the KF can be found in Chapter 1 of [2], while amore complete introductory discussion can be found in [3].

2-The Process to be Estimated

The KF addresses the general problem of trying to estimate the state $x \in \Re^n$ of a discrete-time controlled process that is governed by the linear stochastic difference equation

$$x_{k+1} = A_k x_k + Bu_k + w_k \qquad (1)$$

with a measurement $z \in \Re^m$ that is

$$z_k = H_k x_k + v_k \qquad (2)$$

where z_k is an observation at time k, x_k is the system state at time k. A, B and H are linear scaling. The random variables w_k and v_k represent the process and measurement noise (respectively). They are assumed to be independent (of each other), white, and with normal probability

The justification for eq.(9) is rooted in the probability of the *a priori* estimate $x_k(-)$ conditioned on all prior measurements z_k (Bayes' rule)[4]. For now let it suffice to point out that the KF maintains the first two moments of the state distribution,

$$E[x_k] = \hat{x}_k \quad (11)$$

$$E[(x_k - \hat{x}_k)(x_k - \hat{x}_k)^T = P_k. \quad (12)$$

The *a posteriori* state estimate eq.(9) reflects the mean (the first moment) of the state distribution it is normally distributed if the conditions of eq.(3) and eq.(4) are met. The *a posteriori* estimate error covariance eq.(8) reflects the variance of the state distribution (the second non-central moment).

4- The Discrete Kalman Filter Algorithm

This section will be for a broad overview, covering the "high-level" operation of one form of the discrete KF, After presenting this high-level view, there will be a focus to the specific equations and their use in this version of the filter.

The KF estimates a process by using a form of feedback control: the filter estimates the process state at some time and then obtains feedback in the form of (noisy) measurements. As such, the equations for the KF fall into two groups: *time update* equations and *measurement update* equations. The time update equations are responsible for projecting forward (in time) the current state and error covariance estimates to obtain the *a priori* estimates for the next time step.

The measurement update equations are responsible for the feedback i.e. for incorporating a new measurement into the *a priori* estimate to obtain an improved *a posteriori* estimate.

The time update equations can also be thought of as *predictor* equations, while the measurement update equations can be thought of as *corrector* equations. Indeed the final estimation algorithm resembles that of a *predictor-corrector* algorithm for solving numerical problems.

The specific equations for the time and measurement updates are presented below in eq.(13 & 14) for discrete KF time update, while eq.(15, 16 & 17) are for discrete KF measurement update.

$$\hat{x}_{k+1} = A_k \hat{x}_k + Bu_k \quad (13)$$

$$P_{k+1}(-) = A_k P_k A_k^T + Q_k \quad (14)$$

Again notice how the time update equations above project the state and covariance estimates forward from time step k to step $k+1$. A_k and B are from eq.(1), while Q_k is from eq.(3).

The first task during the measurement update is to compute the KF. Notice that the equation given here as eq.(15) is the same as eq.(10).

$$K_k = P_k(-)H_k^T (H_k P_k(-) H_k^T + R_k)^{-1} \quad (15)$$

$$\hat{x}_k = \hat{x}_k(-) + K(z_k - H_k \hat{x}_k(-)) \quad (16)$$

$$P_k = (1 - K_k H_k) P_k(-) \quad (17)$$

The next step is to actually measure the process to obtain z_k, and then to generate an *a posteriori* state estimate by incorporating the measurement as in eq.(16). Again eq.(16) is simply eq.(9) repeated here for completeness. The final step is to obtain an *a posteriori* error covariance estimate via eq.(17).

After each time and measurement update pair, the process is repeated with the previous *a posteriori* estimates used to project or predict the new *a priori* estimates. This recursive nature is one of the very appealing features of the KF. It makes practical implementations much more feasible than an implementation of another filter which may be designed to operate on *all* of the data *directly* for each estimate. The KF instead recursively distributions. Equations (1&2) can be expressed diagrammatically as in Fig.(1).

In practice, the *process noise covariance* Q and *measurement noise covariance matrices* R are assumed to be constant, and with normal probability distributions

$$p(w) \approx N(0,Q), \quad (3)$$

$$p(v) \approx N(0,R) \quad (4)$$

The $n \times n$ matrix A in the difference eq.(1) relates the state at k time step to the state at step $k+1$, in the absence of either a driving function or process noise. Note that, in practice might change with each time step, but here it is assumed to be constant.

The $n \times l$ matrix B relates the optional control input $u \in \Re^l$ to the state x. The $m \times n$ matrix H in the measurement eq.(2) relates the state to the measurement z_k.

3- The Computational Origins of the Filter

Define $\hat{x}_k(-) \in \Re^n$ to be *a priori state* estimate at step k given knowledge of the process prior to step k, and $\hat{x}_k \in \Re^n$ to be a *posteriori* state estimate at step k given measurement z_k. Then define *a priori* and *a posteriori* estimate errors as

$$e_k(-) \equiv x_k - \hat{x}_k(-), \quad (5)$$

and

$$e_k \equiv x_k - \hat{x}_k. \quad (6)$$

The *a priori* estimate error covariance is then:

$$P_k(-) = E[e_k(-)e_k^T], \quad (7)$$

and the *a posteriori* estimate error covariance is

$$P_k = E[e_k e_k^T]. \quad (8)$$

In deriving the equations for the KF, the beginning will be with the goal of finding an equation that computes an *a posteriori* state estimate $\hat{x}_k$ as a linear combination of an *a priori* estimate $\hat{x}_k(-)$ and a weighted difference between an actual measurement z_k and a measurement prediction $H_k\hat{x}_k(-)$ as shown below in eq.(9).

$$\hat{x}_k = \hat{x}_k(-) + K(z_k - H_k\hat{x}_k(-)) \quad (9)$$

The difference $(z_k - H_k\hat{x}_k(-)$ in eq.(9) is called the measurement *innovation,* or the *residual.* The residual reflects the discrepancy between the predicted measurement $H_k\hat{x}_k(-)$ and the actual measurement z_k. A residual of zero means that the two are in complete agreement.

The $n \times m$ matrix K in eq.(9) is chosen to be the *gain* or *blending factor* that minimizes the *a posteriori* error covariance eq.(8). This minimization can be accomplished by first substituting eq.(9) into the above definition for e_k, substituting that into eq.(8), performing the indicated expectations, taking the derivative of the trace of the result with respect to K, setting that result equal to zero, and then solving for K. One form of the resulting K that minimizes eq.(8) is given by

$$K_k = P_k(-)H_k^T(H_kP_k(-)H_k^T + R_k)^{-1} \quad (10)$$

Equation (8) represents the Kalman gain. The weighting by K is that as the measurement error covariance R_k approaches zero. The actual measurement z_k is "trusted" more and more, while the predicted measurement $H_k\hat{x}_k(-)$ is trusted less and less. On the other hand, as the *a priori* estimate error covariance $P_k(-)$ approaches zero the actual measurement z_k is trusted less and less, while the predicted measurement $H_k\hat{x}_k(-)$ is trusted more and more.

conditions the current estimate on all of the past measurements. Fig.(2) below offers a complete picture of the operation of the filter.

5- Parallel Kalman Filter

In this section, the methodology of systolic and pipelined implementation on KF in real-time is presented. Also a model-based processor for the dynamic estimation problem is also developed.

The state space representation is used as the basic model. The operation of the filter as a predictor-corrector algorithm is followed by the derivation of the discrete KF for estimating the discrete-time states from discrete observations. In order to speed up KF computations, parallel processing is performed at two levels:

1) the predictor and corrector equations of the KF are decoupled so that the predictor and corrector can be computed on separate processors.
2) the measurement data are pipelined into each processor. Therefore, both multiprocessing and pipelining are considered to achieve large improvements in computational speed.

The standard Kalman filtering equations are given by the following:

(a)Predictor

$$\begin{cases}\hat{x}_k(-)=\Phi_{k-1}\hat{x}_{k-1}(+)\\ P_k(-)=\Phi_{k-1}P_k(+)\Phi_{k-1}^T\end{cases} \quad (18)$$

(b)Corrector

$$\begin{cases}\hat{x}_k(+)=\hat{x}_k(-)+K_k[y_k-H_k\hat{x}_k(-)]\\ P_k(+)=[I-K_kH_k]P_k(-)\end{cases} \quad (19)$$

where the Kalman gain is given by:

$$K_k=P_k(-)H_k^T[H_kP_k(-)H_k^T+R_k]^{-1} \quad (20)$$

Note that the equations are inherently sequential since the temporal update (predictor equations) must be evaluated before the observation updates (corrector equations). From a computational point of view, this is not desirable since to evaluate the corrector, a uniprocessor must wait until the predictor has been evaluated. To avoid this difficulty, the predictor-corrector equations can be decoupled to obtain a PKF.

The decoupling of the state predictor and corrector is achieved by forcing the predictor to lead the corrector by one time step, as follows:

(a) The predictor:

$$\hat{x}_{k+1}(-)=\Phi_k\Phi_{k-1}\hat{x}_{k-1}(+) \quad (21)$$

(b) The corrector:

$$\hat{x}_k(+)=\hat{x}_k(-)+K_k[y_k-H_k\hat{x}_k(-)] \quad (22)$$

Let the covariance of the estimation error before and after a measurement update be denoted by:

$$P_{k+1}(-)=E\tilde{x}_{k+1}(-)\tilde{x}_{k+1}^T(-) \quad (23)$$

$$P_k(+)=E\tilde{x}_k(+)\tilde{x}_k^T(+) \quad (24)$$

where

$$\tilde{x}_{k+1}(-)\overset{\Delta}{=}\hat{x}_{k+1}(-)-x_{k+1} \quad (25)$$

$$\tilde{x}_k(+)\overset{\Delta}{=}\hat{x}_k(+)-x_k \quad (26)$$

By direct computation, it can be shown that the covariance of the estimation error before the update is given by:

$$P_{k+1}(-)=\Phi_k\Phi_{k-1}(+)\Phi_{k-1}^T\Phi_k^T \quad (27)$$

Because the form of eq.(22) is the same as the 1[st] equation of eq.(19), the covariance of the estimation error after a measurement update in the PKF is given by

$$P_k(+)=(I-K_kH_k)P_k(-) \quad (28)$$

$$K_k=P_k(-)H_k^T[H_kP_k(-)H_k^T+R_k]^{-1} \quad (29)$$

6- Summary of the decoupled PKF equations

Once the decoupling has been performed, i.e. the predictor and corrector equations in KF have been decoupled, then computations can be performed simultaneously on separate processors. One processor will be selected for predictor equations and one processor for the corrector equations. The algorithm of PKF, whose equations are given below, is shown in Fig.(3)

(a) The predictor is expressed as

$$\begin{cases} \hat{x}_{k+1}(-) = \Phi_k \Phi_{k-1} \hat{x}_{k-1}(+) \\ P_{k+1}(-) = \Phi_k \Phi_{k-1} P_{k-1}(+) \Phi_{k-1}^T \Phi_k^T \end{cases} \quad (30)$$

(b) The corrector is expressed as

$$\begin{cases} \hat{x}_k(+) = \hat{x}_k(-) + K_k[y_k - H_k \hat{x}_k(-)] \\ P_k(+) = [I - K_k H_k] P_k(-) \end{cases} \quad (31)$$

(c) The Kalman gain is expressed as

$$K_k = P_k(-) H_k^T [H_k P_k(-) H_k^T + R_k]^{-1} \quad (32)$$

7- Real-time signal identification

The PKF in Fig(4) represents the simulink implementation to identify the signal in real-time. The signals to be identified are sinusoidal signal and a simulated radar signal[5].

Figure (5) illustrious the signal identification (sinusoidal signal) that results from applying the PKF algorithm. This has been done, by using the Matlab together with the simulink package.

Now, the next example is to replace the sinusoidal signal in Fig.(4) with a simulated radar signal, Fig.(6) shows the actual signal together with the identified one by using the PKF.

8- Conclusions

The effort of this work is to develop the identification capabilities of signals and particularly the radar signal. Also a treatment to signals in real-time was

designed and implemented on the simulink, this was carried out by introducing the parallel Kalman filter.

The advantages of the approach is assessed, hence is provided with motivation for producing estimator that is

robust to noisy and incomplete date. The Implementation of the parallel Kalman filter is to reduce Kalman filter computational time in sequal makes tracking to the signal in real-time.

References

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Fig.(4) Simulink realization to the PKF to identify the sinusoidal signal.

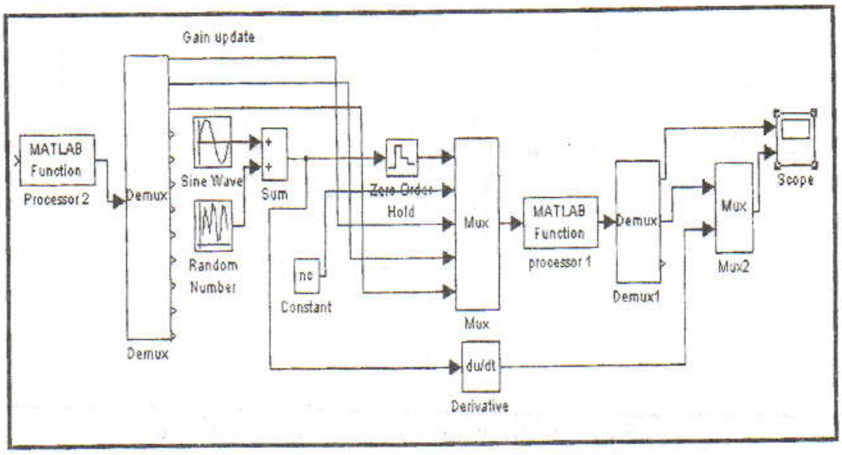

Fig.(5) Sinusoidal signal identification by using the PKF

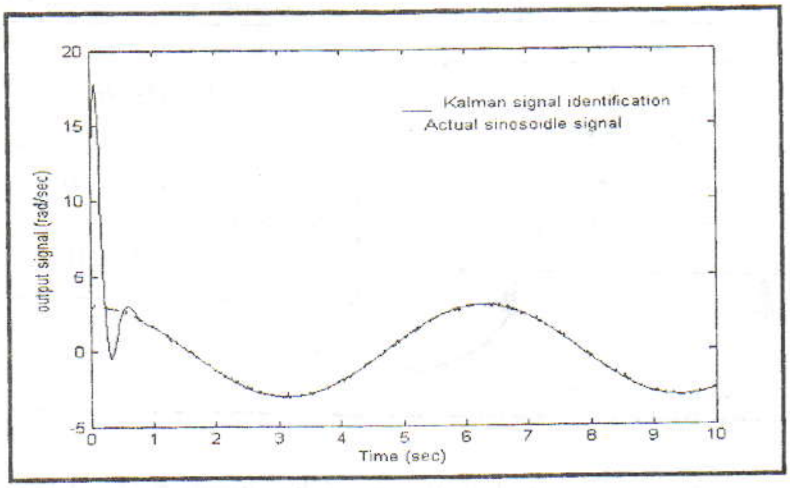

Fig.(6) The actual and the identified sample of radar signal using PKF in real-time

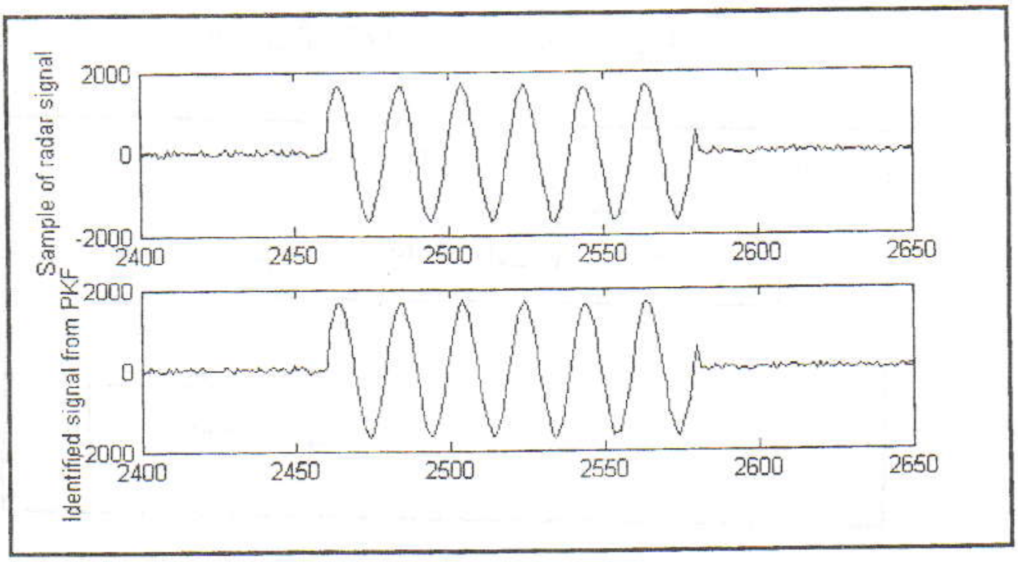

Fig.(4) Simulink realization to the PKF to identify the sinusoidal signal.

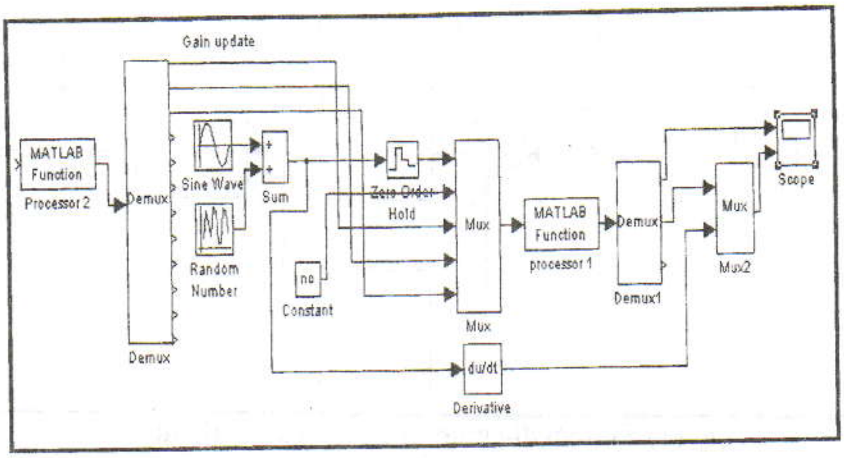

Fig.(5) Sinusoidal signal identification by using the PKF

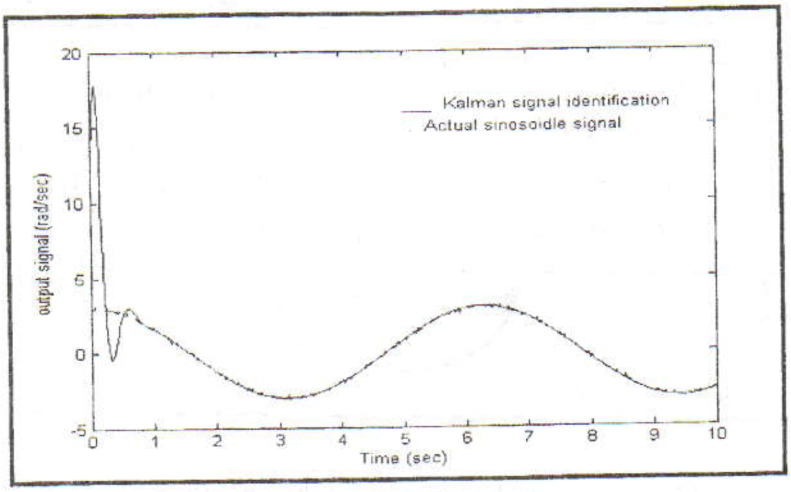

Fig.(6) The actual and the identified sample of radar signal using PKF in real-time