DESIGN OF KALMAN FILTER FOR AUGMENTING GPS TO INS SYSTEMS

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Received on: 10 /12 / 2005
Accepted on: 1 / 6/2006

Abstract

Strapdown Inertial Navigation Systems (SDINS) and Global Positioning Systems (GPS) both can be used for a wide range of navigation functions. Each has its strength and weakness in this respect.

Kalman filter is used to incorporate information from the accelerometers and gyros at high rates and information from GPS measurements at lower rates to improve ballistic missile strapdown navigation systems by GPS aiding. A linearized Kalman filter model is implemented for augmented system based on the navigation frame to estimate the inertial system error based on the measurement error between the GPS and INS systems. A three degree of freedom missile flight simulator with strapdown navigation algorithms was carried out in order to compare the proposed augmented system with other navigation algorithms. The presents results show that the error in both position and velocity is very small compared with other algorithms.

Keywords: Vehicular navigation, GPS/INS integration, Kalman filter

الخلاصة

إن منظومات الملاحة من النوع الثابت (SDINS) وأنظمة تحديد المواقع العالمية (GPS) كلاهما يُمْكِنُ أنْ تُستعملَ لتشكيلة واسعة من وظائفِ الملاحةِ. كُلّ لهُ قوّته وضعفه في هذا المجال.

مرشح Kalman أستعملُ لدَمْج المعلوماتِ مِنْ أجهزة قياس التعجيل (Accelerometers) والبوصلة الجيروسكوبيةِ (Gyros) ذات الترددات العالية مع المعلوماتِ مِنْ مقاييس أنظمة تحديد المواقع العالمية ذات الترددات الأوطأ لتحسين أنظمة ملاحةِ الصاروخ الباليستي بمُسَاعَدَة (GPS). أي نموذج مرشح linearized Kalman مُطبّقٌ للنظام المَدْمُوج مستند على إطار الملاحة أنْ يُخمّنَ خطأ نظام inertial مستند على خطأ المقياسَ بين أنظمة (GPS) ونظام (INS).

محاكي طيران قذيفةِ للدرجة الثالثة من الحرية مع خوارزمياتِ ملاحةِ strapdown نُفّذَ لكي يُقارنَ النظامَ المَدْمُوجَ المُقترَحَ بخوارزمياتِ الملاحةِ الأخرى. من النتائجِ بأنّ ان الخطأ في كلا الموقع والسرعة صغيرةُ جداً مُقارَنتاً بالخوارزمياتِ الأخرى.

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1. Introduction

Since the early 1960's, modern navigation has been making use of the hybrid (integrated) navigation systems, where various electronic sensing devices (sensors) are used side by side, to collect the information necessary to find the "continuous" position of the navigated vehicle, and to reduce inertial sensor errors [1]. The integrated system combining several independent navigation sensors, like inertial measurement unit, Doppler radar, and radio position fixed devices (e.g. TACAN) were used. Sindlinger [2] investigated the optimization of such integrated navigation systems e.g. a "Combined Navigation System". A comprehensive survey of this system presented in [3, 4].

Integrated systems based on GPS and INS generated increased interest in the airborne survey and remote sensing community over the past few years. Where, with full operational GPS capability, it has been recognized that an optimal combination of GPS with inertial navigation brings a number of advantages over stand-alone inertial or GPS navigation [5].

GPS contributes its high accuracy and stability over time, enabling continuous monitoring of inertial sensors errors. Implementation of closed-loop INS error calibration allows continuous on-the-fly error update that bounds INS errors, because the INS have drift characteristics that cause the system error to grow with time. They also exhibit un-damped oscillatory errors that are undesirable [6], leading to decreased estimation accuracy. On the other hand, INS contributes immunity to GPS outage, continuous attitude solution, and reduction of the GPS ambiguity search volume / time [5].

This paper is concerned with the improvement of ballistic missile strapdown navigation system by GPS aiding. And how Kalman filter is required for such applications to smooth and feed the measured information from the GPS unit to the INS system.

Linearized and extended Kalman filters are the two basic ways for the particular combination of inertial data integrated with GPS system. Figure (1) shows the linearized Kalman

filter, or the feed forward configuration, in which the output of inertial system measurements are not used as "measurements" in the Kalman filter. Rather, they are used exclusively to provide the reference trajectory.

Although, both Linearized and extended modes of operation have the same performance within the linearity assumption and may have been used successfully [7], the feedback approach is to be preferred in applications where the mission time is long, as would be true of a ship at sea for many weeks or months [6, 8].

In our application, the mission time is short and, hopefully, the actual trajectory will match the programmed

one closely, so an ordinary linearized Kalman filter will be preferred, where the purpose of this filter is to estimate the inertial system errors based on the measurement error between the GPS and INS systems.

The rest of this paper is organized as four sections, error dynamic equations of inertial navigation systems will be derived first. The GPS measurement equation discussed in the next section, while the Kaman filters equations for the augmented system described in the third section. The last section will have the simulation results and discussions.

2. Ins Error Model

Errors can be developed in the navigation system state through a variety of sources as discussed in [9, 10]; therefore it is important to understand the behavior of the navigation system error.

2.1 Position Error Dynamics

Since the position dynamic equation is a function of both position and velocity [10], thus the partial derivatives of this equation will be:

$$\delta \dot{\underline{r}}^n = F_{11}\delta \underline{r}^n + F_{12}\delta \underline{v}^n \quad \dots (1)$$

where

$$F_{11} = \begin{pmatrix} \frac{\partial \dot{L}}{\partial L} & \frac{\partial \dot{L}}{\partial l} & \frac{\partial \dot{L}}{\partial h} \\ \frac{\partial \dot{l}}{\partial L} & \frac{\partial \dot{l}}{\partial l} & \frac{\partial \dot{l}}{\partial h} \\ \frac{\partial \dot{h}}{\partial L} & \frac{\partial \dot{h}}{\partial l} & \frac{\partial \dot{h}}{\partial h} \end{pmatrix} = \begin{pmatrix} 0 & 0 & \frac{-v_N}{(R_N+h)^2} \\ \frac{v_E \sin L}{(R_E+h)\cos^2 L} & 0 & \frac{-v_E}{(R_E+h)^2 \cos L} \\ 0 & 0 & 0 \end{pmatrix} \quad \ldots (2)$$

$$F_{12} = \begin{pmatrix} \frac{\partial \dot{L}}{\partial v_N} & \frac{\partial \dot{L}}{\partial v_E} & \frac{\partial \dot{L}}{\partial v_D} \\ \frac{\partial \dot{l}}{\partial v_N} & \frac{\partial \dot{l}}{\partial v_E} & \frac{\partial \dot{l}}{\partial v_D} \\ \frac{\partial \dot{h}}{\partial v_N} & \frac{\partial \dot{h}}{\partial v_E} & \frac{\partial \dot{h}}{\partial v_D} \end{pmatrix} = \begin{pmatrix} \frac{1}{R_N+h} & 0 & 0 \\ 0 & \frac{1}{(R_E+h)\cos L} & 0 \\ 0 & 0 & -1 \end{pmatrix} \quad \ldots (3)$$

$[L,\ l,\ h]$: are geodetic positions (latitude, longitude, and height)

$[V_N\ V_E\ V_D] = V^n$: geodetic velocity vector (north, east, and down)

R_N and R_E: are the radii of curvature in the north and east direction, considered as constants.

2.2 Velocity Error Dynamics

The computed value of the velocity dynamic equation can be perturbed; and by collecting the first order terms only, the velocity equations can be reduced to [8]: ... (4)

$$\delta \dot{\underline{v}}^n = F_{21}\delta \underline{r}^n + F_{22}\delta \underline{v}^n + F_{23}\underline{\varepsilon}^n + R_{b2n}\delta f^b + g^n$$

where

$$F_{21} = \begin{pmatrix} -2v_E w_{ie}\cos L - \frac{v_E^2}{(R_E+h)\cos^2 L} & 0 & \frac{-v_N v_D}{(R_N+h)^2} + \frac{v_E \tan L}{(R_E+h)^2} \\ 2w_{ie}(v_N \cos L - v_D \sin L) + \frac{v_E v_N}{(R_E+h)\cos^2 L} & 0 & \frac{-v_E v_D}{(R_E+h)^2} - \frac{-v_N v_E \tan L}{(R_E+h)^2} \\ 2v_E w_{ie}\sin L & 0 & \frac{v_E^2}{(R_E+h)^2} - \frac{v_N^2}{(R_N+h)^2} - \frac{2g_y}{R+h} \end{pmatrix} \quad \ldots (5)$$

$$F_{22} = \begin{pmatrix} \frac{v_D}{R_N+h} & -2w_{ie}\sin L - 2\frac{v_E \tan L}{R_E+h} & \frac{v_N}{R_N+h} \\ 2w_{ie}\sin L + \frac{v_E \tan L}{R_E+h} & \frac{v_D + v_N \tan L}{R_E+h} & 2w_{ie}\cos L + \frac{v_E}{R_E+h} \\ \frac{-2v_N}{R_N+h} & -2w_{ie}\cos L - 2\frac{v_E}{R_E+h} & 0 \end{pmatrix} \quad \ldots (6)$$

$$F_{23} = \begin{pmatrix} 0 & f_D & -f_E \\ -f_D & 0 & f_N \\ f_E & -f_N & 0 \end{pmatrix} \quad \ldots (7)$$

$$g_y = g_0 \left(\frac{R}{R+h} \right)^2 \quad \ldots (8)$$

And $\quad R = \sqrt{R_E R_N} \quad \ldots (9)$

Perturbing the upper Eq.(9) yields:

$$\delta g_y = -2\left(\frac{g_y}{R+h} \right)\delta h \quad \ldots (10)$$

and

w_{ie} : Earth angular velocity (7.2921×10^{-5} rad / sec)

2.3 Attitude Error Dynamics

The attitude error dynamic equations can be derived in the same manner as described in [11]:

$$\dot{\underline{\varepsilon}} = F_{31}\delta \underline{r}^n + F_{32}\delta \underline{v}^n + F_{33}\underline{\varepsilon}^n - R_{b2n}w_{ib}^b \quad \ldots (11)$$

where

$$F_{31} = \begin{pmatrix} -w_{ie}\sin L & 0 & \frac{-v_E}{(R_E+h)^2} \\ 0 & 0 & \frac{v_N}{(R_N+h)^2} \\ -w_{ie}\cos L - \frac{v_E}{(R_E+h)\cos^2 L} & 0 & \frac{v_E \tan L}{(R_E+h)} \end{pmatrix} \quad \ldots (12)$$

$$F_{32} = \begin{pmatrix} 0 & \frac{1}{R_E+h} & 0 \\ \frac{-1}{R_N+h} & 0 & 0 \\ 0 & \frac{-\tan L}{R_E+h} & 0 \end{pmatrix} \quad \ldots (13)$$

$$F_{33} = \begin{pmatrix} 0 & w_D & w_E \\ -w_D & 0 & w_N \\ w_E & -w_N & 0 \end{pmatrix} \quad \ldots (14)$$

2.4 General Ins Error Equations

From the above subsections, the general state equation can be given by:

$$\dot{x}_i(t) = A(t)x_i(t) + G(t)u(t) \quad \ldots (15)$$

where

$x_i(t)$: The nine INS error states = $[\delta \underline{r}^n, \delta \underline{v}^n, \delta \underline{\varepsilon}^n]$

$$A(t) = \begin{pmatrix} F_{11} & F_{12} & 0 \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{pmatrix} \quad \ldots (16)$$

$$G = \begin{pmatrix} 0 & 0 \\ C_b^n & 0 \\ 0 & -C_b^n \end{pmatrix} \quad \ldots (17)$$

$$u = \begin{pmatrix} \delta f^n \\ \delta w_{ib}^b \end{pmatrix} \quad \ldots (18)$$

where

F_{ij} : Matrices defined in previous subsections

δf^n: Accelerometers noise in navigation frame

δw_{ib}^{b}: Gyros noise in navigation frame

C_b^n: Transformation matrix from body to navigation frame

3. Kalman Filter Implementation For The Augmented System

The state equation for the augmented Kalman filter is given by the equations (14)-(17), where the elements of $\underline{u}(t)$ are white noise whose covariance matrix is given by:

$$E\left[\underline{u}(t)\underline{u}(t)^T\right] = Q(t)\delta(t-\tau) \qquad \text{... (19)}$$

where

δ ( ): Denotes the Dirac delta function whose unit is 1/time [11]

Q: Spectral density matrix, and has the form:

$$Q = diag(\sigma_{ax}^2 \;\; \sigma_{ay}^2 \;\; \sigma_{az}^2 \;\; \sigma_{wx}^2 \;\; \sigma_{wy}^2 \;\; \sigma_{wz}^2) \qquad \text{... (20)}$$

where σ_a and σ_w are standard deviations of accelerometers and gyroscopes, respectively.

For discrete Kalman filter, the state equation, which is given in above equation, must have the form: ... (21)

$$X_i(k+1) = F(k)X_i(k) + w(k)$$

where

$X_i(k+1)$: INS error state vector

$F(k)$: State transition matrix

$w(k)$: white noise during the time interval (t_k, t_{k-1})

But the sampling interval in the INS system is very small, the state transition matrix, $F(k)$, can be given by[3, 8]:

$$F(k) = \exp\left(A(t)\Delta t\right) \approx I + A(t)\Delta t \qquad \text{... (22)}$$

where I is an identity matrix, and Δt is the sampling time. The discrete from of the spectral density Q may be given by [6, 12]:

$$Q(k) = E\left[w(k)w(k)^T\right] \approx GQG^T\Delta t \qquad \text{... (23)}$$

The observation equation, for the complete state space model given by [13, 14]:

$$z(k) = H(k)X(k) + v(k) \qquad \text{... (24)}$$

which express the vector measurement, $z(k)$, at time t_k as a linear combination of the state vector, $x(k)$, pulse the random measurement error, $v(k)$, with covariance matrix of:

$$E\left[v(k)v(k)^T\right] = \begin{cases} R(k), & i = k \\ 0, & i \neq k \end{cases} \qquad \text{... (25)}$$

and

$$E\left[w(k)v(i)^T\right] = 0, \qquad \forall i,k \qquad \text{... (26)}$$

So that, the measurement equation can be written as:

$$z(k) = \begin{bmatrix} \underline{r}_{INS}^n - \underline{r}_{GPS}^n \\ \underline{v}_{INS}^n - \underline{v}_{GPS}^n \end{bmatrix} = \begin{bmatrix} L_{INS} - L_{GPS} \\ l_{INS} - l_{GPS} \\ h_{INS} - h_{GPS} \\ \underline{v}_{INS}^n - \underline{v}_{GPS}^n \end{bmatrix} \qquad \text{... (27)}$$

and the H(k), matrix will be a constant values:

$$H(k) = \begin{bmatrix} I_{3\times3} & 0_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & I_{3\times3} & 0_{3\times3} \end{bmatrix} \qquad \text{... (28)}$$

and the covariance matrix for the measurement noise sequence for C/A-code pseudorange data is:

$$R(k) = E\{v_r(k)v_r^T(k)\} = diag(\sigma_L^2 \;\; \sigma_l^2 \;\; \sigma_h^2 \;\; \sigma_{V_N}^2 \;\; \sigma_{V_E}^2 \;\; \sigma_{V_D}^2) \qquad \text{... (29)}$$

From the above model, the Kalman filter can be applied.

In many cases, the estimation problem begins with no prior measurements. Thus, if the process mean is zero, the initial estimate is zero, and the associated error covariance matrix is just the covariance matrix of the states. Therefore ... (30) $P(0)$ can be found from:

$$P(0) = E\{X_i(k)X_i^T(k)\} = diag(\sigma_{\delta L}^2 \;\; \sigma_{\delta l}^2 \;\; \sigma_{\delta h}^2 \;\; \sigma_{\delta V_N}^2 \;\; \sigma_{\delta V_E}^2 \;\; \sigma_{\delta V_D}^2 \;\; \sigma_{\delta\varepsilon_N}^2 \;\; \sigma_{\delta\varepsilon_E}^2 \;\; \sigma_{\delta\varepsilon_D}^2)$$

4. Simulation Results And Discussion

The outputs of the GPS receiver are directly comparable with the INS outputs (i.e. both outputs are position and velocity). Thus, the simulation dose not needs to handle raw GPS data structure and implement a GPS position solution. In other words, this simulation discusses the operation of the Kalman filter for the augmented system only.

Kalman filter for the augmented system added to the operation of terrestrial algorithm as shown in figure (2), where the GPS data can be used to give the correct initial position and velocity.

In simulation we assumed that the GPS provide missile position and velocity in every one second, while the INS data is available in every 0.1 second, and the receiver Kalman filter used to predict the required values (position and velocity) between sampling instants of the GPS receiver.

Figure (3) shows a comparison between the velocity of the typical trajectory and the navigation velocity output. While displacement was compare in figure (4). From the above results, we note that the error in both position and velocity is very small compared with the terrestrial algorithm discussed in [10], although the several types of noise dispersion was selected for the values of σ_a and σ_w.

The position error in augmented system in ECEF coordinates are given in figure (5), where we assumed the discontinuity in GPS reading. From these figures, Kalman filter always stable although the discontinuity in GPS readings.

5. References

1. Vanicek, P. and M. Omerbasic, 1996, "Does a Navigation Algorithm have to use Kalman Filter?" InterNet Site, 1996.
2. Sndlinger, R., 1978, "Investigations on the Optimization of Aided Inertial Navigation System," AGARD No.95, Strapdown Systems, pp. 124-139.
3. Dorobantu, R. and B. Zebhauser, 1999, "Field Evaluation of a Low-Cost Strapdown IMU by Means GPS," AGARD No.95, Strapdown Systems.
4. Amean, Z. S., 2001, "Development of GPS Navigation Aiding for a Ballistic Missile," M.Sc. Thesis, Department of Control and Computers Engineering, University of Technology.
5. Dorota, A. G. and J. Wang, 1998, "Gravity Modeling for High-Accuracy GPS/INS Integration," Navigation, Vol. 45, No.3, pp.209-220.
6. Brown, R. G. and P. Y. C. Hwang, 1992, *Introduction to Random Signals and Applied Kalman Filtering*, 2nd Edition, Johan Wiley and Sons., Inc.
7. Lin, C. F. 1991, *Modern Navigation, Guidance, and Control Processing*, Volume II, by Prentice-Hall, Inc.
8. Lee, H. K., et al, 2001, "Kinematic Positioning With an Integrated GPS/Pseudolite/INS," InterNet Site.
9. Britting, K. R. 1971, *Inertial Navigation Systems Analysis*, Johan Wiley and Sons, Inc.
10. Khadim, A. A., 2000, "Application of Analytical Methods to Improve the Accuracy of Strapdown Navigation System," M.Sc. Thesis, College of Engineering, University of Baghdad.
11. Shin, E. H., 2001, "Accuracy Improvement of Low Cost INS/GPS for Land Applications," M.Sc. Thesis, Department of Geometric Engineering, University of Calgary.
12. Abdul-Kader, R., 2001, "Ph.D. Lectures in Stochastic Control," Electrical and Electronic Eng. Department, Military College of Engineering.
13. Bozic, S. M., 1981, *Digital and Kalman Filtering*, Edward Arnold.
14. Sorenson H. W., 1985, *Kalman Filter; Theory and Application*," the Institute of Electrical and Electronics Engineering, Inc.

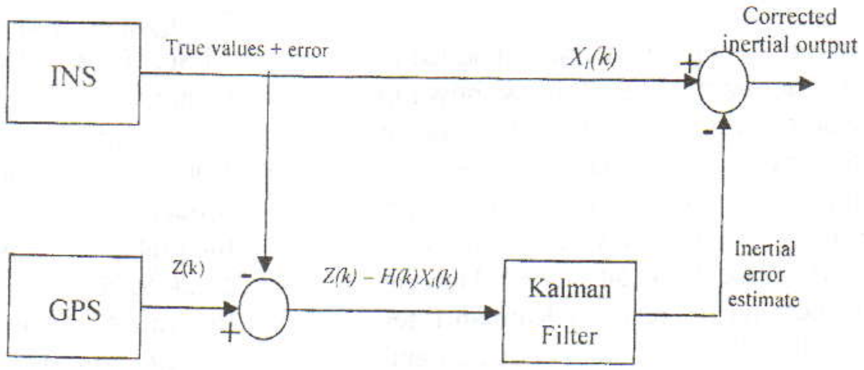

Figure 1. Feed forward configuration

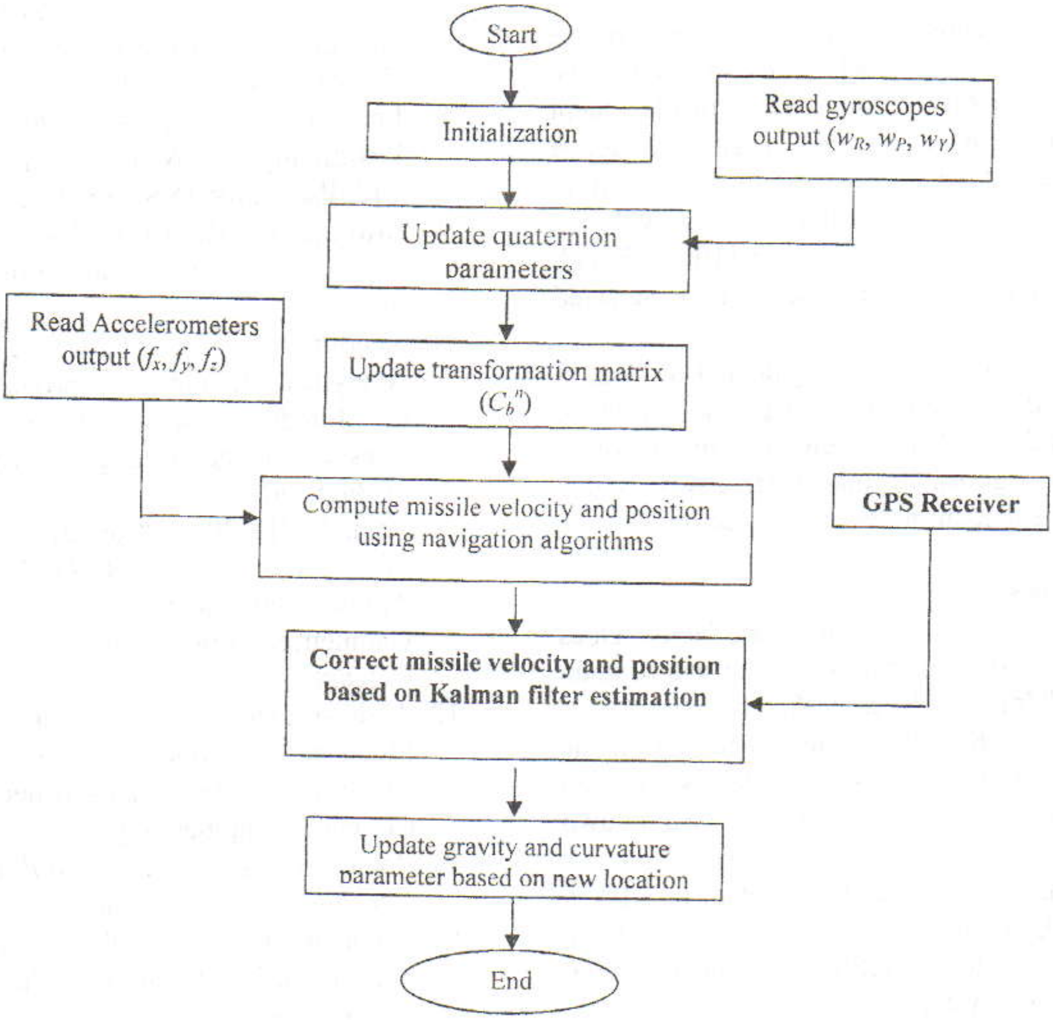

Figure 2. Flow diagram of terrestrial navigation algorithm with augmented Kalman filter

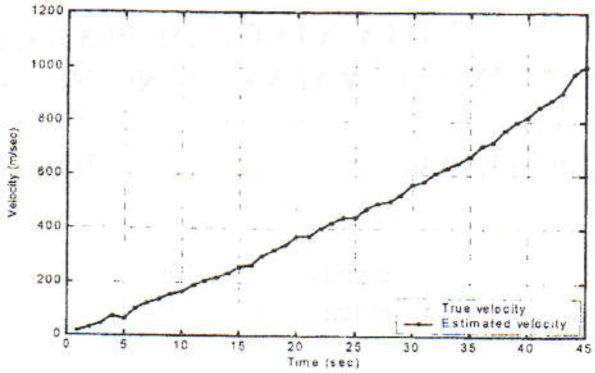

Figure 3. Comparison between the true and the calculated velocity for the augmented system

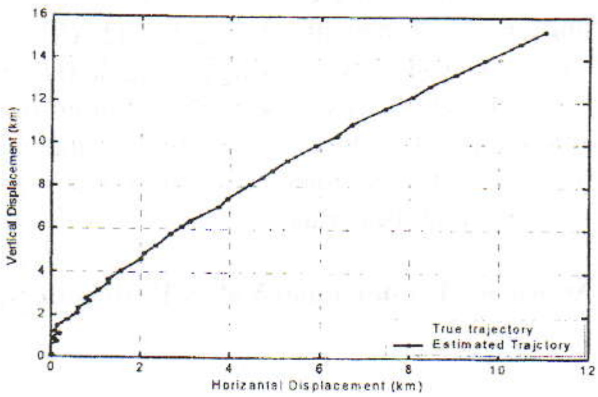

Figure 4. Comparison between the true and the calculated trajectory for the augmented system

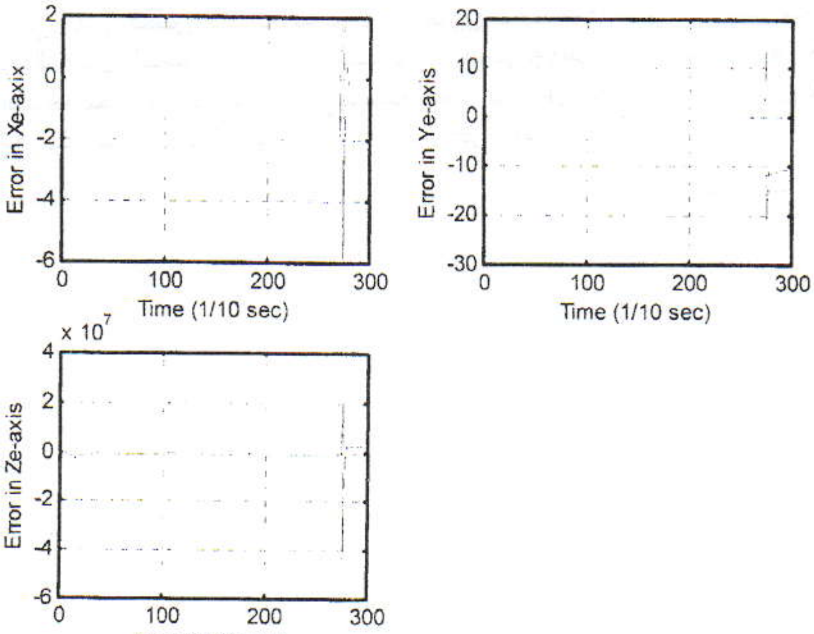

Figure 5. Error trajectories for the augmented system in ECEF coordinates