

## Application of Neural Network Predictive Control to a Stepping Motor

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### Abstract

In this paper, an application of the neural network predictive controller for stepping motor will be presented. This approach has been considered in order to assure high control performance of the system. The advantage of this type of control with respect to the classical PID controller will be illustrated. In the neural network predictive control approach the model of the system to be controlled is first determined in a system identification phase. The system model is used to predict future behavior of the system, and an optimization algorithm is used to select the control input that optimizes future performance.

### الخلاصة

يتضمن هذا البحث تطبيق مسيطر الشبكة العصبية التنبؤي على منظومة محرك الخطوة. وقد تم استخدام هذا النوع من المسيطرات لتحقيق سيطرة عالية الأداء حيث يوضح هذا البحث امكانية هذا النوع في تحسين اداء النظام مقارنة بالمسيطر الاعتيادي ال PID . ولتصميم هذا النوع من المسيطرات يتم تعريف النظام المطلوب السيطرة عليه حيث يستخدم هذا النظام في تنبؤ التصرف المستقبلي للنظام. ولغرض اختيار اشارة السيطرة تستخدم خوارزمية التفضيل التي تقوم باختيار الاداء المستقبلي  
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**Nomenclature**

$N_r$	: number of rotor teeth.
$\lambda$	: tooth pitch (rad).
J	: moment of rotor inertia (Kg. $m^2$ ).
$\theta$	: rotor angular position (rad).
D	: viscous damping coefficient (N.m.s. $rad^{-1}$ ).
$n\Phi_M$	: flux linkage.
$i_A, i_B$	: currents in windings A and B (amper).
C	: coulomb friction coefficient.
$T_L$	: load torque.
r.	: stator resistance (ohm).
V	: d.c. terminal voltage supplied to the stator windings (volt).
L	: self inductance of each stator phase (mH).
M	: mutual inductance between phases (mH).
$\delta\theta$	: deviation of rotor angle from the equilibrium position (rad).
$I_o$	: stationary current (amper).
$\delta i_A, \delta i_B$	: deviation of currents in windings A and B (amper).
$\Theta_o$	: actual rotor position (rad).
$\Theta_i$	: demanded position (rad).
$Y_r$	: the desired response.
$Y_m$	: the network model response.
$u'$	: the tentative control signal.
u	: the optimal plant input.

**1. Introduction**

The control system is one of the most important elements in stepping motor applications. The control systems of stepping motors are classified into open loop and closed loop schemes. In the open loop control scheme there is no feedback information of position to the controller and therefore it is imperative that the motor must respond correctly to each excitation change. If the excitation changes are made too quickly, the motor is unable to move to the new demanded position and consequently there is a permanent error in the actual position compared to the position expected by the controller. The timing of phase control signals for optimum open-loop performance is reasonably straightforward, if the load parameters remain constant. However, in the applications where the load varies significantly, the timing must be set for the worst conditions (largest load) and the control scheme is then non optimal for all other loads [1, 2].

In more sophisticated methods of the open loop control, the variations of load and friction torques with speed

can also be taken into account. Moreover, if higher resolution is needed the microstepping drive method can be used, but then the inaccuracy of the system increases due to the nonlinearity of microstepping. If high accuracy is needed, the closed loop control scheme is recommended. In closed loop stepping motor systems the instantaneous rotor position is detected via a feedback sensor and sent to the control unit. The general block diagram of the closed loop scheme is presented in figure (1). Essentially, there are several approaches how to design the closed loop control system. Most used method is based on a switching angle (lead angle) and it is used in applications with rotary stepping motors coupled with rotary encoders, where the information about position of the rotor against the stator is measured. However, the linear motor drive uses the rotary stepping motor with a lead screw.

In this application, it is impossible to measure the displacement of the rotor due to the motor structure. The linear optical encoder is used to measure the linear displacement. This gives a better precision of the linear drive, because the control variable is

directly measured by a sensor [1]. It is essentially true that for many applications the open loop control is entirely adequate and choosing a closed loop system would be an expensive luxury. Nowadays, the use of microprocessors can decrease the price and if the high accuracy and high reliability of the positioning system is required, the closed loop control looks very attractive [1].

In this paper a neural network predictive controller is to be designed. The neural networks have been applied very successfully in the control of dynamic systems. The universal approximation capabilities of the multilayer perceptron make it a popular choice for modeling systems and for implementing general purpose controllers. There are typically two steps involved when using neural networks for control, the first is the system identification where in which you develop a neural network model of the plant that you want to control. The second is the control design where in which the neural network plant model is used to design the controller [3].

## 2. System modeling

This section will deal with building a mathematical model of the linear motor drive. Physical modeling approach is used for constructing the system model. The principle of physical modeling is to divide the properties of the system to subsystems whose behaviors are known. For a technical system, this means that the laws of nature describing the subsystems are used in general [4].

Basically, the model of the permanent magnet stepping motor consists of two parts, an electrical and a mechanical part [5]. The permanent magnet stepper motor dynamical model includes nonlinearities and contains some physical parameters. The values of physical parameters are not exactly known and can be subjected to some variations, so the model is not very easy to handle for control synthesis. Figure (2) shows the model for a permanent magnet stepping motor. The model has two phases denoted by  $A$  and  $B$ . The rotor has  $(2N_r)$  magnetic poles, while the stator has a set of identical poles and windings equally arranged at intervals of  $(\lambda)$  [2].

The dynamic equations for the motion of the rotor are developed. Let  $T$  be the developed torque by motor windings,  $J$  the inertia of the rotor,  $D$  the coefficient of viscous friction,  $T_f$  represents the detent and the coulomb frictional torque, and  $\theta$  is the rotor angular position. Then, the dynamics of the rotor can be expressed by the following equation:

$$T = J \frac{d^2\theta}{dt^2} + D \frac{d\theta}{dt} + T_f \quad (1)$$

The torques produced by windings  $A$  and  $B$  are given by:

$$T_A = -N_r n \Phi_M i_A \sin(N_r \theta) \quad (2)$$

$$T_B = -N_r n \Phi_M i_B \sin(N_r (\theta - \lambda)) \quad (3)$$

Where  $i_A, i_B$  are the currents in windings  $A$  and  $B$ ,  $N_r$  is the number of the rotor teeth,  $n \Phi_M$  is the flux linkage,  $\theta$  is the rotational angle of the rotor and  $\lambda$  is the tooth pitch in radians.

The mechanical part of the permanent magnet stepper motor model can be expressed by an equation derived from equations (1),(2) and (3) :

$$\begin{aligned}
& J \frac{d^2\theta}{dt^2} + D \frac{d\theta}{dt} + N_r n \Phi_M i_A \sin(N_r \theta) \\
& + N_r n \Phi_M i_B \sin(N_r (\theta - \lambda)) + C \text{sign}\left(\frac{d\theta}{dt}\right) \\
& + T_L = 0
\end{aligned} \tag{4}$$

Where  $J$  denotes the moment of rotor inertia ( $\text{Kg.m}^2$ ),  $D$  denotes the viscous damping coefficient ( $\text{N.m.s.rad}^{-1}$ ),  $C$  represents the coulomb friction coefficient, and  $T_L$  is the load torque [2,5].

The electrical part of a permanent magnet stepper motor model is described by voltage equations for the stator windings [2].

$$\begin{aligned}
V - r i_A - L \frac{di_A}{dt} - M \frac{di_B}{dt} \\
+ \frac{d}{dt} (n \Phi_M \cos(N_r \theta)) = 0
\end{aligned} \tag{5}$$

$$\begin{aligned}
V - r i_B - L \frac{di_B}{dt} - M \frac{di_A}{dt} \\
+ \frac{d}{dt} (n \Phi_M \cos(N_r (\theta - \lambda))) = 0
\end{aligned} \tag{6}$$

Where  $V$  is the DC terminal voltage supplied to the stator windings (volt),  $L$  denotes the self inductance of each stator phase (mH),  $M$  represents the mutual inductance between phases (mH) and  $r$  is stator circuit resistance (ohm). Thus, the

complete model of the permanent magnet stepping motor consists of the rotor dynamic equation (4) and differential equations for current equation (5) and (6). Those equations are nonlinear differential equations. Since it is very difficult to deal with nonlinear differential equations analytically, linearization is needed. Linearization is made with aid of a new variable  $\delta\theta$ , which represents the deviation of the angle from the equilibrium position. The deviation is a function of time  $t$  and very small in magnitude. Figure (3) shows two stator phases, which carry the stationary current  $I_0$  in a direction to create south pole. The equilibrium position of the stator is then at  $\theta = \frac{\lambda}{2}$  [2].

When the rotor oscillates about its equilibrium position, the currents in both motor windings will deviate from the stationary value  $I_0$  by  $\delta i_A$  and  $\delta i_B$  and the angular rotor position can be expressed by:

$$\theta = \frac{\lambda}{2} + \delta\theta \tag{7}$$

The current in both windings are expressed as follows:

$$i_A = I_o + \delta i_A \tag{8}$$

$$i_B = I_o + \delta i_B \tag{9}$$

Then the nonlinearities expressed by sin and cosine functions in equations (4), (5) and (6) will be approximated with knowledge of trigonometric identities and when  $N_r \delta \theta$  is small angle:  $\cos(N_r \delta \theta) \cong 1$  and  $\sin(N_r \delta \theta) = N_r \delta \theta$ .

After substituting those approximations into the motor equations, the linearized model is obtained in the following form (more detailed description is given in [2]):

$$J \frac{d^2(\delta \theta)}{dt^2} + D \frac{d(\delta \theta)}{dt} + 2N^2_r n \Phi_M I_o \cos\left(\frac{N_r \lambda}{2}\right) \delta \theta + N_r n \Phi_M \sin\left(\frac{N_r \lambda}{2}\right) (\delta i_A - \delta i_B) + C \text{sign}\left(\frac{d\theta}{dt}\right) = 0 \tag{10}$$

$$r \delta i_A + L \frac{d(\delta i_A)}{dt} + M \frac{d(\delta i_B)}{dt} - N_r n \Phi_M \sin\left(\frac{N_r \lambda}{2}\right) \frac{d(\delta \theta)}{dt} = 0 \tag{11}$$

$$r \delta i_B + L \frac{d(\delta i_B)}{dt} + M \frac{d(\delta i_A)}{dt} + N_r n \Phi_M \sin\left(\frac{N_r \lambda}{2}\right) \frac{d(\delta \theta)}{dt} = 0 \tag{12}$$

Where:  $\sin\left(\frac{N_r \lambda}{2}\right)$ , and  $\cos\left(\frac{N_r \lambda}{2}\right)$  are constants.

The permanent magnet stepping motor transfer function is derived from equations (10), (11) and (12) with the aid of Laplace transform. The coulomb friction coefficient  $C$  is considered to be zero. The resulting form of the transfer function in two phase excitation in the voltage source drive is:

$$\frac{\Theta_o}{\Theta_i} = \frac{\frac{r}{L} w_{np}^2}{s^3 + \left(\frac{r}{L_p} + \frac{D}{J}\right) s^2 + \left(\frac{rD}{L_p J} + w_{np}^2(1+k_p)\right) s + \left(\frac{r}{L_p}\right) w_{np}^2} \tag{13}$$

Where

$$L_p = L - M, w_{np}^2 = \frac{2N^2_r n \Phi_M I_o \cos\left(\frac{N_r \lambda}{2}\right)}{J}$$

$$k_p = \frac{n \Phi_M \sin^2\left(\frac{N_r \lambda}{2}\right)}{L_p I_o \cos\left(\frac{N_r \lambda}{2}\right)}$$

,  $\Theta_o$  is the Laplace transform of the actual rotor position,  $\Theta_i$  represents the

laplace transform of the demanded position and  $s$  is the Laplace operator. The actual parameters for the linear stepper motor are given in table (1) [6]. Figure (4) shows a linear analysis of the system where the open loop bode plot is plotted. From the bode plot it is clear that the system is relatively stable.

### 3. Controller Design

The neural network predictive controller uses a neural network model of the plant to predict future plant performance. The controller then calculates the control input that will optimize plant performance over a specified future time horizon. The first step in model predictive control is to determine the neural network plant model. Next the plant model is used by the controller to predict future performance.

The first stage is to train a neural network to represent the forward dynamics of the plant. The prediction error between the plant output and the neural network output is used as the neural network training signal. Figure

(5) shows the block diagram of system identification. The neural network plant model uses previous inputs and previous outputs to predict future values of the plant output. Figure (6) shows the structure of the neural network plant model. This network was trained offline using data collected from the operation of the plant. The backpropagation training algorithm was used for network training [7].

The model predictive control is based on the receding horizon technique [8]. The neural network model predicts the plant response over a specified time horizon. The predictions are used by a numerical optimization program to determine the control signal that minimizes the following performance criterion over the specified horizon.

$$J = \sum_{j=N_1}^{N_2} (y_r(t+j) - y_m(t+j))^2 + \rho \sum_{j=1}^{N_u} (u(t+j-1) - u(t+j-2))^2 \quad (14)$$

Where  $N_1, N_2$  and  $N_u$  define the horizon over which the tracking error and the control increments are evaluated. The  $u'$  variable is the tentative control signal,  $y_r$  is the desired response and  $y_m$  is the network model response. The  $\rho$  value determines the contribution that

the sum of the squares of the control increments has on the performance index. Figure (7) shows the block diagram of the model predictive control. The optimization part of the predictive controller determines the values of  $u'$  that minimize  $J$  and then the optimal  $u$  is input to the plant [8].

The response of using PID controller is shown in figure (8). From this figure it is clear that the response is not smooth and any other improvements can not be obtained because the large change in PID controller parameters leads to unstable system. The optimum parameters of PID controller have been determined using the nonlinear control design (NCD) blockset in Matlab [9]. The NCD blockset provides a graphical user interface (GUI) to assist in time domain based control design. With this blockset, we can tune parameters within nonlinear simulink model to meet time domain performance requirements by graphically placing constraints within a time domain window.

Figure (9) shows the response of the system using the neural network predictive controller. An overall improvement with respect to the PID

controller can be observed. The parameters  $N_1, N_2, N_u$  and  $\rho$  are usually chosen empirically. For our system, the parameters were chosen according to the bandwidth of the system. The following set of parameters for  $J$  were chosen:  $N_1 = 1, N_2 = 20, N_u = 2$  and  $\rho = 0.005$ .

#### 4. Conclusion

An application of the neural network predictive controller to a stepping motor system has been presented. This paper compared the performance of the neural network predictive controller with the classical PID controller. It is clear that the performance of the classical PID controller was poor. A neural network predictive controller approach was found to provide significantly improved transient performance.

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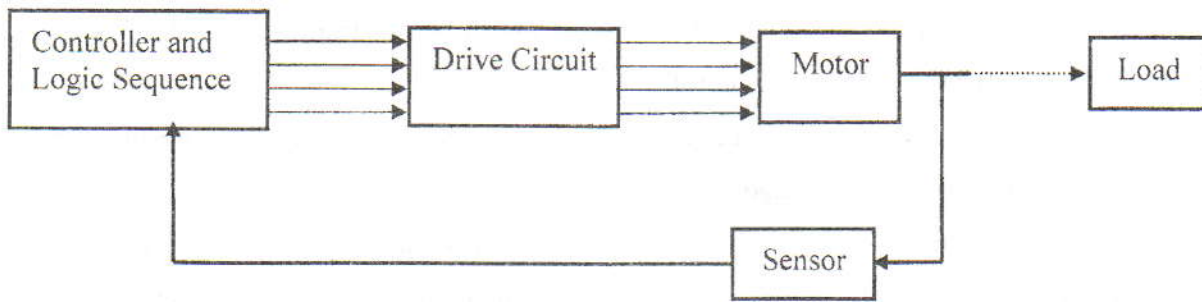


Figure (1): Block diagram of closed loop stepping motor.

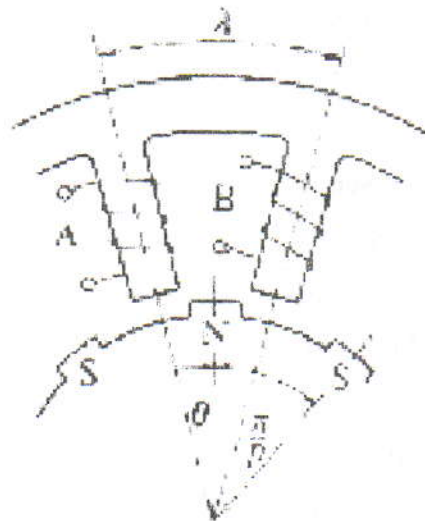


Figure (2): Permanent magnet stepper motor.

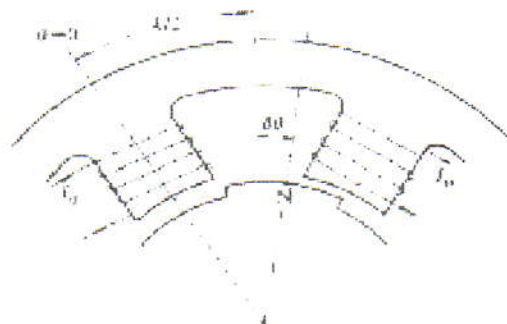


Figure (3): Two stator phases of motor.

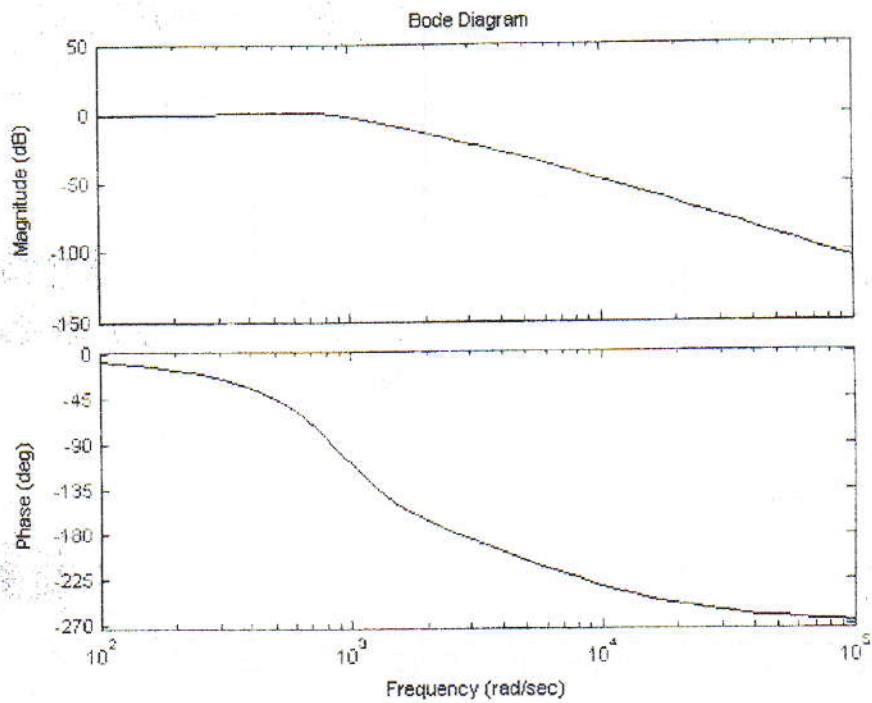


Figure (4): Frequency response of the system.

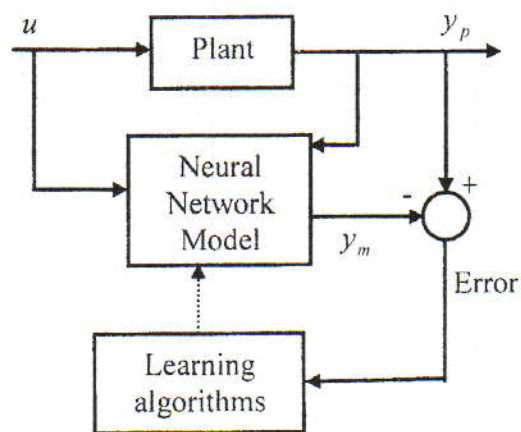


Figure (5): Block diagram of system identification.

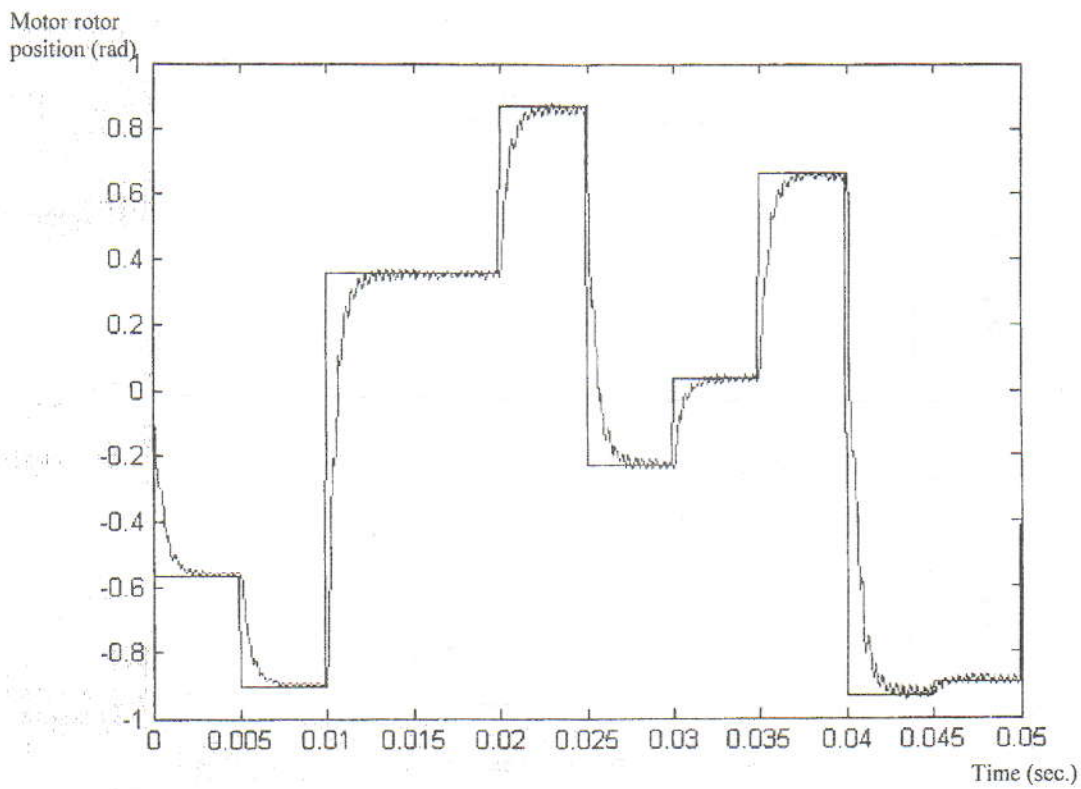


Figure (8): Step response of the system with PID controller.

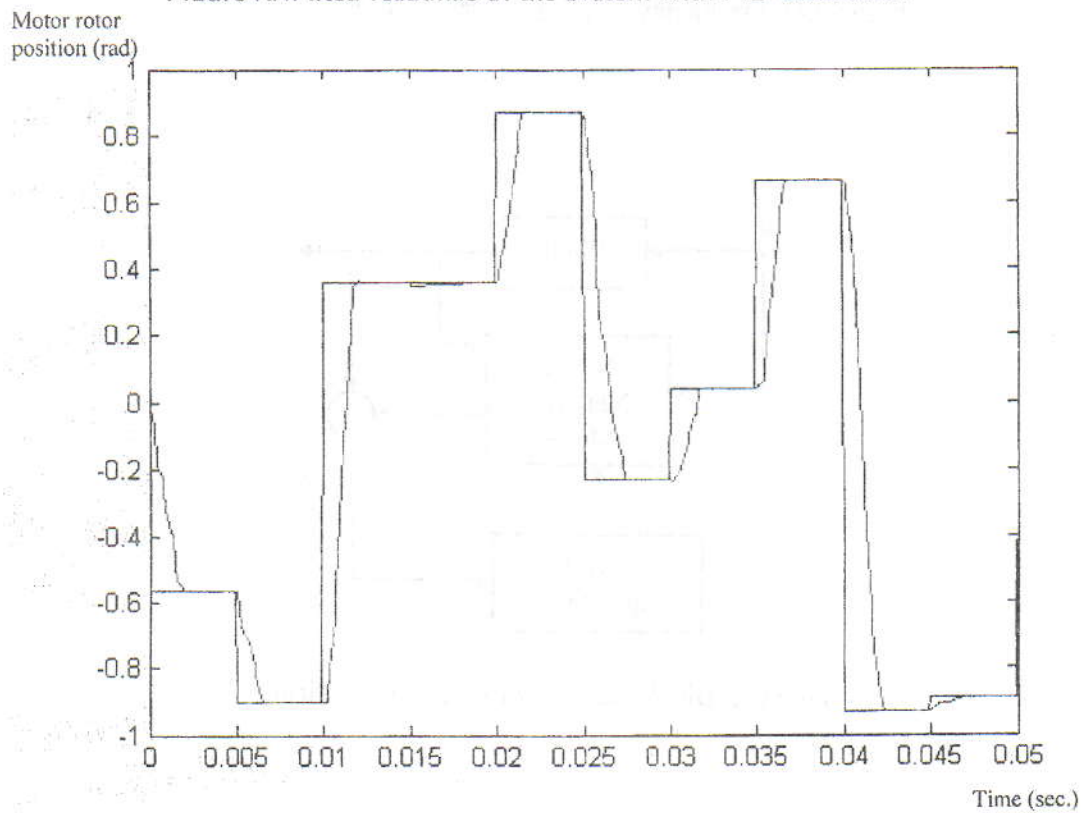


Figure (9): Step response of the system with neural network predictive controller.