$* -$ Connectedness in Intuitionistic Fuzzy Ideal Bitopological spaces

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Abstract:

In This paper we introduce the nation of $*-$ Connectedness in Intuitionistic Fuzzy Ideal Bitopological Space . we obtain several properties of $*-$ Connectedness in Intuitionistic Fuzzy Ideal Bitopological spaces and the relationship between this notion and other related notions.

Keywords:

Intuitionistic Fuzzy Ideal Bitopological Spaces ,

Pairwise $*-$ Connected intuitionistic fuzzy sets ,

Pairwise $*-$ Separated intuitionistic fuzzy sets ,

Pairwise $*-$ Connected intuitionistic fuzzy Ideal Bitopological Space

* – الأتصال في الفضاءات التوبولوجية الثنائيه الحدسية الضبابية ذات المثاليات الحدسية الضبابية

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الملخص :

قدمنا في هذا البحث مفهوم * – الاتصال في الفضاءات التوبولوجية الثنائية الحدسية الضبابية ذات المثاليات الحدسية الضبابية . وقد حصلنا على بعض الخواص حول * – الاتصال في الفضاءات التوبولوجية الثنائية الحدسية الضبابية وبعض العلاقات حول هذا المفهوم وارتباطه مع المفاهيم الاخرى ذات العلاقة .

1. Introduction

The concept of " fuzzy sets " interdused by Zadeh [7] in 1965 . The idea of " intuitionistic fuzzy sets " was firt published by Atanassove [5. , 6] in 1986 , 1988 .

Then Coker [2 , 3] introduced " intuitionistic fuzzy topological space " using intuitionistic fuzzy set in 1996 , 1997 . The notion of " ideal in intuitionistic fuzzy topological space " was introduced by A.Asalam and S . A . Alblowi [1] in 2012.

Kelly introduced the concept of "bitopological space" as extension of topological space [4] in 1963

Mohammed (2015) introduced the notion of " intuitionistic fuzzy ideal bitopological space" [9] .

The purpose of this paper is to introduce and study the notion of " $* -$ connectedness in intuitionistic fuzzy ideal bitopological space " .

We study the notion of " pairwise $* -$ connected intuitionistic fuzzy ideal bitopological space" .

2. Preliminaries :

Definition 2.1. [7] :-

Let X be a non – empty set and $I = [\,0\,,1\,]$ be the closed interval of the real numbers . A fuzzy subset μ of X is defined to be membership function $\mu : X \rightarrow I$, such that $\mu\,(X\,) \in I$ for every $x \in X$ The set of all fuzzy subsets of X denoted by I^X .

Definition 2.2 [5] :-

An intuitionistic fuzzy set (IFs , for short) A is an object have the form : $A = \{< x\,,\mu_A(x\,), \nu_A(x\,) > ; x \in X\,\}$, where the functions $\mu_A : X \rightarrow I$, $\nu_A : X \rightarrow I$ denote the degree of membership and the degree of non – membership of each element $x \in X$ to the set A respectively ,and $0 \leq \mu_A\,(x) + \nu_A(x\,) \leq 1$, for each $x \in X$. The set of all intuitionistic fuzzy sets in X denoted by IFS (x) .

Definition 2.3. [3] :-

$0_{\sim} =< x\,,0\,,1 >$, $1_{\sim} =< x\,,1\,,0 >$ are the intuitionistic sets corresponding to empty set and the entire universe respectively .

Definition 2.4. [2] :-

Let X be a non – empty set . An intuitionistic fuzzy point (IFP , for short) denoted by $x_{(\propto,\beta)}$ *is an intuitionistic fuzzy set have the form*

$x_{(\alpha,\beta)}(y) = \begin{cases} < x , \alpha , \beta > & ; \quad x = y \\ < x , 0 , 1 > & ; \quad x \neq y \end{cases}$ *, where* $x \in X$ *is a fixed point , and* $\alpha , \beta \in [0 , 1]$ *satisfy* $\alpha + \beta \leq 1$ *. The set of all IFPs denoted by IFP (x) . If* $\in IFs\ (x)$ *. We say the* $x_{(\alpha,\beta)} \in A$ *if and only if* $\alpha \leq \mu_A(x)$ *and* $\beta \geq \nu_A(x)$ *, for each* $x \in X$ *.*

Definition 2.5. [2] :-

Let $=< x , \mu_A (x) , \nu_A(x) >$, $B =< X , \mu_B (x) , \nu_B(x) >$ be two intuitionistic fuzzy sets in X . A is said to be quasi – coincident with B (written AqB) if and only if , there exists an element $x \in X$ such that $\mu_A (x) > \nu_B(x)$ or $\nu_A(x) < \mu_B (x)$, otherwise A is not quasi – coincident with B and denoted by $A\tilde{q}B$.

Definition 2.6. [2] :-

Let $x_{(\alpha,\beta)} \in IFP(X)$ and $\in IFS\ (X)$. We say that $x_{(\alpha,\beta)}$ quasi – coincident with A denoted $x_{(\alpha,\beta)}q$ A if and only if , $\alpha > \nu_A(x)$ or $\beta < \mu_A (x)$, otherwise $x_{(\alpha,\beta)}$ is not quasi – coincident with A and denoted by $x_{(\alpha,\beta)}\tilde{q}$ A .

Definition 2.7.[2] :-

Let $x_{(\alpha,\beta)}$ be an intuitionistic fuzzy point in X and $A = \{< x , \mu_A (x) , \nu_A(x) > ,\ x \in X \}$ an IFS in X . Suppose further α and β are real numbers between 0 and 1 . The intuitionistic fuzzy point $x_{(\alpha,\beta)}$ is said to be properly contained in A if and only if , $\alpha < \mu_A (x)$ and $\beta > \nu_A(x)$.

Definition 2.8.[2] :-

An intuitionistic fuzzy point $x_{(\alpha,\beta)}$ is said to be belong to an intuitionistic fuzzy set A in X , denoted by $x_{(\alpha,\beta)} \in A$ if $\alpha \leq \mu_A(x)$ and $\beta \geq \nu_A(x)$.

Proposition 2.9. [3] :-

Let A , B be IFSs and x (α , β) an IFP in X . Then

1- $A\tilde{q}B \Leftrightarrow A \leq B$

2- $AqB \Leftrightarrow A \nleq B^C$,

3- $x_{(\alpha,\beta)} \in A \Leftrightarrow x_{(\alpha,\beta)}\tilde{q}A^C$,

4- $x_{(\alpha,\beta)}\, q\, A \Leftrightarrow x_{(\alpha,\beta)} \notin A^C$.

Proposition 2.10. [8] :-

For A , B ∈ IFS and $x_{(\alpha,\beta)} \in IFP(X)$, we have :

$A \leq B$ if and only if , for $x_{(\alpha,\beta)} \in A$ then $x_{(\alpha,\beta)} \in B$ –i

ii - $A \leq B$ if and only if , for $x_{(\alpha,\beta)}\, q\, A$ then $x_{(\alpha,\beta)}\, q\, B$.

Lemma 2.11. [10] :-

Let A , B and C be intuitionistic fuzzy sets . If $q(A \cup B)$, then CqA or CqB .

Definition 2.12. [3] :-

An intuitionistic fuzzy topology (IFT , for short) on a non empty set X is a family τ of an intuitionistic fuzzy set in X such that

(i) $0_\sim , 1_\sim \in \tau$,

(ii) $G_1 \cap G_2 \in \tau$, for any $G_1 , G_2 \in \tau$,

(iii) $\cup G_i \in \tau$, for any arbitrary family $\{ G_i : i \in J \} \subseteq \tau$.

In this case the pair (X , τ) is called an intuitionistic fuzzy topological space (IFTS , in short) .

Definition 2.13. [3] :-

Let (X, τ) be an intuitionistic fuzzy topological space and $A = \{< x, \mu_A(x), \nu_A(x) >, x \in X\}$ be an intuitionistic fuzzy set in X then , an intuitionistic fuzzy interior and intuitionistic fuzzy closure of A are respectively defined by

$\text{int}(A) = A^{\circ} = \cup \{G : G \text{ is an IFOS in } X \text{ and } G \subseteq A\}$

$\text{cl}(A) = \overline{A} = \cap \{K : K \text{ is an IFCS in } X \text{ and } A \subseteq K\}$.

Definition 2.14. [3] :-

A non – empty collection of intuitionistic fuzzy sets L of a set X is called intuitionistic fuzzy ideal on X (IFI , for short) such that :

(i) If $A \in L$ and $B \leq A \Longrightarrow B \in L$ (heredity)

(ii) If $A \in L$ and $B \in L \Longrightarrow A \vee B \in L$ (finite additivity) . If (X, τ) be an IFTS , then the triple (X, τ, L) is called an intuitionistic fuzzy ideal topological space (IFITS , for short) .

Definition 2.15. [1] :-

Let (X, τ, L) be an IFITS . If $\in$ IFS (X) . Then the intuitionistic fuzzy local function $A^*(L, \tau)$ (A^* , for short) of A in (X, τ, L) is the union of all intuitionistic fuzzy points $x_{(\alpha,\beta)}$ such that :

$A^*(L, \tau) = \vee \{ x_{(\alpha,\beta)} : A \wedge U \notin L$, for every $\in N\left(x_{(\alpha,\beta)}, \tau\right)\}$, where $N\left(x_{(\alpha,\beta)}, \tau\right)$ is the set of all quasi – neighborhoods of an IFP $x_{(\alpha,\beta)}$ in τ . The intuitionistic fuzzy closure operator of an IFS A is defined by $\text{cl}^*(A) = A \vee A^*$, and $\tau^*(L)$ is an IFT finer than τ generated $\text{cl}^*(\cdot)$ and defined as

$\tau^*(L) = \{A : \text{cl}^*(A^C) = A^C\}$.

Lemma 2.16. [8] :-

Let (X , τ , L) be an IFITS and $B \subset A \subset X$. Then

$B^*(\tau_A, L_A) = B^*(\tau, L) \cap A$.

Lemma 2.17. [8] :-

Let (X , τ , L) be an IFITS and $B \subset A \subset X$. Then

$cl_A^*(B) = cl^*(B) \cap A$.

Definition 2.18. [8] :-

An intuitionistic fuzzy set (IFS) A of intuitionistic fuzzy ideal topological space(X , τ , L) is said to be *-dense if $cl^*(A) = X$.

An intuitionistic fuzzy ideal topological space (X , τ , L) is said to be *-hyperconnected if IFS A is *-dense for every IF open subset $A \neq \emptyset$ of X .

Lemma 2.19. [8] :-

Let (X , τ , L) be an IFITS for each $v \in \tau^*, \tau_v^* = (\tau_v)^*$.

Lemma 2.20. [8] :-

Let (X , τ , L) be an IFITS , $A \subset Y \subset X$ and $Y \in \tau$. The following are equivalent

(1) A is *-IF open in Y, (2) A is *-IF open in X .

Proof :- (1)$\Longrightarrow$ (2) let A be *-IF open in Y .Since $Y \in \tau \subset \tau^*$, by lemma (2.19) , A is *-IF open in X .

Let A be *-IF open in X . By lemma (2.19) , $A = A \cap Y$ is *-IF open in X .(2) $\Longrightarrow$ (1) .

Definition 2.21. [8] :-

Two non empty intuitionistic fuzzy sets A and B of an intuitionistic fuzzy ideal topological space (X , τ , L) are said to be intuitionistic fuzzy

$* -$ separated sets (IF $* -$ separated sets , for short) if $cl^*(A)\tilde{q}B$ and $A\tilde{q}cl\,(B)$.

Definition 2.22. [8] :-

An intuitionistic fuzzy set E in intuitionistic fuzzy ideal topological space (X , τ , L) is said to be intuitionistic fuzzy $* -$ connected if it can not be expressed as the Union of two intuitionistic fuzzy $* -$ separated sets . otherwise , E is said to be intuitionistic fuzzy $* -$ disconnected .

If $= X$, then X is said to be intuitionistic fuzzy $* -$ connected space .

Definition 2.23. [8] :-

Let τ_1 and τ_2 be two intuitionistic fuzzy topologies on a non – empty set X . The Triple (X , τ_1 , τ_2) is called an intuitionistic fuzzy bitopological space (IFBTS , for short) , every member of τ_i is called $\tau_i -$ intuitionistic fuzzy open set ($\tau_i -$ IFOS) , $i \in \{1 , 2\}$ and the complement of $\tau_i -$ IFOS is $\tau_i -$ imtuitionistic fuzzy closed set ($\tau_i -$ IFCS) , $i \in \{1 , 2\}$.

Example 2.24.[8] :-

Let $X = \{e , d\}$ and $A , B \in IFS\,(X)$ such that $=< x , (0.3 , 0.1) , (0.5 , 0.6) >$,

$B =< x , (0.2 , 0.4) , (0.7 , 0.3) >$. Let $t_1 = \{0_\sim , 1_\sim , A \}$ and $\tau_2 = \{0_\sim , 1_\sim , B \}$ be two IFTS on X . Then (X , τ_1 , τ_2) is IFBTS .

Definition 2.25.[8] :-

Let (X , τ_1 , τ_2) be an IFBTS , $A \in IFS\,(X)$ and $x_{(\alpha ,\beta)} \in IFP\,(X)$. Then A is said to be quasi – neighborhood of $x_{(\alpha ,\beta)}$ if there exists a $\tau_i -$ IFOS B , $i \in \{1 , 2\}$ such that $x_{(\alpha ,\beta)}qB \leq A$. The set of all quasi –

neighborhoods of $x_{(\alpha,\beta)}$ *in* (X,τ_1,τ_2) *is denoted by :* $N(x_{(\alpha,\beta)},\tau_i), i \in \{1,2\}$.

Definition 2.26.[8] :-

An intuitionistic fuzzy bitopological space (X,τ_1,τ_2) with an intuitionistic fuzzy ideal L on X is called intuitionistic fuzzy ideal bitopological space (X,τ_1,τ_2,L) and denoted by IFLBTS

Example 2.27. [8] :-

Let $X = \{e\}$ *and* $A, B \in IFS(X)$ *such that* $=< X, 0.3, 0.5 >$,
$B =< X, 0.2, 0.4 >$. *Let* (X,τ_1,τ_2) *be an IFLBTS , where* $\tau_1 = \{0_\sim, 1_\sim, A\}$ *and* $\tau_2 = \{0_\sim, 1_\sim, B\}$. *If* $L = \{0_\sim, A, C : C \in IFS(X)$ *and* $C \leq A\}$ *be an IFL on X . Then* (X,τ_1,τ_2) *is IFLBTS .*

Definition 2.28. [8] :-

Let (X,τ_1,τ_2, L) be an IFLBTS and $\in$ IFS (X) . Then the intuitionistic fuzzy local function of A in (x,τ_1,τ_2,L) denoted by $A^*(L,\tau_i), i \in \{1,2\}$ and defined by as follows :

$A^*(L,\tau_i) = \vee\{x_{(\alpha,\beta)} : A \wedge U \notin L$, for every $\in N(x_{(\alpha,\beta)},\tau_i)\}, i \in \{1,2\}$.

Definition 2.29. [8]:-

Let (X,τ_1,τ_2) be an IFBTS and $\in$ IFS (X) . Then intuitionistic fuzzy interior and intuitionistic fuzzy cloure of A with respect to τ_i , $i \in \{1,2\}$ are defined by :

$\tau_i - \text{int}(A) = \vee\{G : G \text{ is a } \tau_i - \text{IFOS}, G \leq A\}$.

$\tau_i - \text{cl}(A) = \wedge\{K : K \text{ is a } \tau_i - \text{IFCS}, A \leq K\}$.

Proposition 2.30.[8] :-

Let (X,τ_1,τ_2) be an IFBTS and $\in$ IFS (X) . Then we have :

(i) $\tau_i - \text{int}(A) \leq A, i \in \{1,2\}$

(ii) $\tau_i - \text{int}(A)$ is a largest $\tau_i -$ IFOS contains in A

(iii) A is a $\tau_i -$ IFOS if and only if $\tau_i - \text{int}(A) = A$

(iv) $\tau_i - \text{int}\left(\tau_i - \text{int}(A)\right) = \tau_i - \text{int}(A)$.

(v) $A \leq \tau_i - \text{cl}(A), i \in \{1,2\}$.

(vi) $\tau_i - \text{cl}(A)$ is smallest $\tau_i -$ IFCS contains A .

(vii) A is a $\tau_i -$ IFCS if and only if $\tau_i - \text{cl}(A) = A$.

(viii) $\tau_i - \text{cl}\left(\tau_i - \text{cl}(A)\right) = \tau_i - \text{cl}(A)$

(ix) $[\tau_i - \text{int}(A)]^c = \tau_i = \text{cl}(A^c), i \in \{1,2\}$.

(x) $[\tau_i - \text{cl}(A)]^c = \tau_i = \text{int}(A^c), i \in \{1,2\}$.

Definition 2.31. [8] :-

We define $* -$ intuitionistic fuzzy closure operator for intuitionistic fuzzy bitopology $\tau_i^*(L)$ as follows :

$\tau_i - \text{cl}^*(A) = A \vee A^*(L, \tau_i)$ for every $A \in \tau_i - \text{IFS}(X)$ .Also , $\tau_i^*(L)$ is called an intuitionistic fuzzy bitopology generated by $\tau_i - \text{cl}^*(A)$ and defined as :

$\tau_i^*(L) = \{A : \tau_i - \text{cl}^*(A^c) = A^c, i \in \{1,2\}\}$.

Note : $\tau_i^*(L)$ finer than intuitionistic fuzzy bitopology τ_i , (i . e $\tau_i \leq \tau_i^*(L)$) .

Remark 2.32. [8] :-

(i) If $L = \{0_\sim\} \Rightarrow A^*(L, \tau_i) = \tau_i - \text{cl}(A)$, for any $A \in \text{IFS}(X)$

$\Rightarrow \tau_i - \text{cl}^*(A) = A \vee A^*(L, \tau_i) = A \vee \tau_i - \text{cl}(A) = \tau_i - \text{cl}(A)$

$\Rightarrow \tau_i^*(\{0_\sim\}) = \tau_i , i \in \{1,2\}$.

(ii) If $L = \text{IFS}(X) \Rightarrow A^*(L, \tau_i) = 0_\sim$, for any $A \in \text{IFS}(X)$

$\Rightarrow \tau_i - \text{cl}^*(A) = A \vee A^*(L, \tau_i) = A \vee 0_\sim = A$

$\Rightarrow \tau_i^*(L)$ is the intuitionistic fuzzy discrete bitopology on X .

3. Main Results

3.1 $*-$ Connectedness in Intuitionistic fuzzy Ideal Bitopological Spaces

Definition 3.1.1 :-

Two non empty $\tau_i -$ intuitionistic fuzzy sets A and B of an intuitionistic fuzzy ideal bitopological space (X, τ_1, τ_2, L), $i \in \{1,2\}$, are said to be intuitionistic fuzzy $*-$ separated sets ($\tau_i -$ IF $*-$ separated sets , for short) , $i \in \{1,2\}$ if

$\tau_i - cl^*(A)\tilde{q}B$ and $A\tilde{q}\ \tau_i - cl(B)$

Propoition 3.1.2 :-

Let A and B be an $\tau_i -$ intuitionistic fuzzy $*-$separated sets in IFLBT (X, τ_1, τ_2, L) , A , B are two non empty $\tau_i -$ intuitionistic fuzzy $*-$separated sets such that $A_1 \leq A$ and $B_1 \leq B$ then A_1 and B_1 are $\tau_i -$intuitionistic fuzzy $*-$separated sets in X , $i \in \{1,2\}$.

Proof :-

Since $A_1 \leq A$ and $B_1 \leq B$, we have

$\tau_i - cl^*(A_1) \leq \tau_i - cl^*(A)$ and $\tau_i - cl(B_1) \leq \tau_i - cl(B)$, Since A ,B are $\tau_i -$ intuitionistic fuzzy $*-$separated then ,

$\tau_i - cl^*(A)\tilde{q}B$ and $\tilde{q}\tau_i - cl(B)$, $i \in \{1,2\}$

Therefore $\tau_i - cl^*(A)\tilde{q}B$ we get $\tau_i - cl^*(A_1)\tilde{q}B_1$

And $\tilde{q}\tau_i - cl(B)$, and also we get $A_1\tilde{q}\tau_i - cl(B_1)$, $i \in \{1,2\}$

Then A_1 and B_1 are $\tau_i -$ IF $*-$separated .

Theorem 3.1.3 :-

Let A be $\tau_i -$intuitionistic fuzzy open set ($\tau_i -$IFOS) , $i \in \{1,2\}$ and B be $*-\tau_i -$ intuitionistic fuzzy open set in intuitionistic fuzzy ideal bitopological space (X, τ_1, τ_2, L) . Then A and B are τ_i −IF ∗ −separated sets in X if and only if ,$A\tilde{q}B$.

Proof :-

(⟹) suppose that AqB , then exists an element $x \in X$ such that $\mu_A(x) > \nu_B(x)$ or $\nu_A(x) < \mu_B(x)$, and since

$A \subseteq \tau_i - cl^*(A)$ and $\subseteq \tau_i - cl(B)$, $i = \{1,2\}$

This implies $\mu_{\tau_i - cl^*(A)}(x) > \nu_B(x)$ or $\nu_{\tau_i - cl^*(A)}(x) < \mu_B(x)$

And $\mu_A(x) > \nu_{\tau_i - cl(B)}(x)$ or $\nu_A(x) < \mu_{\tau_i - cl(B)}(x)$, $i \in \{1,2\}$

Then $\tau_i - cl^*(A)qB$ and $Aq\tau_i - cl(B)$, $i \in \{1,2\}$

This is contradiction . Hence $\tilde{q}B$.

(⟸) Suppose that $\tilde{q}B$.

By proposition (2.9) , we have $A \leq B^c$

Since B^c is τ_i − intuitionistic fuzzy closed set , $i \in \{1,2\}$

Therefore , $\tau_i - cl^*(A) \leq \tau_i - cl^*(B^c) = B^c$, $i \in \{1,2\} \rightarrow \tau_i - cl^*(A) \leq B^c$

Hence by proposition (2.9), we get $\tau_i - cl^*(A)\tilde{q}(B^c)^c$.

Then $\tau_i - cl^*(A)\tilde{q}B \ldots (1)$

Let $\leq B^c$, since B^c is $* - \tau_i$ −IFCS in X .

Therefore , $\tau_i - cl(A) \leq \tau_i - cl(B^c) = B^c$, $i \in \{1,2\}$

Hence by proposition (2.9) , we have $\tau_i - cl(A)\tilde{q}(B^c)^c$,then $\tau_i - cl(A)\tilde{q}B$

Since $A \subseteq \tau_i - cl(A)$ and $\subseteq \tau_i - cl(B)$, $i \in \{1,2\}$

Thus $A\tilde{q}\tau_i - cl(B) \ldots (2)$

From (1) and (2) we get A and B are τ_i − IF ∗ − separated sets in X .

Proposition 3.1.4 :-

Let A be an $*-\tau_i$ $-$IFCS and B is an τ_i $-$IFCS , $i \in \{1,2\}$ in intuitionistic fuzzy ideal bitopological space (X, τ_1, τ_2, L) .

Then A and B are τ_i $-$IF $*-$ Separted sets in X if and only if $\tilde{q}B$.

Proof :-

$(\Longrightarrow)$ suppose that A , B are τ_i $-$IF $*-$ separated sets in X .

$\Rightarrow \tau_i - cl^*(A)\tilde{q}B$ and $\tilde{q}\tau_i - cl(B)$, $i \in \{1,2\}$

Since A is $*-\tau_i$ $-$IFCS , then $\tau_i - cl^*(A) = A$, $i \in \{1,2\}$, we get $A\tilde{q}B$

$(\Longleftarrow)$ Suppose that $A\tilde{q}B$

Since A is $*-\tau_i$ $-$IFCS and B is τ_i $-$IFCS , $i \in \{1,2\}$

Therefore , $\tau_i - cl^*(A) = A$ and $\tau_i - cl(B) = B$, $i \in \{1,2\}$

We get $\tau_i - cl^*(A)\tilde{q}B$ and $A\tilde{q}\tau_i - cl(B)$

Hence A , B are τ_i $-$IF $*-$separated sets in X .

Definition 3.1.5 :-

An τ_i $-$intuitionistic fuzzy set (τ_i $-$IFS) A of intuitionistic fuzzy ideal bitopological space (X, τ_1, τ_2, L) is said to be $*-\tau_i-$ dense if $\tau_i - cl^*(A) = X$, $i \in \{1,2\}$

An IF ideal bitopological space (X, τ_1, τ_2, L) is said to be $*$ $-$hyperconnected if τ_i $-$IFS A is $*-\tau_i$ $-$dense for every τ_i $-$IF open subset $A \neq \emptyset$ of X , $i \in \{1,2\}$.

Theorem 3.1.6 :-

Let (X, τ_1, τ_2, L) be an intuitionistic fuzzy ideal bitopological space and A ,B are τ_i $-$intuitionistic fuzzy sets such that A , $B \subset Y \subset X$. Then A and B are τ_i $-$IF $*-$separated in Y if and only if A , B are τ_i $-$IF $*$ $-$separated in X .

Proof :- It follows from lemma (2.17) that $\tau_i - cl^*(A)\tilde{q}B$ and $A\tilde{q}\tau_i - cl(B)$, $i \in \{1,2\}$.

Proposition 3.1.7 :-

Let A be an τ_i −intuitionistic fuzzy open set (τ_i −IFOS) and B is an $* - \tau_i$ −intuitionistic fuzzy open set ($* - \tau_i$ −IFOS) in IFLBTS $(X , \tau_1 , \tau_2 , L)$. Then the sets $C_A B = A \wedge B^c$ and $C_B A = B \wedge A^c$ are τ_i −IF ∗ −separated in X .

Proof :-

Since $C_A B = A \wedge B^c$, $C_A B \le B^c$

$\tau_i - cl^*(C_A B) \le \tau_i - cl^*(B^c) = B^c$ because B^c is $* - \tau_i$ −IFCS ,

By proposition (2.9) we get

$\tau_i - cl^*(C_A B)\tilde{q}(B^c)^c \Longrightarrow \tau_i - cl^*(C_A B)\tilde{q}B$, $i \in \{1,2\}$

Since $C_B A \le B$

Therefore $\tau_i - cl^*(C_A B)\tilde{q}C_B A \ldots (1)$

$C_B A \le A^c$

$\tau_i - cl(C_B A) \le \tau_i - cl(A^c) = A^c, i \in \{1,2\}$

$\tau_i - cl(C_B A) \le A^c$

$\Longrightarrow \tau_i - cl(C_B A)\tilde{q}(A^c)^c , i \in \{1,2\} \Longrightarrow \tau_i - cl(C_B A)\tilde{q}A , i \in \{1,2\}$

Since $C_A B \le A$

Then $\tau_i - cl(C_B A)\tilde{q}C_A B \ldots (2)$

From (1) and (2) we get , $C_A B$, $C_B A$ are τ_i −IF ∗ −eparated set in X .

Proposition 3.1.8 :-

Let A be an $* - \tau_i$ −intuitionistic fuzzy closed set ($* - \tau_i$ −IFCS) and B be τ_i −intuitionistic fuzzy closed set (τ_i −IFCS) in IFLBTS

(X, τ_1, τ_2, L) . Then the τ_i −IFS $C_A B = A \wedge B^c$ and $C_B A = B \wedge A^c$ are τ_i −IF $*$ −separated sets in X , $i \in \{1,2\}$.

Proof :-

Since A is $* - \tau_i$ −IFCS and B is an τ_i −IFCS , $i \in \{1,2\}$

So $A = \tau_i - cl^*(A)$ and $B = \tau_i - cl(B)$

$C_A B \leq A \Longrightarrow \tau_i - cl^*(C_A B) \leq \tau_i - cl^*(A) = A$, $i \in \{1,2\}$

By proposition (2.9) we get

$\tau_i - cl^*(C_A B) \tilde{q} A^c$

Since $C_B A \leq A^c$, then $\tau_i - cl^*(C_A B) \tilde{q} C_B B$ … (1)

Since $C_B A \leq B \Longrightarrow \tau_i - cl(C_B A) \leq \tau_i - cl(B) = B$, $i \in \{1,2\}$

By proposition (2.9) we get

$\tau_i - cl(C_B A) \tilde{q} B^c$

Since $C_A B \leq B^c$, then $\tau_i - cl(C_B A) \tilde{q} C_A B$ … (2)

$C_A B$, $C_B A$ are τ_i −IF $*$ −separated sets in X .

Theorem 3.1.9 :-

Let (X, τ_1, τ_2, L) be IFLBTS . Then A and B are two τ_i −IF $*$ −separated sets if and only if there exists an τ_i −intuitionistic fuzzy open set (τ_i −IFOS)U and $* - \tau_i$ −intuitionistic fuzzy open set V ($* - \tau_i$ −IFOS) , $i \in \{1,2\}$

Such that $A \leq U$, $B \leq V$, $A \tilde{q} V$ and $B \tilde{q} U$.

Proof :-

($\Longrightarrow$) Suppoe that A , B are τ_i −IF $*$ −separated sets .

$\Longrightarrow \tau_i - cl^*(A) \tilde{q} B$ and $A \tilde{q}\, \tau_i - cl(B)$

Now put $V = (\tau_i - cl^*(A))^c$ and $U = (\tau_i - cl(B))^c$

So U is τ_i −IFOS and V $* - \tau_i$ −IFOS , $i \in \{1,2\}$

Then $V^c\tilde{q}B$ and $A\tilde{q}U^c$

By proposition (2.9) we get $V^c \leq B^c \Longrightarrow B \leq V$ and $A \leq U$

So $A \leq (\tau_i - cl(B))^c$ and $B \leq (\tau_i - cl^*(A))^c$

Since $B \subseteq \tau_i - cl(B)$ and since $\tau_i - cl^*(A) = A \vee A^*(L, \tau_i)$, $i \in \{1,2\}$, then $A \subseteq \tau_i - cl^*(A)$

Then $A \leq V^c$ and $B \leq U^c$

Therefore , $A\tilde{q}V$ and $B\tilde{q}U$.

($\Longleftarrow$) Suppose that there exist U be τ_i −IFos and V be $* - \tau_i$ −IFOS in X such that $A \leq U$,

$B \leq V$, $A\tilde{q}V$ and $B\tilde{q}U$.

Now $U^c$is τ_i −IFCS and V^c is an $* - \tau_i$ −IFcs in X , $i \in \{1,2\}$

Since $A\tilde{q}V$ and $B\tilde{q}U$, then $A \leq V^c$ and $B \leq U^c$.

Since $A \leq U$ and $\leq V$, thus $U^c \leq A^c$ and $V^c \leq B^c$

Since $A \leq V^c \Longrightarrow \tau_i - cl^*(A) \leq \tau_i - cl^*(V^c) = V^c$

Because V^c is $* - \tau_i$ −IFCS

$\Longrightarrow \tau_i - cl^*(A) \leq V^c \leq B^c$, since $B \leq U^c$

$\Longrightarrow \tau_i - cl(B) \leq \tau_i - cl(U^c) = U^c$, because U^c is τ_i −IFCS , $i \in \{1,2\}$

Thus $\tau_i - cl(B) \leq U^c \leq A^c$

By proposition (2.9) $\tau_i - cl^*(A) \leq B^c$,

Then $\tau_i - cl^*(A)\tilde{q}B$ … (1)

$\tau_i - cl(B) \leq A^c \Longrightarrow \tau_i - cl(B)\tilde{q}A$, then $A\, \tilde{q}\tau_i - cl(B)$ … (2)

Hence A , B are τ_i −IF $*$ − separated sets

Definition 3.1.10 :-

An τ_i −intuitionistic fuzzy set E in intuitionistic fuzzy ideal bitopological space (X, τ_1, τ_2, L) is said to be intuitionistic fuzzy $*$ − connected if it can not be expressed as the Union of two intuitionistic fuzzy $*-$ separated sets . Otherwise , E is said to be intuitionistic fuzzy $*-$ disconnected . If $= X$, then X is said to be intuitionistic fuzzy $*-$ connected space . And we shall denoted it by (τ_i $-$IF $*$ $-$connected sets , for short i $\in \{1,2\}$) .

Theorem 3.1.11 :-

Let A and B be τ_i $-$intuitionistic fuzzy $*$ $-$separated sets in an intuitionistic fuzzy ideal bitopological pace (X, τ_1, τ_2, L) and E be a non empty τ_i $-$IF $*$ $-$connected set in X such that $E \leq A \vee B$. Then exactly one of the following conditions holds :

a) $E \leq A$ and $E \wedge B = 0_{\sim}$

b) $E \leq B$ and $\wedge A = 0_{\sim}$.

Proof :-

Let $E \wedge B = 0_{\sim}$

Since $E \leq A \vee B$ then $E \leq A$

Similarly , if $E \vee A = 0_{\sim}$ we have $E \leq B$

Since $E \leq A \vee B$ then $E \wedge A = 0_{\sim}$ and $E \wedge B = 0_{\sim}$ can not hold simultaneously (because $E \neq 0_{\sim}$)

Suppose that $E \wedge B \neq 0_{\sim}$ and $\wedge A \neq 0_{\sim}$.

Then $E \wedge A$ and $E \wedge B$ are τ_i $-$IF $*$ $-$separated set in X such that $E = (E \wedge A) \vee (E \wedge B)$ therefore E is an τ_i $-$intuitionistic fuzzy $*$ $-$disconnectedness of E .

This is contradiction

Hence exactly one of the conditions (a) and (b) must hold .

Theorem 3.1.12 :-

Let E ,F be two τ_i −intuitionistic fuzzy sets of IFLBTS (X, τ_1, τ_2, L) if E is an τ_i −IF ∗ −connected and $E \leq F \leq \tau_i - cl^*(E)$, $i \in \{1,2\}$. Then F is an τ_i −IF ∗ −connected set .

Proof :-

If $= 0_\sim$, then the result is true .

Let $F \neq 0_\sim$ and F is an IF ∗ −disconnected. There exist two τ_i −IF ∗ −separated sets A and B in X such that $F = \vee B$. Since E is an τ_i −IF ∗ −connected and

$E \leq F = E \vee F , E \leq F = A \vee B , E \leq A \vee B$

So by theorem (3.1.11) , we get

$E \leq A$ and $E \wedge B = 0_\sim$ or $E \leq B$ and $E \wedge A = 0_\sim$

Let $E \leq A$ and $E \wedge B = 0_\sim$

$B = B \wedge F \leq B \wedge \tau_i - cl^*(E) \leq B \wedge \tau_i - cl^*(A) \leq B \wedge B^c \leq B$, $i \in \{1,2\}$

It follows that $B = B \wedge B^c$ when $B = 0_\sim$ or $\mu_B(x) = \nu_B(x)$, $\forall x \in X$.

Since $\neq 0_\sim \Longrightarrow \mu_B(x) = \nu_B(x)$, $\forall x \in X$.

Thus , $B_r = X$ where B_0 denotes the support of B .

Now $E \wedge B = 0_\sim$ implies $E_r \wedge B_r = \emptyset \Longrightarrow E_r = \emptyset \Longrightarrow E = \emptyset$

Which is a contradiction

Similarly , if $E \leq B$ and $E \wedge A = 0_\sim$, then we get $E = 0_\sim$ a contradiction

Hence F is an τ_i −intuitionistic fuzzy ∗ −connected .

Theorem 3.1.13 :-

Let A and B be two τ_i −intuitionistic fuzzy ∗ −connected sets which are not τ_i −intuitionistic fuzzy ∗ −separated .Then $A \vee B$ isτ_i −intuitionistic fuzzy∗ −connected set .

Proof :-

Suppose that $A \vee B$ is an τ_i $-$intuitionistic fuzzy $*$ $-$disconnected set $\Longrightarrow$ $A \vee B = G \vee H$ where G and H are τ_i $-$intuitionistic fuzzy $*$ $-$separated sets in X .

Since $A \leq A \vee B$ and $B \leq A \vee B$

Then $A \leq G \vee H$ and $B \leq G \vee H$

By theorem (3.1.11), we get

$A \leq G$ with $A \wedge H = 0_{\sim}$ or $A \leq H$ with $A \wedge G = 0_{\sim}$.

And $B \leq G$ with $B \wedge H = 0_{\sim}$ or $B \leq H$ with $\wedge G = 0_{\sim}$.

If $A \leq G$ and $B \leq H$ or $A \leq H$ and $B \leq G$

We get that A and B are τ_i $-$intuitionistic fuzzy $*$ $-$separated and this contradiction

If $A \leq G$ with $B \wedge H = 0_{\sim}$ and $B \leq G$ with $\wedge H = 0_{\sim}$.

If $A \leq H$ with $A \wedge G = 0_{\sim}$ and $B \leq H$ with $B \wedge G = 0_{\sim}$

We get that

$A \vee B \leq G$ and $H = 0_{\sim}$ or $A \vee B \leq H$ and $G = 0_{\sim}$ which contradiction , therefore , $A \vee B$ is τ_i $-$intuitionistic fuzzy $*$ $-$connected set .

Therom 3.1.14 :-

Let $f: (X, \tau_1, \tau_2, L) \longrightarrow (Y, \tau_1, \tau_2)$ is intuitionistic fuzzy continuous on to mapping , if (X, τ_1, τ_2, L) is an τ_i $-$intuitionistic fuzzy $*$ $-$connected ideal bitopological space .Then (Y, τ_1, τ_2) is also τ_i $-$intuitionistic fuzzy $*$ $-$connected bitopological space .

Proof :-

It is known that connectedness is preserved by intuitionistic fuzzy continuous surjections .

The proof is clear.

Corollary 3.1.15 :-

If IFS A is an τ_i −intuitionistic fuzzy $*$ −connected set in an intuitionistic fuzzy ideal bitopological space (X, τ_1, τ_2, L) . Then $\tau_i - cl^*(A)$, $i \in \{1,2\}$ is τ_i −intuitionistic fuzzy $*$ −connected set .

Proof :-

Since $\tau_i - cl^*(A) = A \vee A^*(L, \tau_i)$, $i \in \{1,2\}$,

Then $\subseteq \tau_i - cl^*(A)$.

Since A is τ_i −IF $*$ −connected set and $A \subseteq \tau_i - cl^*(A)$.

By theorem (3.1.12)

$\tau_i - cl^*(A)$ is an τ_i −IF $*$ −connected set .

Theorem 3.1.16 :-

If $\{\mu_i : i \in N\}$ is a non empty family of τ_i −intuitionistic fuzzy $*$ −connected sets of an IFLBTS (X, τ_1, τ_2, L) with $\bigcap_{i\in I} \mu_i \neq \emptyset$. Then $\bigcup_{i\in I} \mu_i$ is an τ_i −intuitionistic fuzzy $*$ −connected set .

Proof :-

Suppose that $\bigcup_{i\in I} \mu_i$ *is not* τ_i *−IF* $*$ *−connected set .*

Then by definition (3.1.10) , there exist two τ_i −IF $*$ −separated sets H and G , such that

$\bigcup_{i\in I} \mu_i = H \cup G$, since $\bigcap_{i\in I} \mu_i \neq \emptyset$. We have a point x in $\bigcap_{i\in I} \mu_i$.

Since $\in \bigcup_{i\in I} \mu_i$, either $x \in H$ or $x \in G$.

Suppose that $\in X$. Since $x \in \mu_i$ for each $\in N$, then μ_i and H intersect for each $i \in N$.

By theorem (3.1.11) $\mu_i \subset H$ and $\mu_i \wedge G = 0_\sim$ or $\mu_i \subset G$ and $\mu_i \cap H = 0_\sim$.

Suppose that $\mu_i \subset H \Longrightarrow \mu_i \subset H$ for all $i \in N$ and hence $\bigcup_{i\in I} \mu_i \subset H$.

This implies that τ_i −IF ∗ −separated set G is empty .

This is a contradiction .

Suppose that $\mu_i \subset G$. By similar way , we get $H = \emptyset$.

And this is a contradication .

Thus , $\cup_{i\in I} \mu_i$ is an τ_i −intuitionistic fuzzy ∗ −connected set .

Theorem 3.1.17 :-

Suppose that $\{\mu_n : n \in N\}$ is an sequence of τ_i −intuitionistic fuzzy ∗ −connected open sets of an intuitionistic fuzzy ideal bitopological space (X, τ_1, τ_2, L) and $\mu_n \cap \mu_{n+1} \neq \emptyset$ for each $\in N$. Then $\cup_{i\in I} \mu_i$ is τ_i −IF ∗ −connected set .

Proof :-

By induction and theorem (3.1.16)

The $N_n = \cup_{k\leq n} \mu_k$ is τ_i −IF ∗ −connected open set for each $n \in N$

Also , N_n is τ_i −IF ∗ −connected open set .

Thus , $\cup_{n\in N} \mu_n$ is τ_i −IF ∗ −connected set .

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