Free Vibrations of Uniform Pipes Made From Composite Materials at an Internal Flow Under Effect of Additional Boundary Conditions

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Abstract

In this paper the approximate method of Raleigh method can be used to study the effect of additional boundary conditions (clamped – free & clamped – clamped) on the free transverse vibrations of uniform pipes which have length, L (1m) , inner radius, "R_i" (1cm) & thickness, "t" (1mm) made from composite materials, where the resin of unsaturated polyester represented the matrix material reinforced by aligned (E-fibers glass) in the first case and used aligned fiber (Kevlar-49) in the second case. The length of fibers is in the two types, the first type is long fibers (continuous) and the second is short fibers (discontinuous) for different length all at volume fraction of fibers, "f" (0.15 & 0.25). At any construction of the pipe in composite material the natural frequency decreased when the velocity of flow increased from zero to critical velocity also can be observed the pipe at clamped – clamped boundary conditions predicts natural frequency & critical velocity greater than that pipe at clamped – free. The natural frequency and critical velocity increase with increasing volume fraction and length of discontinuous fiber. The value of natural frequency for pipes which have continuous fibers is constant at certain velocity of flow while are variable in pipes which have discontinuous fibers according to ratio between length of short fiber to critical length of discontinuous fiber whereas the natural frequency increase with increasing this ratio. Finally the pipes with Kevlar fiber have high critical velocity and natural frequency compare with pipes for fiber glass.

Key words: internal flow, uniform pipe, composite materials, boundary conditions.

المقدمة

في هذا البحث تم استخدام طريقة رايلي التقريبية لدراسة تأثير الشروط الحدية المضافة (مثبت – مثبت و مثبت – حر) على الاهتزازات الحرة المستعرضة للأنابيب المنتظمة التي تمتلك طول " L" (m١)، نصف قطر داخلي "R_i" (cm١) و سمك "t" (mm١) مصنوعة من المواد المركبة، حيث أن راتنج البولي استر غير المشبع يمثل المادة الأساس التي تـم تقويتهـا بأليـاف الزجاج نوع (E) المصففة في الحالة الأولى وتم استخدام (ألياف الكفلار– ٤٩) المصففة في الحالة الثانية . طول الألياف يكون على نوعين الأول يمثل الألياف الطويلة (المستمرة) والنوع الثاني تكون الألياف القصيرة (غير مستمرة) ولأطوال مختلفة وكل هذا عند كسر حجمي للألياف "f" (٠,١٥ و ٠,٢٥). عند أي تركيب للأنبوب من المادة المركبة فان التردد الطبيعي يقل عندما تزداد سرعة الجريان من الصفر إلى السرعة الحرجة كذلك من الممكن ملاحظة الأنبوب عند شروط حدية(مثبت – مثبت) تتنبأ بتـردد طبيعي وسرعة حرجة اكبر مما علية للأنبوب في حالة (مثبت – حر). التردد الطبيعي يزداد مع زيادة الكسر ألحجمـي وطـول الألياف الغير مستمرة. عند سرعة معينة للجريان قيمة التردد الطبيعي للأنابيب التي تمتلك ألياف مستمرة تكون ثابتة بينما تكـون متغيرة للأنابيب ذات الألياف الغير مستمرة حسب النسبة بين طول الليف القصير إلى الطول الحرج لليف غير المستمر حيث إن التردد الطبيعي يزدرد مع زيادة هذه النسبة. وأخيرا الأنابيب ذات ألياف الكفلار تمتلك سرعة حرجة وتردد طبيعي أعلى مقارنـة بالأنابيب ذات ألياف الزجاج.

الكلمات المفتاحيه، جريان داخلي، أنبوب منتظم، مواد مركبة، شروط حدية

List of Symbols .

A Cross section area (m^2).
A_P. Cross section area of pipe (m^2).
A_w Cross section area of water (m^2).
a&c_1 Constants.
E Modulus of elasticity (N/m^2).
E_c Modulus of elasticity of composite materials of pipe (N/m^2).
E_f Modulus of elasticity of fiber (N/m^2).
E_m Modulus of elasticity of matrix (N/m^2).

L Length of the pipe (m).
I Second moment of area (m^4)
I_p Second moment of area of composite pipe (m^4).
m_w Mass of fluid per unit length (kg/m).
m_p Mass of pipe per unit length (kg/m).
t Thickness of pipe (mm).
R_i Inner radius at clamped end.
R_i Inner radius at free end.
V Velocity of water inside the pipe (m/sec).
V_c Critical velocity of fluid flows in the pipe (m/sec).
Y_1 Displacement (amplitude of pipe) (m).
ρ_c Mass density of composite materials for pipe (kg/m^3).
ρ_f Mass density of fiber (kg/m^3).
ρ_m Mass density of matrix (kg/m^3).
ρ_p Mass density of pipe material (kg/m^3).
ω Natural frequency of pipe at velocity of flow V (rad/sec).
ω_1 Fundamental natural frequency of pipe in absence of flow (rad/sec).

1.Introduction

Advancements in materials bonding techniques have led to the use of reinforced composite pipelines where have found applications in wide range of industries such as aerospace, automotive pipe lines , etc. which are consist of two or more phases that are usually processed separately and then bonded, resulting in properties that are different from those of either of the component materials. Polymer matrix composites generally combine high strength , high stiffness fiber s (graphite, Kevlar, glasses, etc.) with low density matrix materials (epoxy, polyester, polyvinyl, etc.) to produce strong and stiff materials that are lightweight. The literatures for composites materials to study natural frequency are absent except a little studies but the predominantly studies involve to study of mechanical properties, (**Zou,2005),** presented a state variable model developed for the analysis of fluid induced vibrations of composite pipeline systems (simply sported, clamped and clamped – simply sported pipelines and investigated by ansys software. (**Hani,2008),** investigated the compression and hardness test for composite materials consist from epoxy resin and natural rubber in different ratio of volume fracture. (**Ali ,2009),** studied the effect of changing the reinforcement percentage by fibers on mechanical properties, for composite material consists of epoxy reinforced by biaxial woven roving Kevlar fibers which included impact strength, tensile strength and hardness.(**Sahama,2010),** prepared composite materials from the unsaturated polyester resin as matrix reinforced by woven glass fiber kind (E -glass) as first group and the second group reinforced with woven of Kevlar fiber kinds (49) to test number of mechanical properties (tensile, compression, impact, flexural strength, shear stress and hardness) which done at room temperature. **(Hiba,2010),** studied the effect of fibers on the damping behaviors of composite materials by using epoxy and polyester risen as a matrices for composite materials with carbon, glass and copper as reinforced materials with different of volume fraction. Mechanical properties that be studied (vibration damping, deflection, stiffness, natural frequency and damping period. (**Amar,2011)**, prepared the polyester matrix reinforced with short glass fibers and $Al2O3$ particles with different weight fraction the measured (bending , impact and hardness) the results showed the mechanical properties which improve with increasing weight fraction. (**Zainab,2012),** studied the mechanical for properties for pipes where the specimen cut from pipes made from composite materials were tested under internal pressure loading for carbon and glass fiber /epoxy composites. **(Thaier,2013),** investigated the effect of support on the natural frequency and critical flow velocity of straight pipe conveying laminar flowing fluid by considering the supports as compliant material with linear and rotational springs.(**Ganesa,2014**), presented free vibration analysis of glass fiber reinforced polymer composite materials had done experimental method and FEM analysis for free vibration characteristics.

In this paper, approximate method of Raleigh method is used to obtained frequency equation for uniform pipe made from composite materials at different boundary conditions (Clamped – Free & Clamped – Clamped) made from composite materials where unsaturated polyester resin as matrix reinforced by continuous and discontinuous fibers glass kind (E-glass) as first group of model and the second group of model reinforced with continuous and discontinuous fibers Kevlar kind (49) instead of glass fibers to estimate the natural frequency of vibrations and critical velocity of flow at different values of volume fractions of fibers as well at different length of discontinuous fibers in addition computation natural frequency at diverse velocity of flow.

2.Theoretical analysis

Consider an punctually uniform cross section pipe of length L, the thickness t and the inner radius R_i at any position of part of length of pipe for different boundary conditions as shown in (Fig. 1).

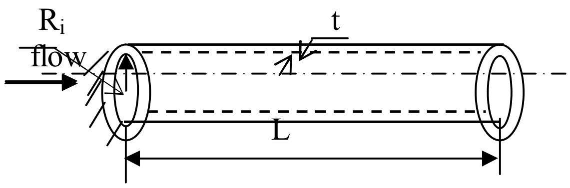

Fig. (1-a) Composite pipe at clamped-free

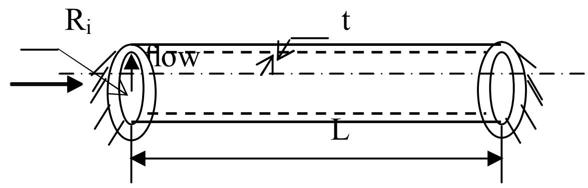

Fig. (1-b) Composite pipe at clamped-clamped

In uniform cross section of pipe at any length of pipe, $A_p = \pi\ (R_o^2 - R_i^2)$, where $R_o = (R_i+t)$, therefore, $m_p = \rho_p * A_p$, here the density of pipe (ρ_p) is represented the density of composite materials (ρ_c) which are component the pipe where its value change with the volume fracture of fiber (f) therefore the density of pipe from the below formula can be calculated:-

$$\rho_c = \rho_f * f + \rho_m * (1 - f) \quad (1)$$

therefore,

$$m_p = \rho_c * A_p \quad (2)$$

and

$$I_p = \pi/4(R_o^4 - R_i^4) \quad (3)$$

Now the procedure of Rayeigh is applied derive the natural frequency for transverse motion of uniform cross section of cantilever pipe (clamped - free) and the pipe at (clamped–clamped) of boundary conditions. Consider the term approximation for clamped – free of boundary condition **[Benoraya, 1998].**

$$Y_1(x) = a\left[1 - \cos\left(\frac{\pi x}{2L}\right)\right] \quad (4)$$

But consider the following term solution for (clamped –clamped) boundary conditions,**[Desai, 1971].**

$$Y_1(x) = c_1\left[1-\cos\left(\frac{2\pi x}{L}\right)\right] \qquad (5)$$

In this method equates the maximum potential energy to the maximum kinetic energy during motion, can be obtained, **[Benoraya,1998].**

$$\omega_1 = \frac{\int_0^L E\,I(x)\,\left[Y_1''(x)\right]^2\,dx}{\int_0^L m(x)\,\left[Y(x)\right]^2\,dx} \qquad (6)$$

Substitute the expression for $Y_1(x)$ and it's the second differentiations in above equation for two boundary conditions and after integration the approximate of natural frequency can be yields:-

$$\omega_1^2 = \frac{(1.9149)^2}{L^2}\sqrt{\frac{E\,I_p}{m}} \qquad \text{(clamped – free)} \qquad (7)$$

$$\omega_1^2 = \frac{(4.73)^2}{L^2}\sqrt{\frac{E\,I_p}{m}} \qquad \text{(clamped – clamped)} \qquad (8)$$

Where E represented the modulus of elasticity of composite materials of pipe and depended on volume fraction (f) therefore the modulus of elasticity for composite materials (E_c)**, [Ashraf, 2001]** can be written as follow:-

$$E_c = E_f * f + E_m * (1 - f) \qquad (9)$$

Equation (9) is really for long (continuous) fibers in order to compute the modulus of elasticity for composite materials also in the present study can be use short (discontinues) fibers whereas (E_c) can evaluate as follow:-

$$E_c = E_f\left(1-\frac{l_c}{2l}\right) * f + E_m * (1 - f) \qquad (10)$$

Where (l) length of short fiber and (l_c) critical length of fiber which is depend on the diameter of fiber (d), fiber – matrix bonds strength (τ_c) and fiber yield strength (σ_f). For short fiber $l < 15l_c$, where in this study can be taken as (l = 1, 4, 8, 12) from l_c.

(m)represented the total mass of system per unit length which consists from mass of pipe and mass of fluid (water), $m_{tot} = m_p + m_w$, where($m_w = \rho_w * A_w$), therefore the equations (7) &(8) can be written as follow:-

$$\omega_1^2 = \frac{(1.9149)^2}{L^2}\sqrt{\frac{E_c\,I_p}{m_{tot}}} \qquad (7\text{-a})$$

$$\omega_1^2 = \frac{(4.73)^2}{L^2}\sqrt{\frac{E_c\,I_p}{m_{tot}}} \qquad (8\text{-a})$$

The estimation of these equations provides us natural frequency ω_1^2 for the pipe carrying fluid which not moved. In order to conception the natural frequency of pipe when the fluid moved at any velocity, firstly should be determined the critical velocity of fluid from the flowing equations **[Ivan, 2010]**,

$$Vc = 1.9149/L \sqrt{EI/\rho_w A_w} \quad \text{(clamped – free)} \qquad (11)$$

$$Vc = 4.73/L \sqrt{EI/\rho_w A_w} \quad \text{(clamped – clamped)} \qquad (12)$$

The above equations are really for uniform pipe but in the present study can be used composite pipe therefore must change equation in order to realized the current study.

$$Vc = 1.9149/L \sqrt{E_c I_p/\rho_w A_w} \quad \text{(clamped – free)} \qquad (11\text{-a})$$

$$Vc = 4.73/L \sqrt{E_c I_p/\rho_w A_w} \quad \text{(clamped – clamped)} \qquad (12\text{-a})$$

Thus the natural frequency of pipe at any velocity of fluid can be found out from the following equation :-

$$\frac{\varpi}{\varpi_1} = \sqrt{1 - \frac{V}{Vc}} \qquad \textbf{[Blivens, 1993]}, \qquad (13)$$

3.Results and Discussion

Table (1) shows the mechanical properties of the materials which can be used in this paper and table(2) shows the values of critical velocity of flow of water through composite pipe which have long (continuous) fibers for different values of volume fracture and boundary conditions by using equations (11-a & 12-a). Figures (2 to 5) show the critical velocity as a function of the length ratio of short (discontinues) fibers obtain for variation values. It is obviously seen that the critical velocity (Vc) increased with the increase length ratio (l/lc) & volume fracture (f). These manners interpret for changing the value of modulus of elasticity (Ec) that is count on the critical velocity where increased with increasing the modulus of elasticity for composed materials. Also can be seen the critical velocity of flow through pipe at clamped – clamped bigger than that at clamped – free because of the stiffness of the system becomes greater than that at clamped --free. In the other hand the critical velocity through composite pipe which have fibers Kevlar is greater resulting of the modulus elasticity is higher than that which fibers glass. Figures (6 to 13) show the natural frequency as a function of velocity of flow (V=0 to V= Vc) get for different length of fibers and values of volume fracture as well for different boundary conditions can be observed at the same value of l & f the natural frequency decrease with increasing velocity of flow because of the deformation which obtains in the wall of pipe as a result of pressure of water which flow through the pipe therefore occur centrifugal force due to acceleration of fluid through the curvature of deformation of the pipe and it produces a reduction of the natural frequency and caused flail pipe for clamped – free and buckling pipe for clamped – clamped. In the other hand at the any value of velocity (V) the natural frequency of system increases with increasing the value of f & l/lc due to increasing the modulus of elasticity therefore caused increasing the stiffness of composite material. Lastly the natural frequency of pipe which fibers Kevlar is greater than that pipe for fiber glass. Product of increasing the modulus of elasticity. Finally the system for clamped-clamped which has natural frequency is greater than that for clamped free due to increasing the stiffness.

4.Conclusion

Conclusions of the present study can be summarized as follows; the boundary conditions of the pipes effect on stiffness of the system subsequently generate the vibration of pipe is upper at clamped – clamped. The natural frequency increases with decreasing flow velocity, increasing the length ratio of fibers & volume fracture. In other hand, the critical velocity of flow increases through the pipes with increasing length ratio of fibers & volume fracture. The modulus of elasticity of pipes increase with increasing volume fracture that cause increasing the natural frequency of system.

Table (1) Mechanical properties of materials, [Daniel, 2014]

Material	Modulus of elasticity (E) GN/m^2	Density ρ kg/m^3
Polyester	2.5	1380
E-glass	72	2500
Kevler-49	154	1470

Table (2) Critical velocity (Vc) of flow at different composite materials of pipes for continues fibers (m/sec).

Boundary Conditions	Volume Fracture(f)	Fiber Glass (E)	Fiber Kevlar (49)
Clamped-Free	0.15	23.45	32.759
=	0.25	29.078	41.44
Clamped-Clamped	0.15	57.92	80.92
=	0.25	71.827	102.37

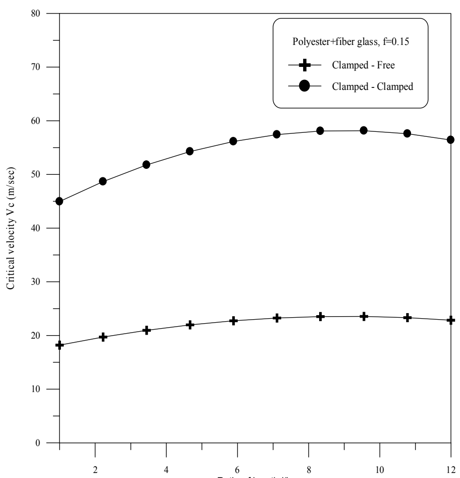

Fig. (2): Critical velocity as a function of length of fibers glass for f=0.15 .

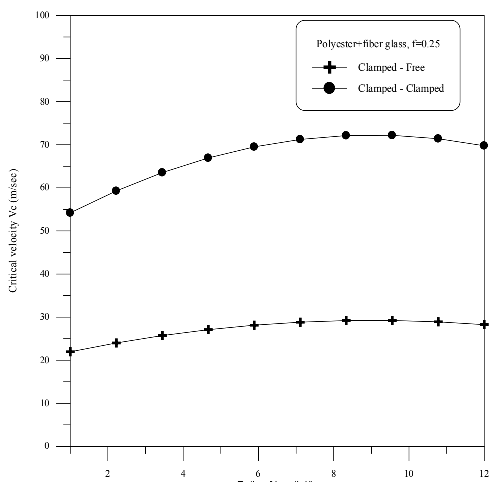

Fig. (3): Critical velocity as a function of length of fibers glass for f=0.25 .

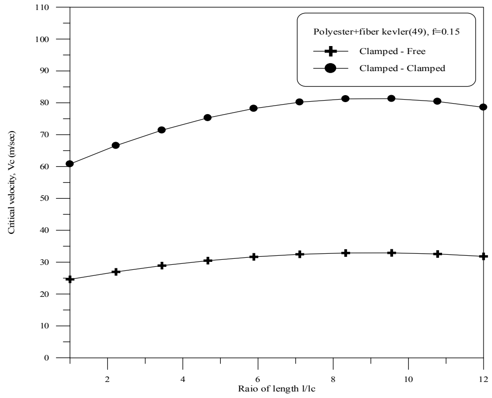

Fig. (4): Critical velocity as a function of length of fibers Kevlar for f=0.15 .

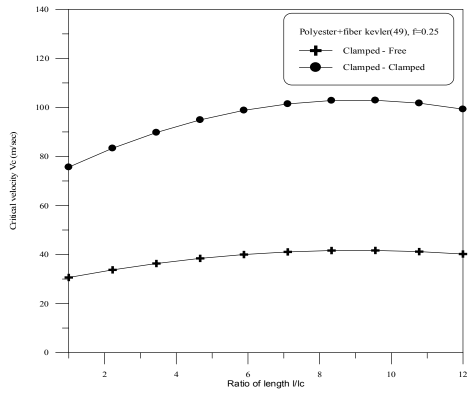

Fig. (5): Critical velocity as a function of length of fibers Kevlar for f=0.25 .

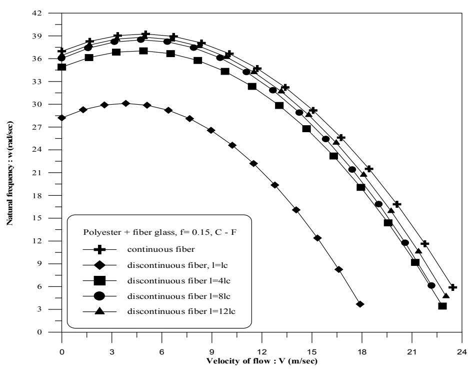

Fig. (6): Natural frequency as a function of velocity of flow for different length of fibers glass for 1st mode at clamped- free for f=0.15 .

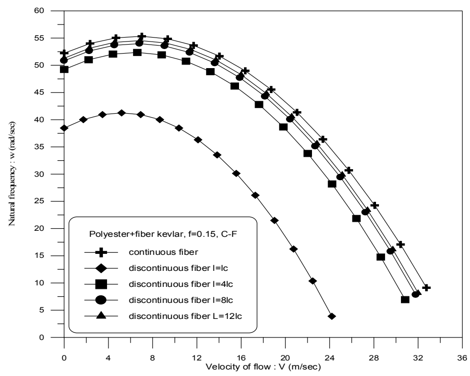

Fig. (7): Natural frequency as a function of velocity of flow for different length of fibers Kevlar for 1st mode at clamped - free for f=0.15 .

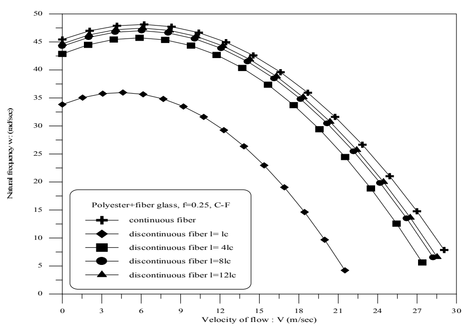

Fig. (8): Natural frequency as a function of velocity of flow for different length of fibers glass for 1st mode at clamped- free for f=0.25 .

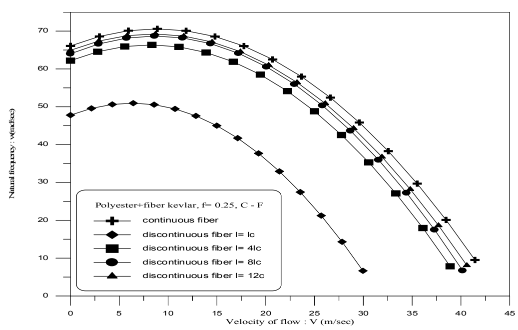

Fig. (9): Natural frequency as a function of velocity of flow for different length of fibers Kevlar for 1st mode at clamped - free for f=0.25 .

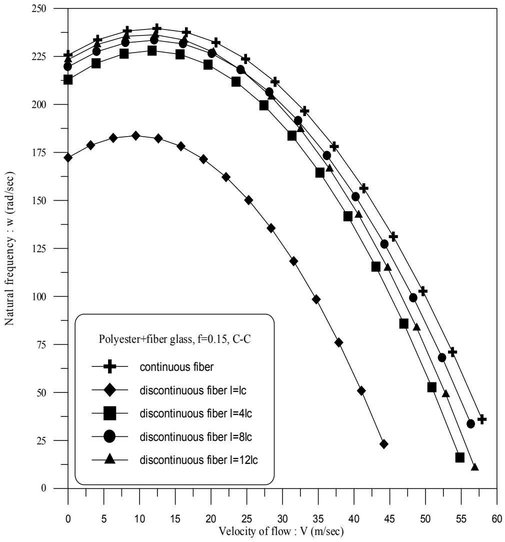

Fig. (10): Natural frequency as a function of velocity of flow for different length of fibers glass for 1st mode at clamped- clamped for f=0.15 .

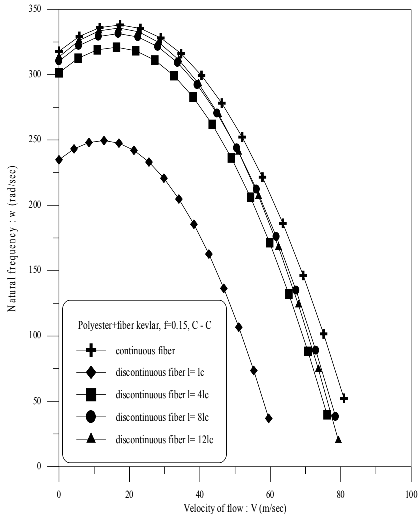

Fig. (11): Natural frequency as a function of velocity of flow for different length of fibers Kevlar for 1st mode at clamped- clamped for f=0.15 .

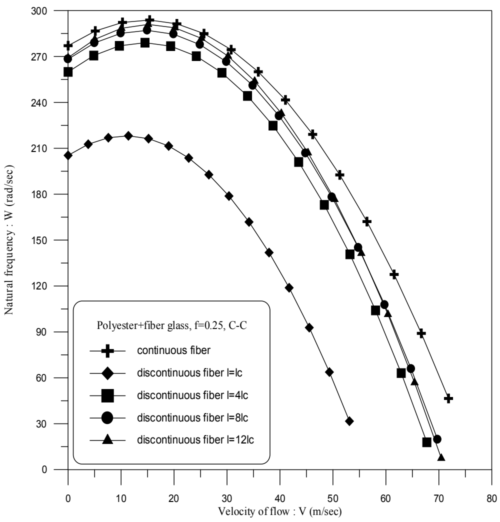

Fig. (12): Natural frequency as a function of velocity of flow for different length of fibers glass for 1st mode at clamped- clamped for f=0.25 .

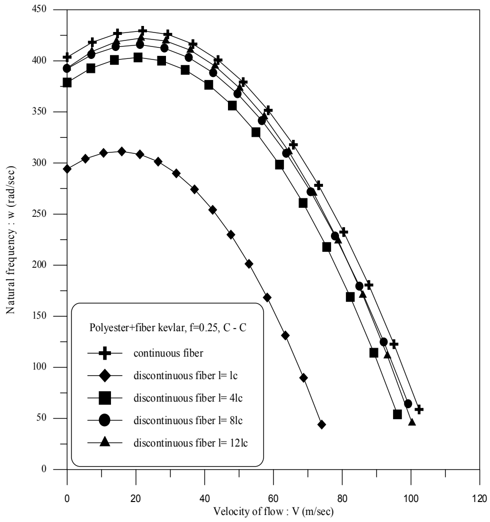

Fig. (13): Natural frequency as a function of velocity of flow for different length of fibers Kevlar for 1st mode at clamped- clamped for f=0.25 .

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