References:

1/ Hapuarachi, K.P.and Wronski ,A.(1955)."Control Charts For Non-Response Rates In The Canadian Labour Force Survey. Ottawa, Ontario, KIA 0T6, Canada."

2/ Woodal, W.H.and Faltin, F.W.(1993). "Auto-correlated data And SPC, ASQC, Statistics Division " Newsletter,Vol.13,No.4,PP(18-21).

3/ Montgormey ,D.C.& Mastrangelo, C.M.(1991). "Some Statistical Process Control Methods For Auto-correlated data , Journal Of Quality Technology ", Vol.23,No.3, PP.(179-204).

4/ Box, G.E.P & Jenkins G.M.(1976) ."Time Series Analysis: Forecasting & Control" Holden-Day.

5/ Burr,I.W.(1976) ." Statistical Quality Control Methods", Marcel Decker , New- york.

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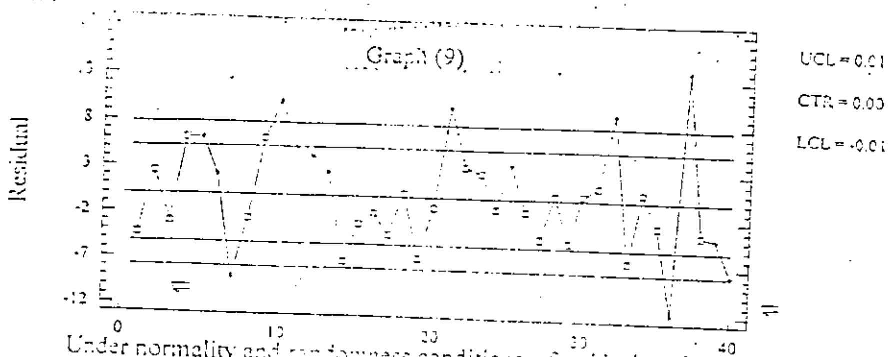
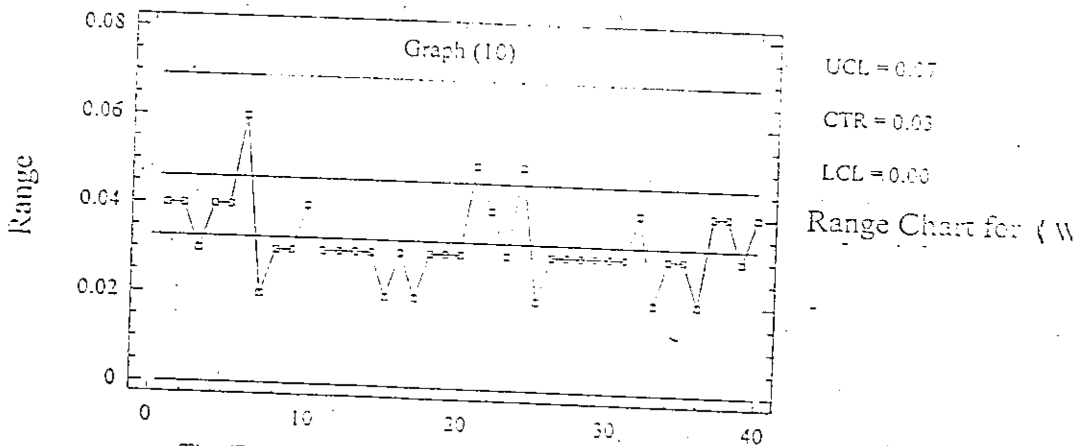
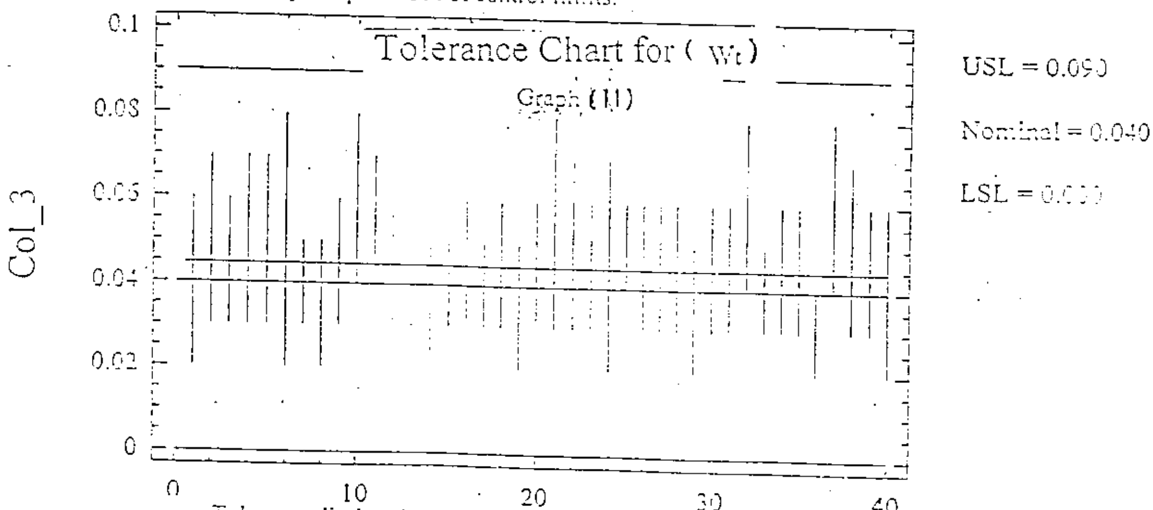
ARIMA Residual Chart for (Wt)



Under normality and randomness conditions, of residuals series ,generated From ARIMA(1,0,1). the impure weights series {Wt}. is out of control. See the chart shows (6) beyond points out of upper and lower control limits.



The (R-chart) for {Wt} , their are no. Beyond point out of control limits.



Tolerance limits, that used for constructing a new structure control chart For controlling impure weight/cigarette during production process in Sul-aimani cigarettes factory , does not allowed the impure rate more than (0.09)grams/ciugarette.

A: Estimates for the model ARIMA (1,0,1), for the series{Wt}.

B: Gives the ARIMA chart for residuals, as given in section (5).
There are (6), beyond points, (the process out of control).

C: Gives the upper and lower limits, for(r-chart) with zero number out of control points (zero beyond points) .

D: Initial estimates for (40), consecutive residuals.

E: Box-Pierce test value(Q = 4.82877), for residuals autos, after modeling the series {Wt},with ARIMA (1,0,1). Which gives a strong indicator that the residuals, generated from the suggested model is random , this attains to , the goodness of fit about the model ARIMA (1,0,1).

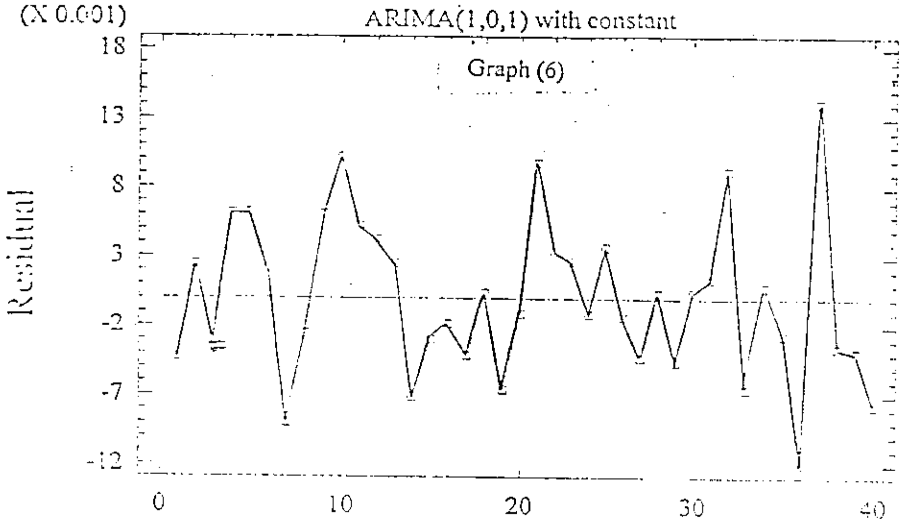
F: Capability index, illustrated in detail in the section(11).

Note: The statistical packages(statgraph , minitab, spss),are used in applications.

Table (4)

A	ARIMA Model Summery Parameter Estimate Constant 0.0434137 AR(1) 0.0249987 MA(1) -0.192458	E	Box-Pierce Test = 4.82874
B	ARIMA Chart ---------------- UCL: +3.0sigma = 0.00780627 Centerline = 0.0 LCL:- 3.0 sigma =-0.00780627 6 beyond limits	F	Capability Indices for Cc Specifications USL = 0.05 Nominal = 0.04 LSL = 0.0 Cp = 0.578333 Cpk = 0.126613 Cpk (upper)= 0.126613 Cpk (lower) = 1.03005
C	Range Chart -------------- UCL: +3.0 sigma = 0.0692453 Centerline = 0.03275 LCL: - 3.0 sigma = 0.0 0 beyond limits		
D	Estimates ------------ Process mean = 0.0445268 Process sigma = 0.0144092 Mean range = 0.03275		

Residual Plot for (Wt):



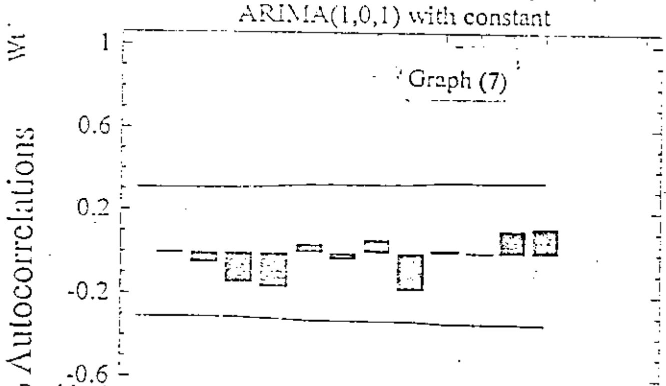


Table (6) ARIMA(1,0,…
residuals

Lag	Autocorrel…
1	-0.0062625…
2	-0.0541791
3	-0.147242
4	-0.169527
5	0.0401831
6	-0.0333546
7	0.0646306
8	-0.178999
9	0.015961
10	0.00219473
11	0.111311
12	0.125175

Residuals autocorrelations, after modeling {Wt}, ARIMA(1,0,1), shows no pattern exist , and also their partials, which gives a strong evidence that the stationary is achived.

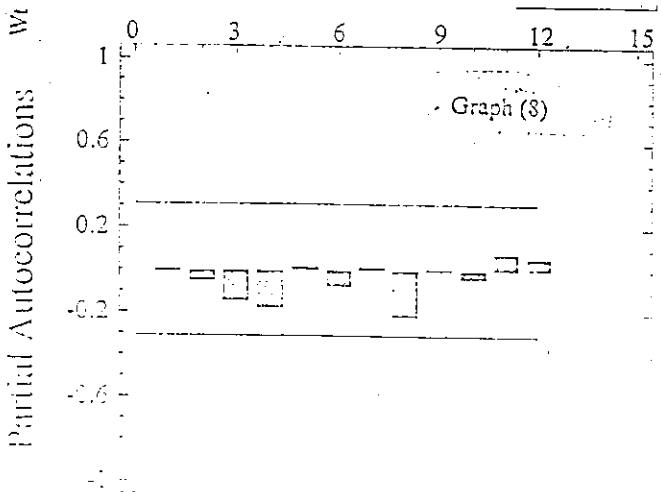
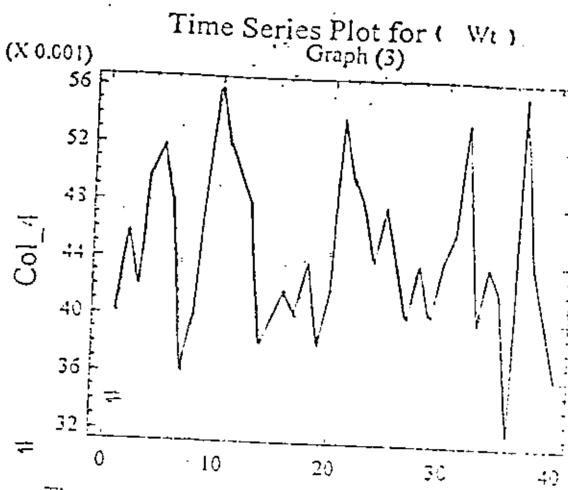


Table (7) ARIMA(1,0,…
residuals Par…

Lag	Autocorrela…
1	-0.0062625…
2	-0.0542204
3	[illegible]
4	[illegible]
5	[illegible]
[illegible]	[illegible]



The mean sub-group impure weights series {Wt, t = 1,2,...,40},
Each sub-group has (5) observation.

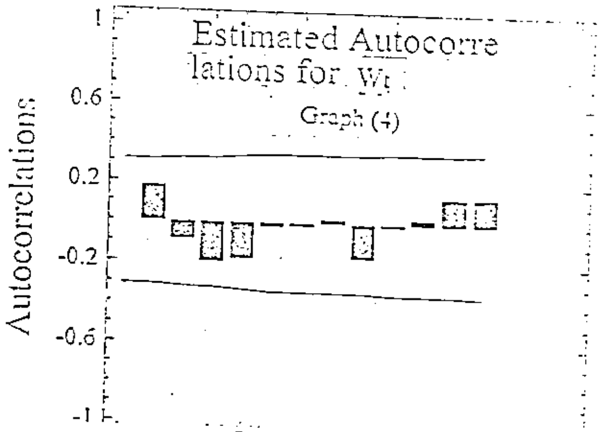


Table (2)

Lag	Estimated Autocorrelation
1	0.179259
2	-0.0864804
3	-0.196905
4	-0.179718
5	-0.0194547
6	-0.0157475
7	0.0201267
8	-0.170711
9	-0.0104473
10	0.0294119
11	0.136979
12	0.140666

Est. autocorrelations for the series {Wt}, showing no pattern, with K =12, successive time lags as well as they are within the normality range.

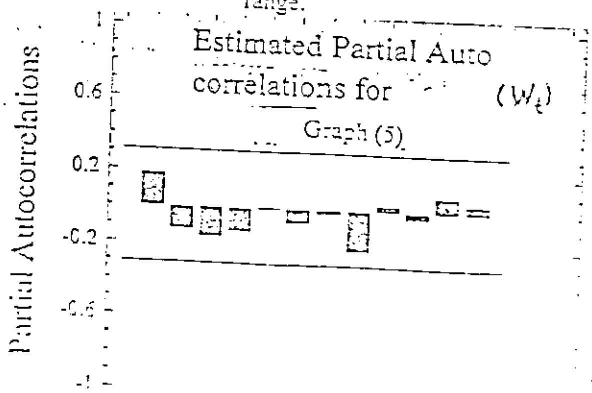


Table (3)

Lag	Estimated Partial Autocorrelation
1	0.179259
2	-0.124619
3	-0.164457
4	-0.132317
5	0.00008178
6	-0.0775657
7	-0.0214515
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]

Est. partial [illegible]
[illegible]

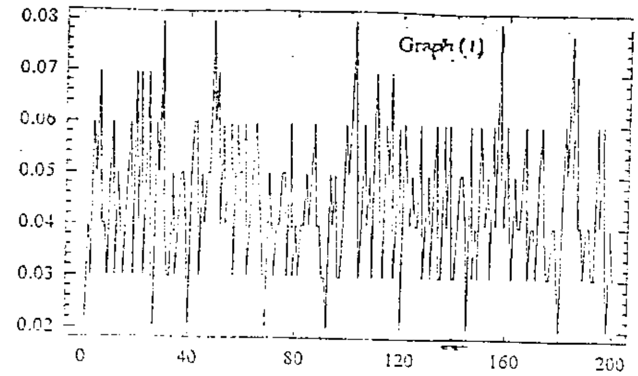
31	5	0.00139587	0.0
32	5	* 0.00916769	0.04
33	5	- 0.00652805	0.02
34	5	0.000842708	0.03
35	5	- 0.00269585	0.03
36	5	* - 0.0119487	0.02
37	5	* - 0.0140859	0.04
38	5	- 0.0035246	0.04
39	5	- 0.00383532	0.03
40	5	- 0.00767553	0.04

Table (5)

ARIMA Chart Report
All Subgroups
*** = Beyond Limits**

Subgroups	Size	Residual	Range
1	5	-0.00429464	0.04
2	5	0.00241287	0.04
3	5	-0.00302804	0.03
4	5	0.00611911	0.04
5	5	0.00615867	0.04
6	5	0.00210106	0.06
7	5	* -0.00901802	0.02
8	5	-0.00257808	0.03
9	5	0.00608251	0.03
10	5	* 0.0101657	0.04
11	5	0.00522988	0.03
12	5	0.00427982	0.03
13	5	0.00251266	0.03
14	5	- 0.00709724	0.03
15	5	- 0.00299775	0.02
16	5	- 0.00183673	0.03
17	5	- 0.00411017	0.02
18	5	0.000377369	0.03
19	5	- 0.00658629	0.03
20	5	- 0.00109609	0.03
21	5	* 0.00974729	0.05
22	5	0.00336041	0.04
23	5	0.00268961	0.03
24	5	- 0.00113129	0.05
25	5	0.00370406	0.02
26	5	- 0.00132653	0.0
27	5	- 0.00425836	0.03
28	5	0.000405889	0.0
29	5	- 0.00459178	0.03
30	5	0.000470058	0.0

Time Series Plot for (Xt)



The origin series {Xt ,t =1,2,….,200},initially shows
that the stationary in mean held.

Estimated Autocorrelations for (Xt)

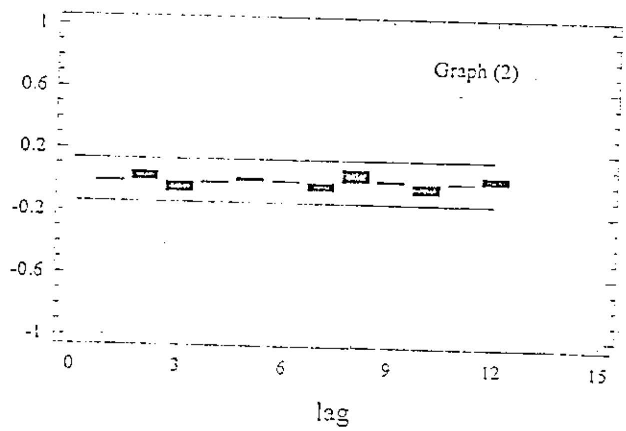


Table (1)

Estimated Autocorrelations

Xt

Lag	Autocorrelation
1	-0.00779169
2	0.0612203
3	-0.0741672
4	-0.0146236
5	0.0259634
6	0.00232927
7	-0.057325
8	0.0330625
9	0.015456
10	-0.0721901
11	-0.000716121
12	0.0497773

Est. autocorrelations for {Xt},with K = 12. successive time lags.
giving a strong evidence as well as box-pierce statistic Q = 1.6.
that the series {Xt}. stationary in origin.

6 / From the tolerance limits (9-a) , and capability index , the suggested control limits for the impure tobacco weight / cigarette, in Sulaimani factory is:

$$[\,UCL, LCL\,] = [\,0.09\,,\,0.0\,]$$

7 / To construct a suitable control chart for data under consideration, (R-chart) is not capable to illustrate the periodic production and controlling the process , but ARMA Residuals chart was more proper to dealing with the data having auto-correlation structure.

(a) :The estimated model for the series {Wt} is:

$$W_t = 0.0434137 + 0.0249987\, W_{t-1} - a_t + 0.192458\, a_{t-1}$$

See table (4) .

11 : Capability Index :

The efficiency for calculated tolerance limits (4-e) ,can be tested with a such measure named , (capability index , Cp) , which has been calculated to summarize the comparison of several specification limits , and selecting a better one , to be a new ARMA residuals control chart. This measure calculated as a proportion of distance between upper and lower specificity limits (UCL - LCL) , to six times standard division of the series . For tolerance limits (4-e), the capability index , Cp= 1.037 , required that the suggested control limits (4-e), is a good control limits for impure weights rate / cigarette in the cigarette production process in Sulaimani factory , with standard division of residuals series , (Oa = 0.0144).

12 : Conclusions :

1 / The estimated ARMA (1,1) model [a] , is an adequate one to describe the impure weight means {Wt} behaviors , to construct an identically and normally distributed residuals series.

2 / Estimates of the parameters for model , made the invertability conditions satisfied.

3 / The origin series , (impure weight, Xt) ,(n=200), is originally stationary in mean and variance, therefore the mathematical transformations (Box-Cox transformations) , and differences did not use to achieve stationary .

4 / The model, was very helpful to achieve a non auto – correlated residuals series , which is an important tool to fit a suitable ARMA residual control chart , graph (9) .

5 / The beyond points , graph (9), gave evidences that the time series at those points are out of control. It is helpful to examine why these states occurred, and search about the casual variations to adjust and improve the production process.

10 : Tolerance Limits :

Tolerance Limits identify the capability of ARMA residuals chart, to be adjusted to make the beyond points in the chart's UCL, LCL, by suggesting a new control limits, this can be done , by calculating several tolerance limits with respect to different specification values until to receive a new control limits, which makes all time periods observations in residuals charts are in control.

From the graph (11), the optimal tolerance limits is given by :

(UCL, LCL) = (0.09 , 0.00) ---------- (10-a)

---

a: specification values to compute optimal tolerance limits, can be determined by taking some arbitrary constants , until an optimal tolerance limits calculated , and about the nominal value, can be chosen as nearly as possible to the process's mean (target value) .

9 : Constructing ARMA Residuals Chart :

To construct ARMA (1,1) residuals chart ,let (r1 , r2.,.,.,.,.,r40), table (5) , be the residuals Series.and (m = 40), is subgroups number, then the residuals mean can define by :

$$\bar{r} = \sum_{i=1}^{40} ri \; / \; m \qquad \text{-------- (9-a)}$$

$$Sr = \left[\sum_{i=1}^{40} (ri - \bar{r})^2 / (n - 1) \right]^{1/2} \qquad \text{---------- (9-b)}$$

are the mean and standard division , for residuals .

Then the center line (r = 0) using shewahart control limits criteria, the upper and lower control limits given by :

$$\bar{r} \pm \; 3 * Sr \qquad \text{---------- (9-c)}$$

Then :

$$(UCL, LCL) = (+ 0.017, \; - 0.017) \qquad \text{---------- (9-d)}$$

Six beyond points are observed in ARMA residuals chart, is a strong evident that the process, [means impure weights / cigarette , {Wt}] ,out of control. See ARMA residuals chart, graph (9). So the production process needs some revisions to detect those casual variations , that made this uncontrolled .

8 : ARMA Residuals Chart :

In constructing control charts by variables, the observations must be identically normally distributed, so the quality control chart of generally traditional Shewahard's chart type, are applicable for normally uncorrelated (independent) individuals.

But the series {Wt} ,was occurred under sequential time periods auto-correlated , therefore the independency condition among time periods was vanished, so (R- chart) is in applicable one.

An appropriate control chart should be constructed from the normally independent residuals series (at) generated from the model ARMA (1,0,1) , (estimates model parameters shown in table (4).

---

a : Beyond points (*) , are an indicators for out of control points.

b : warning limits , calculated as follow:

$$(UWL , LWL) = R \pm 2^* \hat{\sigma}_w$$

$\hat{\sigma}_w$: standard division for the series (W_t) .

9 : Constructing ARMA Residuals Chart :

To construct ARMA (1,1) residuals chart ,let (r1 , r2.,.,.,.,.,r40), table (5) , be the residuals Series.and (m = 40), is subgroups number, then the residuals mean can define by :

$$\bar{r} = \sum_{i=1}^{40} ri \;/\; m \qquad \text{-------- (9-a)}$$

$$Sr = [\sum_{i=1}^{40} (ri - r)^2 / (n - 1)^{½} \qquad \text{---------- (9-b)}$$

are the mean and standard division , for residuals .

Then the center line (r = 0) using shewahart control limits criteria, the upper and lower control limits given by :

$$r \pm 3 * Sr \qquad \text{---------- (9-c)}$$

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In constructing control charts by variables, the observations must be identically normally distributed, so the quality control chart of generally traditional Shewahard's chart type, are applicable for normally uncorrelated (independent) individuals.

But the series {Wt} ,was occurred under sequential time periods auto-correlated , therefore the independency condition among time periods was vanished, so (R- chart) is in applicable one.

An appropriate control chart should be constructed from the normally independent residuals series (at) generated from the model ARMA (1,0,1) , (estimates model parameters shown in table (4).

---

a : Beyond points (*) , are an indicators for out of control points.

b : warning limits , calculated as follow:

$$(UWL, LWL) = R \pm 2^* \hat{\sigma}_w$$

$\hat{\sigma}_w$: standard division for the series (W_t) .

6 : Serially Correlated Data Control Chart:

As an initial quality control analysis by variables, assume first that the means series weights {Wt} ,are uncorrelated, to make use of one of the very important and common Shewahart charts, to make sure if the process (Wt) , is in or out of control , named by (R-chart).

7 : Constructing (R- chart) :

The ranges of each subgroups, are calculated [see table (5)] ,the center line of (R- chart) , is given by:

$$R^{-} = \sum_{I-1}^{M} Ri / m \qquad \text{i : 1,2,3,.,.,.,.,.,40} \qquad \text{----------} \qquad \text{(7-a)}$$

The upper control limit (UCL), and the lower (LCL), are given by:

$$R^{-} + 3 * \sigma^{\wedge}$$

Then R-chart's control (Action) limits for series {Wt}.

UCL = 0.0692453
LCL = 0.0000000

See graph (10) , there is no (beyond points) [(a)] , so the chart indicates that the process is in control.

Also about (warning) [(b)] , upper and lower limits , having no two or more consecutive points of the series {Wt}, gives a strong evident that the process is in control.

d / Define an impure weight / cigarette control limit from an optimal tolerance limits to observe and control the periodic production to reach the better quality product in the future .

5 : ARMA (p,q) Model Identification :

From the origin data (Xt) , (n= 200) , and estimated autocorrelation coefficients table (1) , with [time lag (k) = 12] ,graphs (1,2) are respectively shown stationary and randomness of the origin series , [Box-Pierce statistic Q = 1.6, with having no trend for these coefficients was a good evident for randomness of {Xt}] .

After sketching the series {Wt} , with it's estimated autocorrelation and partial- autocorrelation , table (2) ,graphs (3,4) ,also shown stationary and randomness for {Wt} ,under having no trend among auto's with (Q =4.828) .

To specify the order of ARMA (p, q) ,the estimated autocorrelation and partial auto's initially determining constant model , graphs (4,5) , table(2,3) . After checking iteratively more than one ARMA model with deferent orders ,all statistical measures of modeling goodness of fit , calculated from generated residuals ,using suggested ARMA (1.1) model see graph (6) ,and the results shown in table (4). [MSE =3.38* E-5 ,Q=4.83] , indicates that the model ARMA (1.1) is an adequate model, to represent the behaviors of the series {Wt} .

---

a: Box – pierce test statistic for originally stationary time series data is given by :

$$Q = n * \sum_{K=1}^{MAX(K)} \hat{\sigma}^{2}(K)$$

0(k) : estimated auto's , k= 1,2,3,.,.,.,.,., k. k, is maximum lag time.
$Q \sim x^2$, with, K d.f and ∞ level of significant .

1. Data description:

A time sequence of (n = 200) cigarettes , was taken as an origin series [X_t] , divided to (40) subgroups , each contains (j= 5) cigarettes for (40) successive time periods , form sulaimani cigarettes factory Kurdistan region iraq . This production named by (SAN – CIGARETTES) .

The random variable which was taken as a univariate time series variable was , [X_t , t = 1,2,3,....,200] , represented the (impure weight rate / cigarette) with grams , weights measured by a sensitive scale with two decimals for precision .

[W_t , t = 1,2,3,....40] . is the averages impure weights subgroup time series .

2. Study's assumption:

The first main assumption is , to recognize stationary and randomness behaviors for the origin series [X_t] , as a time correlated series , and also for the series [Wt] .

The second is disconcidane the impure weights in an empirical manner , with other products , because of using low level quality of tobacco and the oldness age for production lines during production process. And finally the tolerance limits should be helpful to identify the capability of ARIMA residuals chart for improving adjusting production.

4: Objectives of the study :

a / Determining ARIMA (p,d,q) model , for (40) subgroups , each observation represents the average impure weight for each subgroup every one hour production, { Wt} .

b / Estimating the identified model in (1) , by maximum likelihood method in Box-Jenkins univariate time series analysis approach.

c/ Specifying ARIMA residuals chart , for the variable under consideration to detect if the process is in or out of control , by observing the residual time periods, and comparing it with the upper and lower ARIMA residual control chart .

Constracting Control Charts, for cigarettes Production, in Sulaimani Cigarettes Factory

NAWZAD M. AHMAD*

1.Abstract :

The quality control in manufacturing processes , is a very important statistical work , to improve the quality production . or at least remaining it within a such acceptance region with respect to some restriction that the process contains .
Cigarettes , is one of productions which is seriously damages health , with a great risk on humans being life , as well as it causes environment an that on national income . so it is necessary to keep the quality of this product to be in control , especially with the impure rate that the cigarette contains , to reduce damages , and improving quality .
ARIMA residuals charts is a special control chart , used to detect , and specify the quality behaviors in data time correlated processes , if they are in or out of control . This kind of charts is better than others (R-chart) , it is helpful for specifying adjusting the quality limits during production , and observing production process .

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