Turbulent Natural Convection Inside an Inclined L – Shaped Enclosure

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Abstract.

Turbulent flow natural convection heat transfer in a two dimensional L – shaped enclosure is numerically analyzed using the finite volume method to solve the system of the governing equations (continuity, momentum and energy equations). Turbulence is modeled by a low Reynolds number (k – ε) model. The procedure used in calculations of the flow field are based on SIMPLE algorithm and a computer program in FORTRAN 90 is built to solve the finite difference equations by tri – diagonal – matrix algorithm (TDMA). Results are presented in the form of the average Nusselt number Nu for Prandtl number Pr= 0.713, a range of Rayleigh number Ra (10^9-10^{13}), a range of orientation angle θ (0- 360°) and aspect ratio A (0.1–0.5). The results show that for A = 0.3, little variation of Nu is observed with θ for Ra =10^9 - 10^{11} but for Ra = 10^{12} – 10^{13}, Nu show a variation which is entirely θ dependent. The maximum heat transfer is at θ = 90° and the minimum at θ = (180° - 270°). For A = 0.5, also for Ra = 10^9 – 10^{11} little variation of Nu is observed with θ and at Ra = 10^{12} – 10^{13} Nu show a variation which is entirely θ dependent. The minimum rate of heat transfer is at θ = 225°.
A correlation has been set up to give the average Nusselt number variation with Ra, θ and A. For the knowledge of the author, no previous results on turbulent thermal convection in this (L – shaped) geometry exist, so a comparison of the streamlines and isotherms against recent experimental measurements, obtained for a square cavity. Satisfactory agreement was observed

Key Words: Turbulent flow, natural convection, k – ε turbulence model, inclined enclosure, finite volume.

Introduction:

A number of studies dealing with different kinds of enclosures have been performed and the solutions for both the flow and thermal fields have been presented. However most of the previous studies for example [Rahman et. al., 2003], [Seok et. al., 2006] [Edimilson and his group, 2008] and [Ravnik and his group, 2008] were focused on the analysis of the fluid motion in a rectangular enclosure, and smaller number of researches considered the enclosure with an irregular shape as [Papanicolaou et. al., 2005], [Tanmay and his group, 2008], [EL Hasson and his group, 2006], which arise in many practical studies (e.g., solar heating, solidification, nuclear waste disposal, double wall thermal insulation, underground cable systems, ventilation of rooms, solar energy collection,…etc.). In the vast majority of cases, a small number of studies have considered the L – shaped geometry with laminar flow. [da Silva et. al., 2005] documented the geometric optimization space, height and lengths of L and C – shaped channels in laminar natural convection subject to global constrains. The objective was to maximize the heat transfer rate from the hot wall to the coolant fluid. Three configurations were considered with different boundary conditions and was performed numerically by using the finite element technique in the range (10^5<Ra<10^{10}) for Pr = 0.7. [Seydaet.al., 2006] reported numerically the buoyancy induced flow and heat transfer characteristics inside an inclined L – shaped enclosure. A control volume based finite volume method was applied. Results were presented in the form of the average Nusselt number for a range of inclination angle, θ (0 -360°) Rayleigh number Ra (1 – 10^5) and aspect ratio, A (0.1 – 0.5). Since to the knowledge of the author, no previous results on turbulent thermal convection in this geometry exist, the validation of the numerically code was performed by comparing velocity and temperature profiles against recent experimental measurements, obtained for a square cavity. Satisfactory agreement was observed

Geometry Under Analysis:

The problem considered is shown schematically in **Fig.(1)** and refers to the two dimensional flow of a Boussinesq fluid of Prandtl number Pr = 0.713 in L – shaped enclosure of infinite depth along the z – axis (the third dimension of the enclosure is considered long enough for the flow to be considered 2D).
The no slip condition is applied on the velocity at all walls, the hot and cold walls are considered to be isothermal and the others are adiabatic. Heat transfer through the walls causes density changes to the fluid in the cavity and leads to buoyancy – driven recirculation. The flow is steady state, turbulent for Rayleigh (10^9< Ra < 10^{13}).

Mathematical Model:

In practical flows the mean density may vary and the instantaneous density always exhibits turbulent fluctuations. [Versteeg and Malalasekera, 1995] stated that small density fluctuations do not appear to affect the flow significantly. In free turbulent flows the velocity fluctuations can easily reach values of 20% of the mean velocity. In such circumstances, the density fluctuations begin to affect the turbulent motion. To investigate the effect of the pertinent fluctuations the authors replace the flow variable **u** (velocity vector) and pressure p by the sum of a mean and fluctuating component.

u = U + u'; v = V + v'; p = P + p'

In Cartesian tensor notation the equations describing the mean flow and temperature within the enclosure are.

Continuity:

$$\frac{\partial \overline{U_i}}{\partial x_i} + \frac{\partial u_i'}{\partial x_i} = 0 \qquad (1)$$

Where $\overline{U_i}$ = mean velocity components

u_i' = velocity component fluctuations

Momentum:

$$\frac{\partial}{\partial x_j}\left(\rho\, U_j U_i\right) = \frac{-\partial P}{\partial x_i} + \rho\, g_i + \frac{\partial}{\partial x_j}\left(\mu\left[\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right] - \rho \overline{u_i u_j}\right) \qquad (2)$$

Thermal energy in the control volume:

$$\frac{\partial}{\partial x_j}\left(\rho\, c_p\, U_j \Theta\right) = \frac{\partial}{\partial x_j}\left(K\frac{\partial \theta}{\partial x_j} - \rho\, c_p\, \overline{u_j \theta}\right) \qquad (3)$$

The turbulent stresses and heat fluxes are given by:

$$\rho \overline{u_i u_j} = \frac{2}{3}\delta_{ij}\rho\, k - \mu_t\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right) \qquad (4)$$

$$\rho\, \overline{u_j \theta} = -\frac{\mu_t}{\sigma_\theta}\frac{\partial \theta}{\partial x_j} \qquad (5)$$

Where:

$\mu_t = c_\mu\, \rho\, k^2 \tilde{\varepsilon}$, and σ_θ, the turbulent Prandtl number is taken = 0.713.

$$c_\mu = 0.09\ \exp\left(-\frac{3.4}{\left(1 + {R_t}/{50}\right)^2}\right)$$

$$c_{\varepsilon 1} = 1.44, \quad c_{\varepsilon 2} = 1.92\left(1 - 0.3\ \exp\left(-R_t^2\right)\right) \quad and \quad R_t = {\rho\, k^2}/{\mu\, \tilde{\varepsilon}}$$

$$\sigma_k = 1.0, \quad \sigma_\varepsilon = 1.3 \ and \ \sigma_\theta = 0.9$$

δ_{ij} is the kronecker delta δ_{ij} =1 if i=j and δ_{ij} = 0 if $i \neq j$

The part of the kinematics' energy dissipation rate associated with spectral transfer (the turbulence energy) k and $\tilde{\varepsilon}$ are obtained from [Ince and Launder, 1989]:

$$\frac{\partial}{\partial x_j}\left(\rho\, U_j k\right) = \frac{\partial}{\partial x_j}\left(\left(\mu + \frac{\mu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right) + P_k - \rho\, \varepsilon \qquad (6)$$

$$\frac{\partial}{\partial x_j}\left(\rho\, U_j\, \tilde{\varepsilon}\right) = \frac{\partial}{\partial x_j}\left(\left(\mu + \frac{\mu_t}{\sigma_\varepsilon}\right)\frac{\partial \tilde{\varepsilon}}{\partial x_j}\right) + c_{\varepsilon 1}\frac{\tilde{\varepsilon}}{k}P_k - c_{\varepsilon 2}\, \rho\frac{\tilde{\varepsilon}^2}{k} + \frac{2\mu\, \mu_t}{\rho}\left(\frac{\partial^2 U_i}{\partial x_k^2}\right)^2 \qquad (7)$$

Where:

$$\varepsilon = \tilde{\varepsilon} + 2\upsilon\left(\frac{\partial k^{1/2}}{\partial x_k}\right)^2 \qquad (8)$$

$$P_k = -\rho\, \overline{u_i u_j}\frac{\partial U_j}{\partial x_j} - \alpha\frac{\rho}{\Theta}\overline{u_i \theta}\, g_i \qquad (9)$$

Nusselt Number Calculation

The most important characteristic of the flow is the rate of heat transfer across the cavity. Nusselt number on the hot wall at y = 0 is given by [Syda et. al., 2006]:

$$Nu_{av} = \frac{1}{abc}\int_{abc} Nu_L\, ds \qquad (10)$$

$$Nu_L = \left(\frac{K}{\Delta T}\right)\frac{\partial T}{\partial n}\bigg|_{at\ hot\ wall} \qquad (11)$$

The above equations were solved on a square mesh by the finite volume method outlined in [Versteeg and Malalasekera, 1995].

Numerical Grid Generation.

All two dimensional computations reported here were performed on a carefully graded (51 x 51) mesh **Fig.(2)** which is being the finest achievable with the computer resources available. Finite volume equations are derived by integration of the differential equations over an elementary control volume or cell surrounding a grid node. Upwind differencing is used in the convective terms and the integrated source term is linearized. Both these practices are widely used to enhance numerical stability.

The equations are solved by line – by – line procedure which is similar to Stones Strongly Implicit Method but free from parameters requiring case to case adjustment and so less complex and slower. The pressure correction equation is solved in a whole field manner, two – dimension simultaneous.

RESULTS AND DISSCUSION:

The modeled equations and the boundary conditions were used to predict turbulent thermal convection within an L- shaped enclosure **Fig. (1).**The governing parameters in the problem were Ra, aspect ratio A and angle of orientation θ for Pr = 0.713 results are presented in the form of representative streamlines and isotherms and average Nusselt number values.

Streamlines and Isotherms:

For aspect ratio A = 0.5 and Ra = 10^{13}, **Fig. (3a – e)** presents the streamlines and isothermal lines for angle of orientations θ = (0° - 225°) as mentioned in the figure. When θ = 0° or θ = 90° (which is the mirror images of the streamlines and isotherms of θ = 0°) a natural clockwise circulation inside the vertical position of the enclosure exists due to the temperature difference between the hot bottom wall and the cold wall. A counter clockwise cell in the horizontal part of the enclosure exists due to the hot bottom wall and the upper cold wall. Two symmetric Benard cells appear when θ = 45° due to the hot bottom walls and cold top walls. This is the most unstable position for fluid inside the enclosure. In contrast, the most stable position for the fluid inside the enclosure is for θ = 225°.

For θ = 135°, the flow circulations follows the enclosure geometry near the walls with no Benardcell exist. The streamlines and isotherms for θ = 315° are the mirror images for the flow and thermal fields for θ = 135°. when θ = 180° and 270° no Benard cell appears due to the cold bottom wall and hot top wall which permits a complete horizontal penetration of circulatory fluid in the vertical portion of the enclosure where a circulation exists due to the horizontal temperature difference.

Rate of Heat Transfer:

For a constant aspect ratio (A=0.5), the average Nusselt number is presented as a function of the Rayleigh number in **Fig. (4)**for nine selected values of θ. Nu increase with Ra but has a minimum value at θ = 225°.

In **Fig. (5)**, Nu is plotted as a function of θ at different Ra for A = 0.3. For Ra = $10^9 - 10^{11}$ little variation of Nu is observed with θ and when convection heat transfer increase at Ra = $10^{12} - 10^{13}$ Nu show a variation which is entirely θ dependent. The maximum heat transfer is at θ = 90° and the minimum at θ = (180° - 270°).

For A = 0.5, Nu is plotted as a function of θ at different Ra in **Fig. (6).** Again for Ra = $10^9 - 10^{11}$ little variation of Nu is observed with θ and when convection heat transfer increase at Ra = $10^{12} - 10^{13}$ Nu show a variation which is entirely θ dependent. The minimum rate of heat transfer is at θ = 225°.

For aspect ratio range A (0.1 – 0.5) Nu is presented as a function of θ in **Fig.(7)** for Ra = 10^{13} . Nu decreases with increasing inter wall spacing linearly.

In **Figs. (8 and 9)** Nu is plotted against aspect ratio for different values of Ra and for θ = (0° and 45°) respectively. It is clear that Nu decreases with increasing inter wall spacing and increase with Rayleigh number.

No previous results on turbulent thermal convection for (L – shaped) geometry exist, so a comparison of the streamlines and isotherms against recent experimental measurements, for a square cavity is set up as shown in **Figs. (10 and 11)** and **Table(1)** for the present study and [Markatos and Pericleous, 1984] respectively.

A correlation equation for Nu against Ra, A and θ had been set up using DGA program and curve fitting method (Least square method) and is given as follows:

$$Nu = a\, A^{N1}\, Ra^{N2}\, \theta^{N3}$$

Where:

a = 1.5717, N1 = 0.587169,
N2 = 0.204479, N3 = 0.22972739

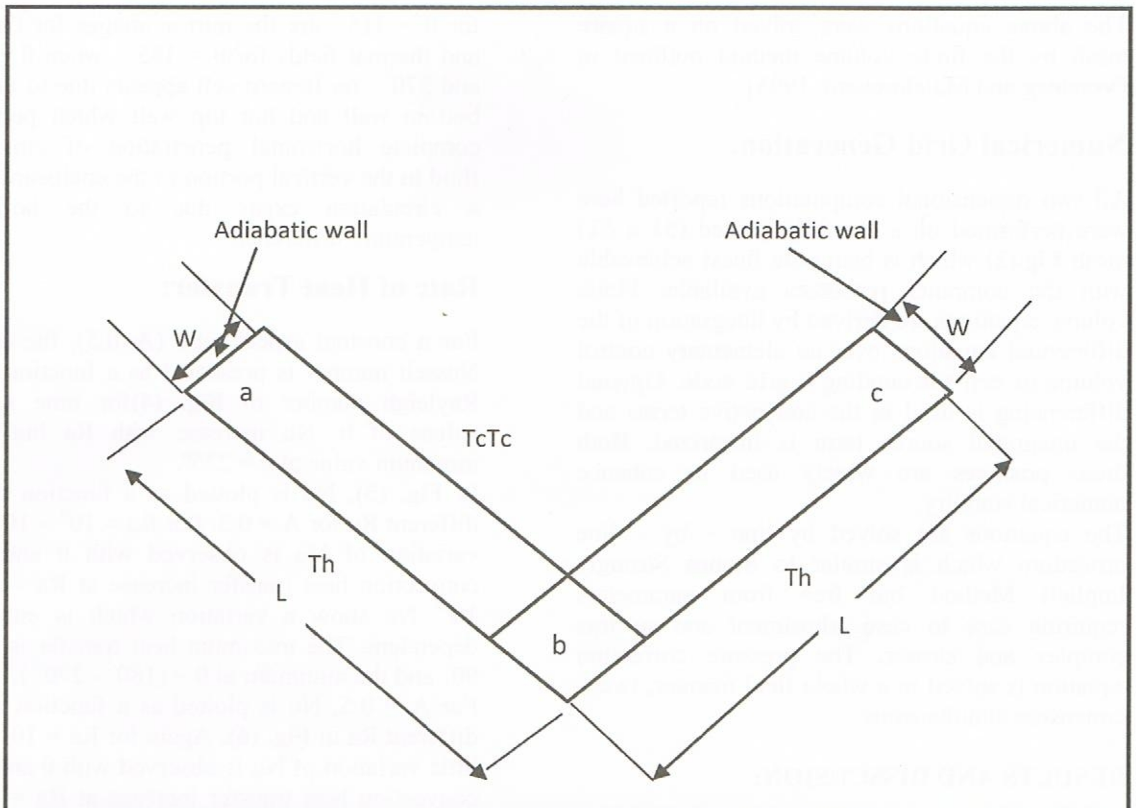


Fig. (1) Schematic diagram of the problem under consideration

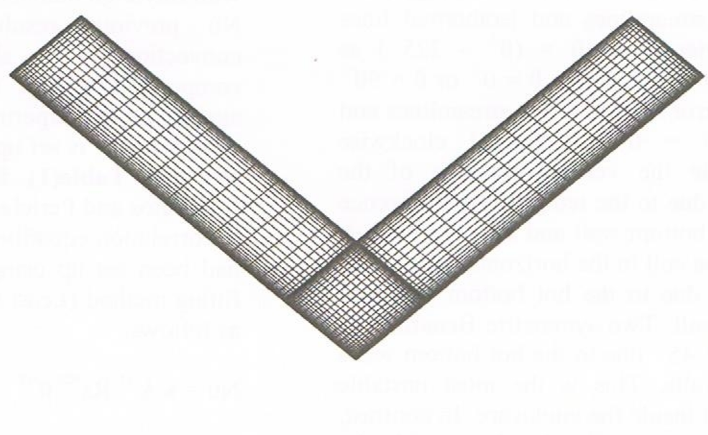
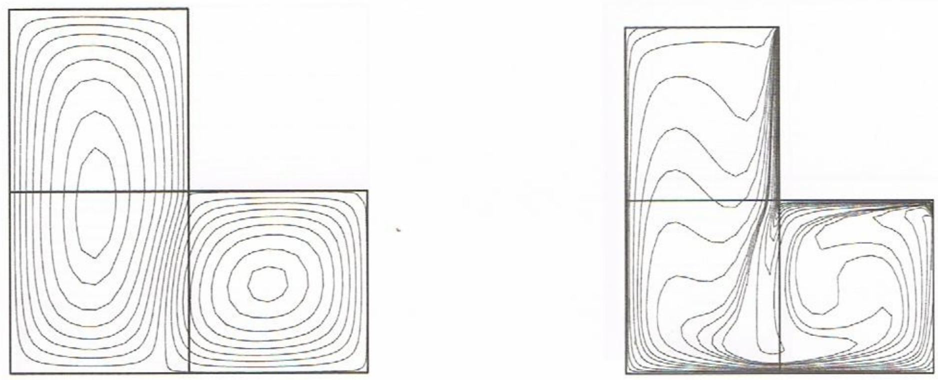
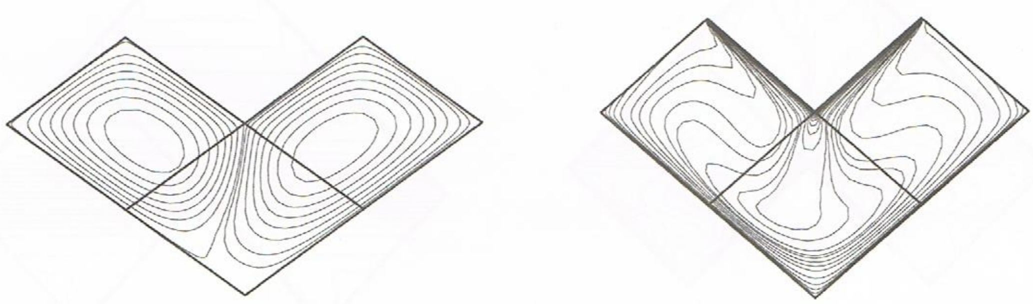


Fig. (2) Grid used for aspect ratio A = 0.2



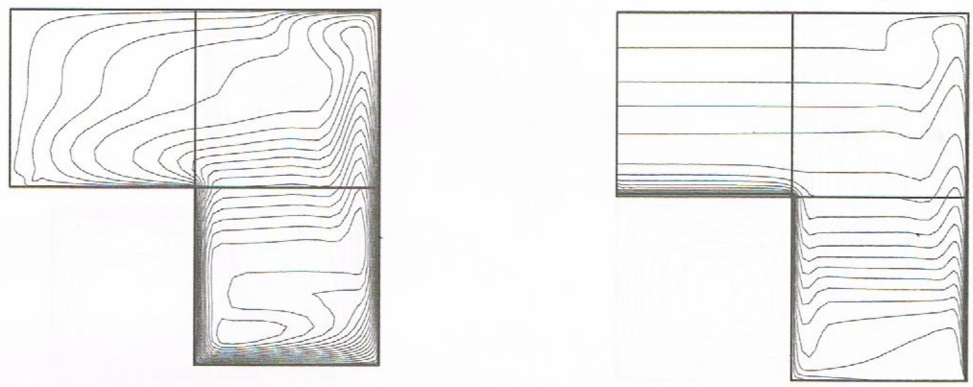
(a) Streamlines and isotherms for θ = 0°



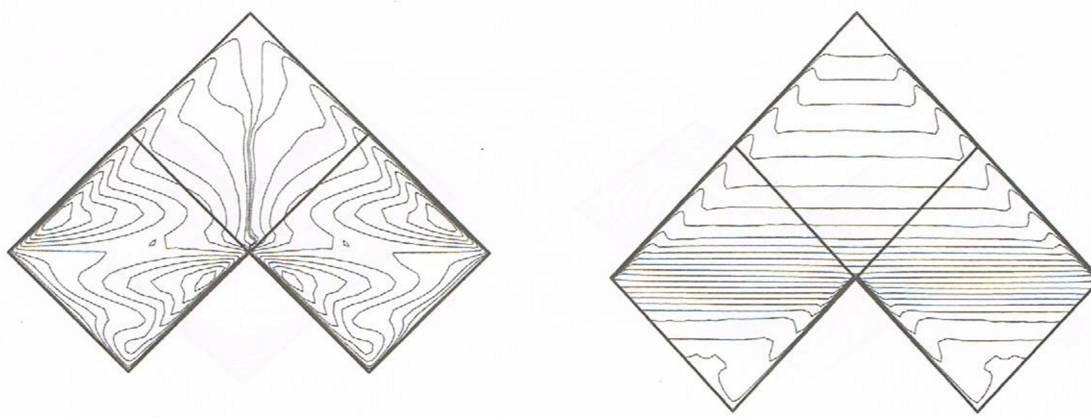
(b) Streamlines and isotherms for θ = 45°

(c) Streamlines and isotherms for θ = 135°

Fig. (3a - e) streamlines and isotherms with $Ra = 10^{13}$, A = 0.5 for different values of θ



(d) Streamlines and isotherms for θ = 180°



(e) Streamlines and isotherms for θ = 225°
Fig. (3a – e) continued

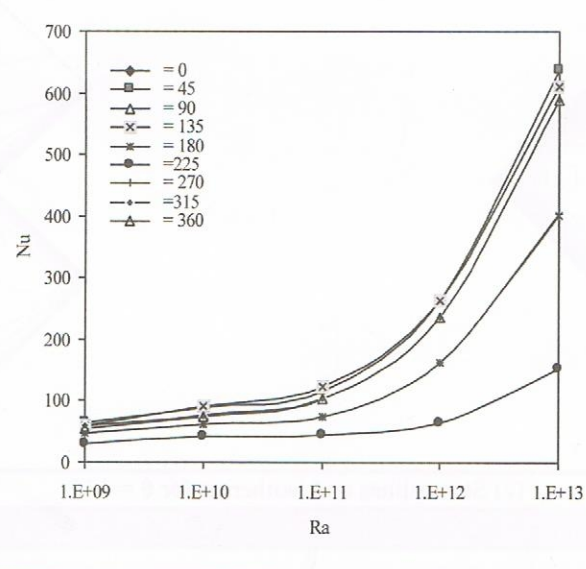


Fig. (4) Variation of Nusselt number against Ra for A= 0.5 and different values oθ

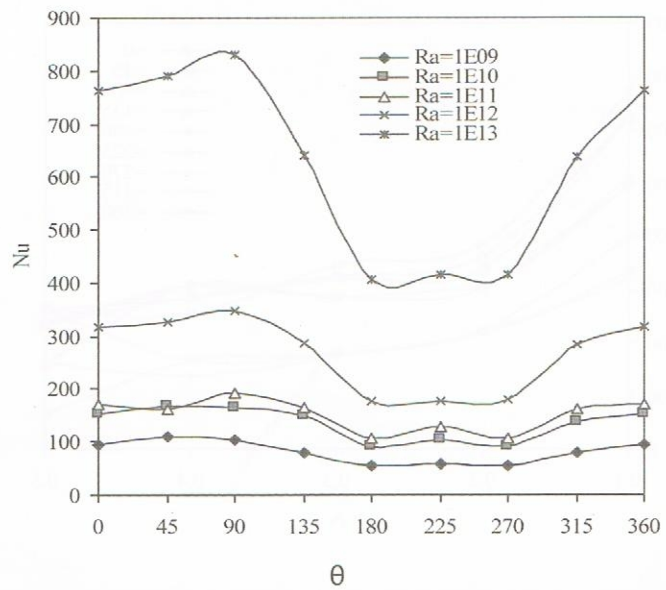


Fig. (5) Variation of Nusselt number against θ for A= 0.3 and different values of Ra

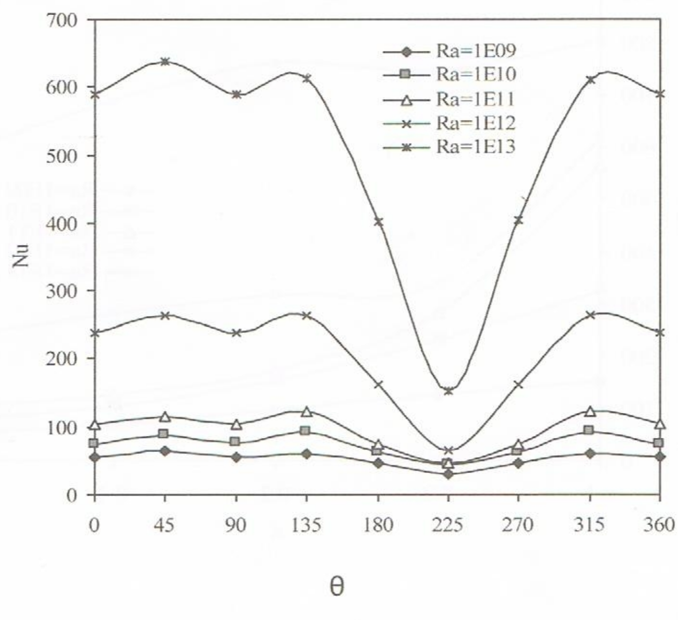


Fig. (6) Variation of Nusselt number against θ for A= 0.5 and different values of Ra

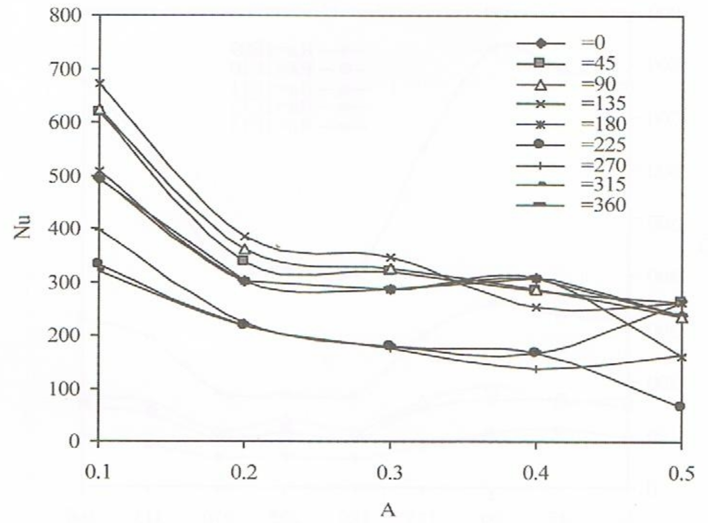


Fig. (7) Variation of Nusselt number against A for Ra $=10^{13}$ and different values of θ

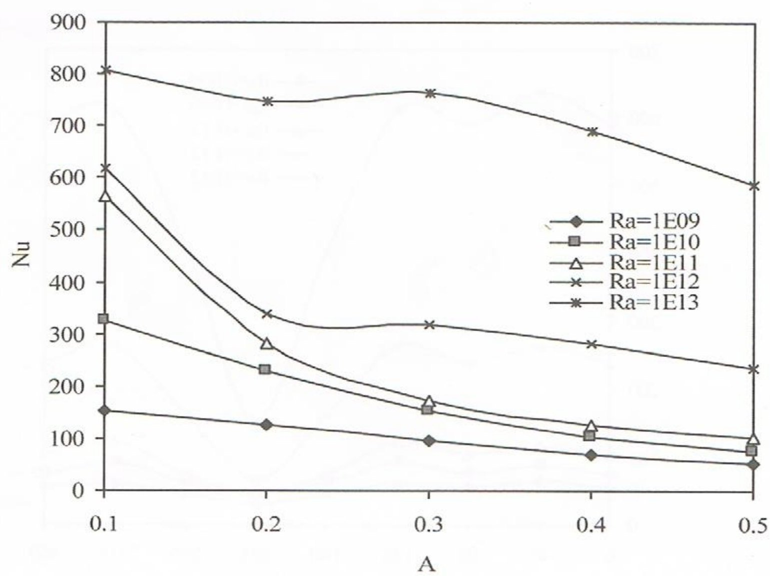


Fig. (8) Variation of Nusselt number against A for $\theta = 0°$ and different values of Ra

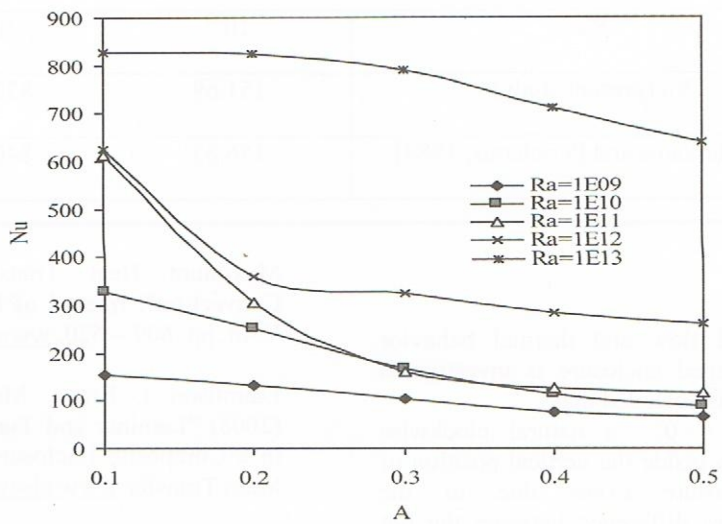


Fig. (9) Variation of Nusselt number against A for $\theta = 45°$ and different values of Ra

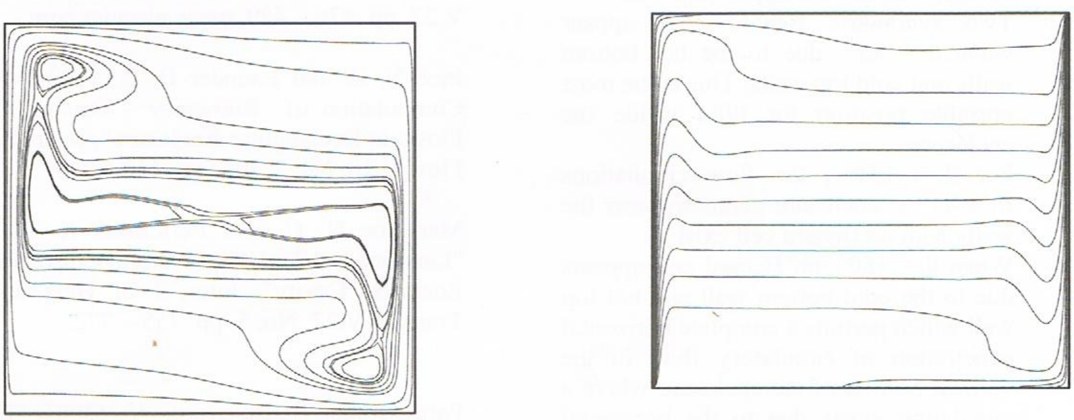


Fig. (10) streamlines and isotherms for present study with $Ra = 10^{10}$

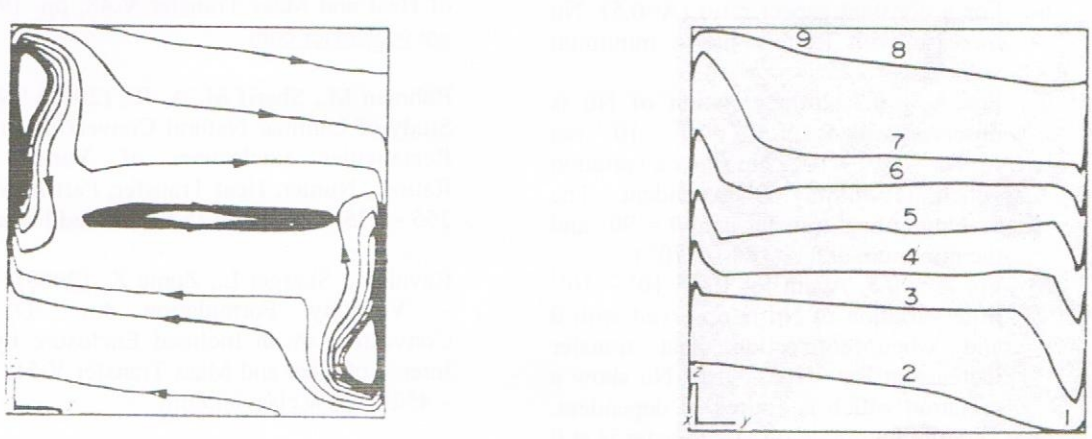


Fig. (11) streamlines and isotherms for [Markatos and Pericleous, 1984] with $Ra = 10^{10}$

Table(1) Comparison of Nusselt number for the present study and for [Markatos and Pericleous, 1984]

Ra	10^{10}	10^{12}
Nu (present study)	151.69	832.44
Nu [Markatos and Pericleous, 1984]	156.85	840.13

Conclosions:

Buoyancy induced flow and thermal behavior inside an L – shaped enclosure is investigated numerically and it is cocluded that:.

1- When $\theta = 0^\circ$ a natural clockwise circulation inside the vertical position of the enclosure exists due to the temperature difference between the hot bottom wall and the cold wall. A counter clockwise cell in the horizontal part of the enclosure exists due to the hot bottom wall and the upper cold wall.
2- Two symmetric Benard cells appear when $\theta = 45^\circ$ due to the hot bottom walls and cold top walls. This is the most unstable position for fluid inside the enclosure.
3- For $\theta = 135^\circ$, the flow circulations follows the enclosure geometry near the walls with no Benard cell exist.
4- When $\theta = 180^\circ$ no Benard cell appears due to the cold bottom wall and hot top wall which permits a complete horizontal penetration of circulatory fluid in the vertical portion of the enclosure where a circulation exists due to the horizontal temperature difference.
5- The most stable position for the fluid inside the enclosure is for $\theta = 225^\circ$.
6- For a constant aspect ratio (A=0.5). Nu increase with Ra but has a minimum value at $\theta = 225^\circ$.
7- For A = 0.3, little variation of Nu is observed with θ for Ra $=10^9$ - 10^{11} but for Ra = $10^{12} - 10^{13}$, Nu show a variation which is entirely θ dependent. The maximum heat transfer is at $\theta = 90^\circ$ and the minimum at $\theta = (180^\circ - 270^\circ)$.
8- For A = 0.5, Again for Ra = $10^9 - 10^{11}$ little variation of Nu is observed with θ and when convection heat transfer increase at Ra = $10^{12} - 10^{13}$ Nu show a variation which is entirely θ dependent. The minimum rate of heat transfer is at $\theta = 225^\circ$.

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NOMENCLATURE:

Symbol	Description	Units
c_p	Specific heat at constant pressure	kJ/kg ȯȯC
g	Gravitational acceleration	m/s^2
g_i	Gravitational acceleration vector	m/s^2
k	Turbulent kinetic energy	
K	Thermal conductivity	W/m ȯȯC
P	Mean pressure	N/m^2
P_k	Generation rate of turbulent kinetic energy	
Pr	Prandtl number	
Ra	Rayleigh number	
U, V	Mean velocity components in x, y directions	m/s
U_i, U_j	Mean velocity in directions x_i and x_j	m/s
u', v'	Velocity fluctuation in x, y direction	m/s
$u'v'$	Reynolds shear stress components	m^2/s^2
x	Horizontal coordinate with origin at left (hot) surface	m
x_i	Cartesian space coordinate	m
y	Vertical coordinate with origin at lower edge of cavity	m

GREEK SYMBOLS

Symbol	Description	Units
α	Dimensionless volumetric expansion coefficient	
$\Delta\Theta$	Temperature excess above ambient or midplane value	ȯȯC
ε	Turbulence energy dissipation rate	
$\bar{\varepsilon}$	Part of ε associated with special transfer $\bar{\varepsilon} = \varepsilon - 2\upsilon\left(\partial k^{1/2} / \partial x_j\right)^2$	
θ	Temperature fluctuation	
Θ	Local mean temperature (absolute scale)	
μ	Dynamic viscosity	kg/m,s
μ_t	Turbulent viscosity	kg/m,s
υ	Kinematic viscosity	m^2/s
ρ	density	kg/m^3
σ_k, σ_ε	Constants for the k- ε model	
σ	Laminar Prandtl number $\sigma = \mu c_p / \lambda$	
σ_t	Turbulent Prandtl number $\sigma_t = \mu_t c_p / \lambda$	

الحمل الطبيعي لجريان مضطرب داخل محتوى مائل بشكل حرف L

د. منال هادي الحافظ

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الخلاصة:

تم التحليل العددي لإنتقال الطاقة الحرارية بالحمل الطبيعي لجريان مضطرب في محتوى ثنائي البعد بشكل حرف L بإستخدام طريقة الحجوم المحددة لحل المعادلات الحاكمة (معادلة الإستمرارية, الزخم والطاقة). تمت نمذجت الإضطراب باستخدم نموذج $(k - \varepsilon)$ لرقم رينولد واطيء. الطريقة المستخدمة لحسابات حقل الجريان بنيت على أساس خوارزمية (SIMPLE algorithm) وتم بناء برنامج (FORTRAN 90) لحل المعادلات باستخدام خوارزمية المصفوفة القطرية (TDMA). بينت النتائج من خلال معدل رقم نسلت Nu لمدى رقم رايلي $Ra(10^9 - 10^{13})$ وزاوية ميل θ $(0 - 360)$ ونسبة باعية $A(0.1 - 0.5)$. أظهرت النتائج لنسبة باعية $(A = 0.3)$ زيادة قليلة في Nu مع θ لرقم رايلي $Ra = 10^9 - 10^{11}$ ولكن تعتمد كلية على الزاوية θ لرقم رايلي $Ra = 10^{12} - 10^{13}$ أعلى انتقال في الطاقة الحرارية عند $\theta = 90$ والأقل عند الزوايا 180^o إلى 270^o. ولنسبة باعية $(A = 0.5)$ أيضا زيادة قليلة في Nu مع θ لرقم رايلي $Ra = 10^9 - 10^{11}$ ولكن تعتمد.