Solve the Advection, KdV and K(2,2) Equations by using Modified Adomian Decomposition Method.

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Abstract:

In this work, we proposed a new application of modified Adomian decomposition method (MADM) to solve the advection, KdV and K(2,2) equations depend on the idea of El-Kalla [7]. We prove the convergence of MADM applied to these equations. Our results are compared with those obtained by Adomian decomposition method (ADM). The numerical results show that the present method has high accuracy, fast convergence and large convergence region.

Keywords: Modified Adomian decomposition method and advection, KdV, K(2,2) equations.

1-Introduction:

Nonlinear differential equations are encountered in such various felids as physics, chemistry, biology, mathematics and engineering. Most nonlinear models of real-life problems are still very difficult to solve either numerically or theoretically.

There has recently been much attention devoted to the search for better and more efficient solution methods for determining a solution (analytical or numerical) of nonlinear models. Recently, a lot of attention has been focused on the application of the Adomian decomposition method [1,8,9]. Moreover, the ADM was developed to solve nonlinear models [3,10,13], one of these modification has been proposed by notable research El-Kalla [7]. He applied this modification to solve nonlinear ordinary differential equations.

In this paper, we introduce a new application by using this modification to solve the advection, KdV and K(2,2) equations. The present paper has been organized as follows: In Section 2 we give an analysis of the method; in Section 3 we apply this method for solving our problems; in section 4 we will prove the convergence of this method applied to these equations; in section 5 we will present results discussion and in the last section we give some conclusions.

2-Analysis of the method:

General properties of the decomposition methods can be found in [3,10,12,13]. Some of these are outlined as follows:

Suppose that we need to solve the following equation:

$$G(u) = f \quad , \tag{1}$$

where G is a nonlinear operator from a Hilbert space H into H, f is given function in H; and u is unknown in H.

The principle of the decomposition methods is based on the decomposition of the nonlinear operator G in the following form:

$$G = L + R + N \ ,$$

where $L + R$ represents linear terms and N represents nonlinear terms, L invertible with L^{-1} as inverse. Using that decomposition, equation (1) is equivalent to

$$u = \theta + L^{-1} f - L^{-1}(R(u)) - L^{-1}(N(u)) \quad , \tag{2}$$

where θ satisfies $L(\theta) = 0$.

In ADM, u is represented as the infinite of series

$$u = \sum_{n=0}^{\infty} u_n \ ,$$

and the nonlinear function $N(u)$ is decomposed as follows:

$$N(u) = \sum_{n=0}^{\infty} A_n \quad , \tag{4}$$

where the A_n 's are polynomials of $u_1, u_2, ..., u_n$ called Adomian's polynomials and are calculated by the formula:

$$A_n = \frac{1}{n!} \frac{d^n}{d\lambda^n} [N(\sum_{i=0}^{n} \lambda^i u_i)]_{\lambda=0} \qquad n = 0,1,2,... \tag{5}$$

Thus (2) can be rewritten as follows:

$$\sum_{n=0}^{\infty} u_n = \theta + L^{-1} f - L^{-1}(R(\sum_{n=0}^{\infty} u_n)) - L^{-1}(\sum_{n=0}^{\infty} A_n) \quad .$$

(6)

Each term of the series $\sum_{n=0}^{\infty} u_n$ is given by recurrent relations

$$\left.\begin{array}{l} u_0 = \theta + L^{-1} f \\ u_n = -L^{-1}(R(u_{n-1})) - L^{-1}(A_{n-1}) \qquad n \geq 1 \end{array}\right\}$$

(7)

The relations (7) are called Adomian algorithm.

The crux of Adomian algorithm depend on calculate the Adomian polynomials, the Adomian polynomials are not unique [3,10,12,13]. In [12] are defined as the following form:

$$\left.\begin{array}{l} A_0 = N(u_0) \\ A_1 = u_1 (\frac{\partial}{\partial u_0}) N(u_0) \\ A_2 = u_2 (\frac{\partial}{\partial u_0}) N(u_0) + (\frac{u_1^2}{2!})(\frac{\partial^2}{\partial u_0^2}) N(u_0) \\ A_3 = u_3 (\frac{\partial}{\partial u_0}) N(u_0) + u_1 u_2 (\frac{\partial^2}{\partial u_0^2}) N(u_0) + (\frac{u_1^3}{3!})(\frac{\partial^3}{\partial u_0^3}) N(u_0) \\ \quad \vdots \end{array}\right\}$$

(8)

By depending on the idea of [7], we can define the nonlinear function $N(u)$ as follows:

$$N(u) = \sum_{n=0}^{\infty} \overline{A}_n$$

(9)

where the $\overline{A}_n$ are functions that depend on $u_0, u_1, \ldots, u_n$. They are determined by

$$\left.\begin{aligned} \overline{A}_0 &= N(u_0) \\ \overline{A}_n &= N(\sum_{i=0}^{n} u_i) - N(\sum_{i=0}^{n-1} u_i) \qquad n \geq 1 \end{aligned}\right\}$$

(10)

Substitute (3) and (9) into (2) yields

$$\sum_{n=0}^{\infty} u_n = \theta + L^{-1} f - L^{-1}(R(\sum_{n=0}^{\infty} u_n)) - L^{-1}(\sum_{n=0}^{\infty} \overline{A}_n)$$

(11)

The u_n's can be determined by the recurrent relations

$$\left.\begin{aligned} u_0 &= \theta + L^{-1} f \\ u_n &= -L^{-1}(R(u_{n-1})) - L^{-1}(\overline{A}_{n-1}) \qquad n \geq 1 \end{aligned}\right\}$$

(12)

The relations (9)-(12) are called modified Adomian decomposition method (MADM).

3-Applications:

In this part, we apply the ADM and the MADM to the equations of advection, Kdv and K(2,2), respectively, in the next three problems.

Problem1:

Consider the homogeneous advection equation [4],

$$u_t - \frac{1}{2}(u^2)_x = 0, \qquad 0 < x \leq 1, \qquad t > 0 \qquad ,$$

(13)

with initial condition $u(x,0) = 0.2x^2$

The explicit analytic solution is given by $u(x,t) = \frac{(1-0.4xt) - \sqrt{1-0.8xt}}{0.4t^2}$. **(14)**

Now, we solve Eq. (13) using ADM and MADM. In this problem

$$N(u) = (\frac{1}{2}u^2)_x,\ f = 0,\ R(u) = 0 \text{ and } \theta = u(x,0) = 0.2x^2$$

We use the recursive relations given by (7) to obtain the terms of the decomposition series (3). In this case by using (8), we obtain

$$\left.\begin{aligned} A_0 &= (\frac{1}{2}u_0^2)_x \\ A_1 &= (u_0 u_1)_x \\ A_2 &= (u_0 u_2 + \frac{1}{2}u_1^2)_x \\ A_3 &= (u_0 u_3 + u_1 u_2)_x \\ &\vdots \end{aligned}\right\}$$

(15)

and so on, the rest of the polynomials can be constructed in a similar manner.

Knowing $\{A_n\}$ terms leads to the calculation of the $\{u_n\}$ terms by using relations (7)

$$\left.\begin{aligned} u_0 &= \frac{1}{5}x^2 \\ u_1 &= \frac{2}{25}x^3 t \\ u_2 &= \frac{1}{25}x^4 t^2 \\ u_3 &= \frac{14}{625}x^5 t^3 \\ &\vdots \end{aligned}\right\}$$

(16)

Substituting these individual terms in (3), we have

$$u(x,t) = \frac{1}{5}x^2 + \frac{2}{25}x^3 t + \frac{1}{25}x^4 t^2 + \frac{14}{625}x^5 t^3 + \cdots$$

(17)

Now, we solve Eq. (13) by MADM. By using Eq. (10), $\{\overline{A}_n\}$ terms can be computed as:

$$\left.\begin{array}{l} \overline{A}_0 = (\frac{1}{2}u_0^2)_x \\ \overline{A}_1 = (u_0u_1 + \frac{1}{2}u_1^2)_x \\ \overline{A}_2 = (u_0u_2 + u_1u_2 + \frac{1}{2}u_2^2)_x \\ \overline{A}_3 = (u_0u_3 + u_1u_3 + u_2u_3 + \frac{1}{2}u_3^2)_x \\ \quad \vdots \end{array}\right\}$$

(18)

and so on.

Substituting relations (18) into recurrent relations (12) gives

$$\left.\begin{array}{l} u_0 = \frac{1}{5}x^2 \\ u_1 = \frac{2}{25}x^3t \\ u_2 = \frac{1}{25}x^4t^2 + \frac{4}{625}x^5t^3 \\ u_3 = \frac{2}{125}x^5t^3 + \frac{49}{6250}x^6t^4 + \frac{164}{78125}x^7t^5 + \frac{6}{15625}x^8t^6 + \frac{16}{546875}x^9t^7 \\ \vdots \end{array}\right\}$$

(19)

Putting these individual terms in Eq. (3), one obtains

$$u(x,t) = \frac{1}{5}x^2 + \frac{2}{25}x^3t + \frac{1}{25}x^4t^2 + \frac{14}{625}x^5t^3 + \frac{49}{6250}x^6t^4 + \frac{164}{78125}x^7t^5$$
$$+ \frac{6}{15625}x^8t^6 + \frac{16}{546875}x^9t^7 + \cdots$$

(20)

Problem2:

Consider the following KdV equation [6],

$$u_t + (3u^2)_x + u_{xxx} = 0, \qquad -12 \le x < 12, t > 0,$$

(21)

with the initial condition $u(x,0)=\frac{1}{2}\sec h^2(\frac{x}{2})$,

The exact solution is given by

$$u(x,t)=\frac{1}{2}\sec h^2(\frac{x-t}{2}) \quad ,$$

(22)

Here, we solve Eq. (21) by the ADM and the MADM, respectively. In this problem

$$N(u)=(3u^2)_x,\ f=0,\ R(u)=u_{xxx} \text{ and } \theta=u(x,0)=\frac{1}{2}\sec h^2(\frac{x}{2}),$$

We again use the recursive relations given by (7) to obtain the terms of the decomposition series (3). In this case, by using the relations (8), we have

$$\left.\begin{aligned} A_0 &= (3u_0^2)_x \\ A_1 &= (6u_0u_1)_x \\ A_2 &= (6u_0u_2+3u_1^2)_x \\ &\vdots \end{aligned}\right\}$$

(23)

and so on,

Substituting the relations (23) into recurrent relations (7) yields

$$\left.\begin{aligned} u_0 &= \frac{1}{2}\sec h^2(\frac{x}{2}) \\ u_1 &= \frac{1}{2}\sec h^2(\frac{x}{2})\tanh(\frac{x}{2})t \\ u_2 &= \frac{1}{8}[-\sec h^2(\frac{x}{2})+3\sec h^2(\frac{x}{2})\tanh^2(\frac{x}{2})]t^2 \\ &\vdots \end{aligned}\right\}$$

(24)

The $\{u_n\}$ terms are known, so the solution is given by

$$u(x,t)=\frac{1}{2}\sec h^2(\frac{x}{2})+\frac{1}{2}\sec h^2(\frac{x}{2})\tanh(\frac{x}{2})t$$

$$+\frac{1}{8}[-\sec h^2(\frac{x}{2})+3\sec h^2(\frac{x}{2})\tanh^2(\frac{x}{2})]t^2$$

(25)

Now we solve Eq. (21) by the MADM. By using Eq. (10), $\{\overline{A}_n\}$ terms can be computed as:

$$\left.\begin{array}{l} \overline{A}_0=(3u_0^2)_x \\ \overline{A}_1=(6u_0u_1+3u_1^2)_x \\ \overline{A}_2=(6u_0u_2+6u_1u_2+3u_2^2)_x \\ \quad\vdots \end{array}\right\}$$

(26)

and so on,

Using the recursive relations in (12) gives the first few components as follows:

$$\left.\begin{array}{l} u_0=\frac{1}{2}\sec h^2(\frac{x}{2}) \\ u_1=\frac{1}{2}\sec h^2(\frac{x}{2})\tanh(\frac{x}{2})t \\ u_2=\frac{1}{8}[-\sec h^2(\frac{x}{2})+3\sec h^2(\frac{x}{2})\tanh^2(\frac{x}{2})]t^2 \\ \quad+\frac{1}{8}[\sec h^4(\frac{x}{2})\tanh(\frac{x}{2})-5\sec h^6(\frac{x}{2})\tanh(\frac{x}{2})+\sec h^4(\frac{x}{2})\tanh^3(\frac{x}{2})]t^3 \\ \vdots \end{array}\right\}$$

(**27)**

Substituting these individual terms in Eq. (3), we have

$$u(x,t)=\frac{1}{2}\sec h^2(\frac{x}{2})+\frac{1}{2}\sec h^2(\frac{x}{2})\tanh(\frac{x}{2})t+\frac{1}{8}[-\sec h^2(\frac{x}{2})$$
$$+3\sec h^2(\frac{x}{2})\tanh^2(\frac{x}{2})]t^2+\frac{1}{8}[\sec h^4(\frac{x}{2})\tanh(\frac{x}{2})$$
$$-5\sec h^6(\frac{x}{2})\tanh(\frac{x}{2})+\sec h^4(\frac{x}{2})\tanh^3(\frac{x}{2})]t^3+\cdots$$

(28)

<u>Problem</u>: 3

Consider the following K(2,2) equation [2]

$$u_t + (u^2)_x + (u^2)_{xxx} = 0, \qquad 0 \le x \le 10, t > 0,$$

(29)

with the initial condition $u(x,0) = x$,

The exact solution is given by

$$u(x,t) = \frac{x}{1+2t} \qquad ,$$

(30)

We solve Eq. (29) by the ADM and MADM, respectively. In this problem

$$N(u) = (u^2)_x + (u^2)_{xxx}, \; f = 0, \; R(u) = 0 \text{ and } \theta = u(x,0) = x,$$

To obtain the solution by ADM, we use the relations (8). In this case the Adomian polynomials is given by

$$\left.\begin{aligned}
A_0 &= (u_0^2)_x + (u_0^2)xxx \\
A_1 &= (2u_0u_1)_x + (2u_0u_1)_{xxx} \\
A_2 &= (2u_0u_2 + u_1^2)_x + (2u_0u_2 + u_1^2)_{xxx} \\
A_3 &= (2u_0u_3 + 2u_1u_2)_x + (2u_0u_3 + 2u_1u_2)_{xxx} \\
&\vdots
\end{aligned}\right\}$$

(31)

and so on,

The A_n's have been known, so the $\{u_n\}$ terms can be determined by using the recursive relations (7). Simple calculation leads to

$$\left.\begin{aligned}
u_0 &= x \\
u_1 &= -2xt \\
u_2 &= 4xt^2 \\
u_3 &= -8xt^3 \\
&\vdots
\end{aligned}\right\}$$

(32)

Putting these individual terms in (3), we have

$$u(x,t) = x - 2xt + 4xt^2 - 8xt^3 + \cdots$$
(33)

By using Eq. (10), we can be calculated $\{\overline{A}_n\}$ terms as:

$$\left.\begin{aligned}
\overline{A}_0 &= (u_0^2)_x + (u_0^2)_{xxx} \\
\overline{A}_1 &= (2u_0u_1 + u_1^2)_x + (2u_0u_1 + u_1^2)_{xxx} \\
\overline{A}_2 &= (2u_0u_2 + 2u_1u_2 + u_2^2)_x + (2u_0u_2 + 2u_1u_2 + u_2^2)_{xxx} \\
\overline{A}_3 &= (2u_0u_3 + 2u_1u_3 + 2u_2u_3 + u_3^2)_x + (2u_0u_3 + 2u_1u_3 + 2u_2u_3 + u_3^2)_{xxx} \\
&\vdots
\end{aligned}\right\}$$
(34)

and so on,

Substituting relations (34) into recursive relations (12) yields

$$\left.\begin{aligned}
u_0 &= x \\
u_1 &= -2xt \\
u_2 &= 4xt^2 - \frac{8}{3}xt^3 \\
u_3 &= -\frac{16}{3}xt^3 + \frac{32}{3}xt^4 - \frac{32}{3}xt^5 + \frac{64}{9}xt^6 - \frac{128}{63}xt^7 \\
&\vdots
\end{aligned}\right\}$$
(35)

Substituting these individual terms in (3), we obtain

$$u(x,t) = x - 2xt + 4xt^2 - 8xt^3 + \frac{32}{3}xt^4 - \frac{32}{3}xt^5 + \frac{64}{9}xt^6 - \frac{128}{63}xt^7 + \cdots$$
(36)

4- <u>Convergence of MADM</u>:

A series is often of no use if it is convergent in a rather restricted region, and thus proving convergence of the solution series is very important. To demonstrate the convergence of the MADM for nonlinear partial differential equations (13), (21) and

(29) let us consider the Hilbert space H defined as $H = L^2(\Omega)$: where $\Omega = (a,b) \times [0,T]$, and using the set of applications:

$$u : \Omega \to R \text{ with } \int_{\Omega} u^2(x,s)\,ds\,d\tau < +\infty,$$

the scalar product

$$\langle u,v\rangle = \int_{\Omega} u(x,s)v(x,s)\,ds\,d\tau,$$

(37)

and the associated norm

$$\|u\|^2 = \int_{\Omega} u^2(x,s)\,ds\,d\tau.$$

(38)

Now, we want to prove the convergence of the MADM applied to problems (1), (2) and (3) by using similar approach of the [8,11]. To prove that, we will prove the following two hypotheses are satisfied:

$\mathrm{H}_1 : \langle L(u) - L(v), u - v\rangle \geq k\|u - v\|^2, k > 0, \forall u, v \in H,$

H_2 : For any $M > 0, \exists$ a constant $C(M) > 0$, such that for $u,v \in H$ with $\|u\| \leq M, \|v\| \leq M$,

we have $\langle L(u) - L(v), w\rangle \leq C(M)\|u - v\|\|w\|$ for every $w \in H$.

<u>Theorem</u>(1): The MADM applied to (13) converges towards a particular solution.

<u>Proof</u>: To prove this theorem, we will verify the conditions H_1 and H_2, firstly, we will verify the convergence hypothesis H_1 for the operator $L(u)$. From (13) we have

$$L(u) - L(v) = \frac{1}{2}\frac{\partial}{\partial x}[u^2 - v^2],$$

therefore,

$$\langle L(u)-L(v),u-v\rangle=\frac{1}{2}\langle\frac{\partial}{\partial x}[u^2-v^2],u-v\rangle,$$

(39)

by Schwartz inequality, definition of scalar product and the properties of the differential operator $\frac{\partial}{\partial x}$ in H, then there exist a constant $\delta>0$ such that

$$\langle-\frac{\partial}{\partial x}[u^2-v^2],u-v\rangle\le\left\|\frac{\partial}{\partial x}[u^2-v^2]\right\|\|u-v\|$$

$$\le\delta\left\|u^2-v^2\right\|\|u-v\|$$

$$=\delta\|(u-v)(u+v)\|\|u-v\|$$

$$\le 2\delta M\|u-v\|^2,$$

where $\|u\|\le M,\|v\|\le M$. Therefore

$$\langle\frac{\partial}{\partial x}[u^2-v^2],u-v\rangle\ge 2\delta M\|u-v\|^2,$$

(40)

Substitute (40) in (39) we get:

$$\langle L(u)-L(v),u-v\rangle\ge\delta M\|u-v\|^2=k\|u-v\|^2,$$

where $k=\delta M$ then the hypothesis H_1 holds.

Secondly, we verify the convergence hypothesis H_2 for the operator $L(u)$. For that we have:

$$\langle L(u)-L(v),w\rangle=\langle\frac{\partial}{\partial x}(u^2-v^2),w\rangle$$

$$\le\delta M\|u-v\|\|w\|$$

$$\le C(M)\|u-v\|\|w\|,$$

where $C(M)=\delta M$, hence the hypothesis H_2 holds.

<u>Theorem</u>(2): The MADM applied to (21) converges towards a particular solution.

<u>Proof</u>: To prove this theorem, firstly, we will verify the convergence hypothesis H_1 for the operator $L(u)$. From (21) we have

$$L(u)-L(v)=-\frac{\partial^3}{\partial x^3}[u-v]-3\frac{\partial}{\partial x}[u^2-v^2],$$

therefore,

$$\langle L(u)-L(v),u-v\rangle=\langle-\frac{\partial^3}{\partial x^3}[u-v],u-v\rangle+3\langle-\frac{\partial}{\partial x}[u^2-v^2],u-v\rangle,$$

(41)

by the definition of scalar product and the properties of the differential operator $\frac{\partial}{\partial x}$ and $\frac{\partial^3}{\partial x^3}$ in H, then there exist constants $\delta_i,\ i=1,2$ such that

$$\langle-\frac{\partial^3}{\partial x^3}[u-v],u-v\rangle\geq\delta_1\|u-v\|^2,$$

(42)

and according the Schwartz inequality, we get

$$\begin{aligned}\langle-\frac{\partial}{\partial x}[u^2-v^2],u-v\rangle&\leq\left\|\frac{\partial}{\partial x}[u^2-v^2]\right\|\|u-v\|\\&\leq\delta_2\left\|u^2-v^2\right\|\|u-v\|\\&=\delta_2\|(u-v)(u+v)\|\|u-v\|\\&\leq2\delta_2M\|u-v\|^2,\end{aligned}$$

where $\|u\|\leq M,\|v\|\leq M$, then we have

$$\langle\frac{\partial}{\partial x}[u^2-v^2],u-v\rangle\geq 2\delta_2 M\|u-v\|^2,$$
(43)

by substitute (42) and (43) in (41) we obtained:

$$\langle L(u)-L(v),u-v\rangle\geq\delta_1\|u-v\|^2+6\delta_1 M\|u-v\|^2$$

$$=(\delta_1+6\delta_2 M)\|u-v\|^2$$

$$=k\|u-v\|^2$$

where $k=\delta_1+6\delta_2 M>0\Rightarrow\delta_1>-6\delta_2 M.$

Thus, hypothesis H_1 holds.

Now verify hypothesis H_2

$$\langle L(u)-L(v),w\rangle=\langle-\frac{\partial^3}{\partial x^3}[u-v]-3\frac{\partial}{\partial x}[u^2-v^2],w\rangle,$$

This yields

$$\langle L(u)-L(v),w\rangle\leq\left\|-\frac{\partial^3}{\partial x^3}[u-v]\right\|\|w\|+\left\|-3\frac{\partial}{\partial x}[u-v]\right\|\|w\|$$

$$\leq\delta_1\|u-v\|\|w\|+6\delta_2 M\|u-v\|\|w\|$$

$$\leq C(M)\|u-v\|\|w\|,$$

where $C(M)=\delta_1+6\delta_2 M$ and, therefore, H_2 fulfilled.

<u>Theorem</u>(3): The MADM applied to K(2,2) equation (29) converges towards a particular solution.

<u>Proof</u>: we will prove the convergence hypothesis H_1 for the operator $L(u)$. From (29) we have

$$L(u)-L(v)=\frac{\partial}{\partial x}[u^2-v^2]-\frac{\partial^3}{\partial x^3}[u^2-v^2],$$

therefore,

$$\langle L(u)-L(v),u-v\rangle=\langle-\frac{\partial}{\partial x}[u^2-v^2],u-v\rangle+\langle-\frac{\partial^3}{\partial x^3}[u^2-v^2],u-v\rangle,$$

(44)

by Schwartz inequality, definition of scalar product and the properties of the differential operator $\frac{\partial}{\partial x}$ and in H, then there exist a constant $\delta_1>0$ such that

$$\langle-\frac{\partial}{\partial x}[u^2-v^2],u-v\rangle\leq\left\|\frac{\partial}{\partial x}[u^2-v^2]\right\|\|u-v\|$$

$$\leq\delta_1\left\|u^2-v^2\right\|\|u-v\|$$

$$=\delta_1\|(u-v)(u+v)\|\|u-v\|$$

$$\leq 2\delta_1 M\|u-v\|^2,$$

where $\|u\|\leq M,\|v\|\leq M,$ then we have

$$\langle\frac{\partial}{\partial x}[u^2-v^2],u-v\rangle\geq 2\delta_1 M\|u-v\|^2,$$

(45)

similarly by Schwartz inequality, definition of scalar product and the properties of the differential operator $\frac{\partial^3}{\partial x^3}$ and in H, then there exist a constant $\delta_2>0$ such that

$$\langle-\frac{\partial^3}{\partial x^3}[u^2-v^2],u-v\rangle\leq\left\|\frac{\partial^3}{\partial x^3}[u^2-v^2]\right\|\|u-v\|,$$

$$\leq\delta_2\left\|u^2-v^2\right\|\|u-v\|$$

$$= \delta_2 \|(u-v)(u+v)\| \| u-v\|$$

$$\leq 2\delta_2 M \| u-v\|^2,$$

where $\|u\| \leq M, \|v\| \leq M$, then we have

$$\langle \frac{\partial^3}{\partial x^3}[u^2 - v^2], u-v\rangle \geq 2\delta_2 M \|u-v\|^2,$$

(46)

by substitute (45) and (46) in (44) we obtained:

$$\langle L(u) - L(v), u-v\rangle \geq 2\delta_1 M \|u-v\|^2 + 2\delta_2 M \|u-v\|^2$$

$$= (2\delta_1 M + 2\delta_2 M)\|u-v\|^2$$

$$= k\|u-v\|^2$$

where $k = 2\delta_1 M + 2\delta_2 M > 0 \Rightarrow \delta_2 > -\delta_1$, then the hypothesis H_1 holds.

Now verify hypothesis H_2

$$\langle L(u) - L(v), w\rangle = \langle -\frac{\partial}{\partial x}[u^2 - v^2] - \frac{\partial^3}{\partial x^3}[u^2 - v^2], w\rangle,$$

This yields

$$\langle L(u) - L(v), w\rangle \leq \left\| -\frac{\partial}{\partial x}[u-v] \right\| \|w\| + \left\| -\frac{\partial^3}{\partial x^3}[u-v] \right\| \|w\|$$

$$\leq 2\delta_1 M \| u-v\| \|w\| + 2\delta_2 M \| u-v\| \|w\|$$

$$\leq C(M) \| u-v\| \|w\|,$$

where $C(M) = 2\delta_1 M + 2\delta_2 M$, then the hypothesis H_2 holds.

<u>Remark</u>:

From theorems (1), (2) and (3), we could prove that the solutions given by MADM are convergence toward the particular solutions.

Similarly we can prove the convergence of ADM, but it is appears from applications that $\sum_{n=0}^{\infty} \overline{A}_n$ is a much better approximation to $N(u)$ than $\sum_{n=0}^{\infty} A_n$.

5- <u>Results and discussion</u>:

In this part, we present the comparison of the approximate solutions obtained by ADM and MADM with exact solution. According to the Tables 1, 2 and 3, we can see that the absolute errors of MADM ($|\boldsymbol{u} - \overline{\Phi}_5|$ and $|\boldsymbol{u} - \overline{\Phi}_7|$**)** are less than the absolute errors of ADM ($|\boldsymbol{u} - \Phi_5|$ and $|\boldsymbol{u} - \Phi_7|$**)**. Figures 1,2 and 3 show the comparisons between the results of the exact solutions, ADM and MADM. From figure 1, it can be clearly seen that there is very good agreement among them and the results obtained from the exact, ADM and MADM have the same shapes for all ($0 < t \leq 1$). Figures 2 and 3 show, when we increase slightly the range of t, the shapes of the ADM solutions deferent from the shapes of the exact solutions. On the other hand, the MADM solutions have the same shapes as the exact solutions even for the large range of t. Therefore, based on these present comparisons, we can see that the accuracy of the MADM is remarkable. Furthermore, the MADM has larger convergence region and faster convergence rate for the series solution than the ADM has.

Table 1: Comparison of ADM and MADM solutions for problem 1.

		ADM		MADM	
t	**x**	$\lvert\boldsymbol{u} - \Phi_5\rvert$	$\lvert\boldsymbol{u} - \Phi_7\rvert$	$\lvert\boldsymbol{u} - \overline{\Phi}_5\rvert$	$\lvert\boldsymbol{u} - \overline{\Phi}_7\rvert$
0.1	**0.1**	**4.6161e-015**	**3.8871e-015**	**2.0691e-015**	**1.77808e-015**
	0.4	**1.4211e-010**	**1.0423e-013**	**3.1122e-011**	**5.62744e-015**
	0.7	**7.2894e-009**	**1.5518e-011**	**1.6351e-009**	**2.05669e-014**
	1	**9.0364e-008**	**3.9281e-010**	**2.0770e-008**	**5.52836e-013**
0.	**0.1**	**2.7288e-011**	**2.9852e-014**	**6.0233e-012**	**2.63244e-015**
	0.4	**4.9745e-007**	**8.6808e-009**	**1.2427e-007**	**2.87437e-011**

5	0.7	2.8205e-005	1.5169e-006	8.0348e-006	1.18459e-008
	1	3.9331e-004	4.3508e-005	1.2886e-004	6.73930e-007
1	0.1	9.0364e-010	3.9284e-012	2.0770e-010	5.52856e-014
	0.4	1.8761e-005	1.3210e-006	5.5940e-006	1.31054e-008
	0.7	1.2976e-003	2.8535e-004	5.2024e-004	1.01868e-005
	1	2.6126e-002	1.2187e-002	1.4822e-002	1.61571e-003

Table 2: Comparison of ADM and MADM solutions for problem 2.

		ADM		MADM	
t	x	$\left\lvert u-\Phi_5 \right\rvert$	$\left\lvert u-\Phi_7 \right\rvert$	$\left\lvert u-\overline{\Phi}_5 \right\rvert$	$\left\lvert u-\overline{\Phi}_7 \right\rvert$
0.5	-12	2.9503e-009	1.7894e-011	3.1246e-015	8.4703e-022
	-6	1.0202e-006	3.1555e-009	3.7225e-012	6.4811e-019
	0	4.4659e-005	1.4565e-006	4.5137e-008	5.5511e-017
	6	1.1698e-006	3.0012e-009	4.4692e-012	1.7347e-018
	12	3.4856e-009	2.0274e-011	3.4181e-015	3.3881e-021
1	-12	8.7471e-008	2.1611e-009	3.3843e-012	1.6941e-021
	-6	3.0554e-005	4.0408e-007	3.8631e-009	1.0842e-018
	0	2.6095e-003	3.4192e-004	4.6848e-005	1.2052e-010
	6	4.0091e-005	3.5812e-007	1.3362e-008	1.7347e-017
	12	1.2219e-007	2.7755e-009	9.6122e-012	2.0329e-020
1.5	-12	6.1800e-007	3.4931e-008	1.6971e-010	2.2446e-020
	-6	2.1760e-004	6.8429e-006	1.8646e-007	2.6672e-016
	0	2.5926e-002	7.6923e-003	2.1600e-003	6.4557e-007
	6	3.2529e-004	5.3754e-006	3.2245e-007	9.8463e-015
	12	1.0225e-006	5.0895e-008	2.2232e-012	2.7105e-021
2	-12	2.4323e-006	2.4815e-007	4.6813e-009	2.8167e-017
	-6	8.6194e-004	5.0488e-005	4.9262e-006	5.5585e-013
	0	1.2335e-001	6.5543e-002	5.1801e-002	8.8874e-004
	6	1.4563e-003	3.1484e-005	1.0580e-005	1.6594e-011
	12	4.7778e-006	4.1076e-007	6.8393e-009	4.0129e-017

Table 2: Comparison of ADM and MADM solutions for problem 3.

ADM MADM

t	x	$\left\|u-\Phi_5\right\|$	$\left\|u-\Phi_7\right\|$	$\left\|u-\overline{\Phi}_5\right\|$	$\left\|u-\overline{\Phi}_7\right\|$
0.1	1	5.3333e-005	2.1333e-006	1.6277e-006	3.8691e-009
	4	2.1333e-004	8.5333e-006	6.5111e-006	1.5474e-008
	7	3.7333e-004	1.9333e-005	1.3944e-005	2.7084e-008
	10	5.3333e-004	2.3333e-005	1.2777e-005	3.8691e-008
0.5	1	5.0000e-001	5.0000e-001	4.7003e-003	1.6443e-004
	4	2.0000e+000	2.0000e+000	1.8802e-006	6.5774e-004
	7	3.5000e+000	3.5000e+000	3.2902e-002	1.1510e-003
	10	5.0000e+000	5.0000e+000	4.7003e-002	1.6443e-003
0.75	1	6.8344e+000	3.0375e+000	2.3586e-002	1.4861e-003
	4	2.7337e+001	1.2150e+001	9.4346e-002	5.9444e-003
	7	4.7841e+001	2.1262e+001	1.6511e-001	1.0403e-002
	10	6.8344e+001	3.0375e+001	2.3586e-001	1.4861e-002

6- Conclusions:

The modified Adomian decomposition method (MADM) is carried out successfully for finding the approximate solutions of advection equation, KdV equation and K(2,2) equation. The obtained solutions are compared with exact solutions and ADM solutions, three solved applications show that the results of the present method have high precision, fast convergence rate for the series solution and large convergence region. It really has advantage over the ADM. For computation and plot, software Mathematica 6 has been used.

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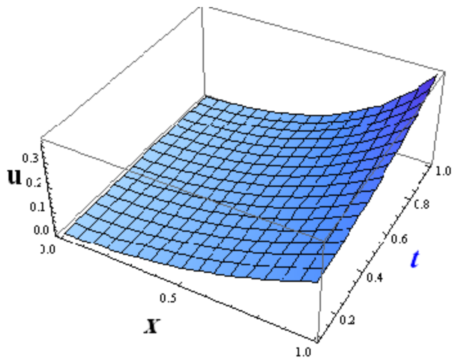
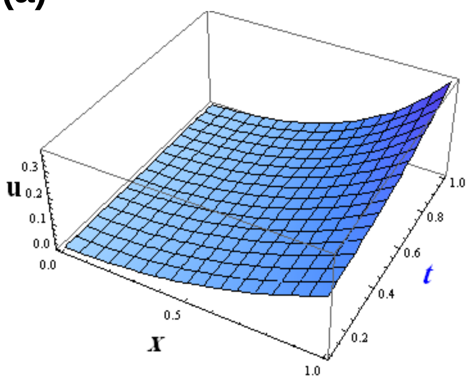
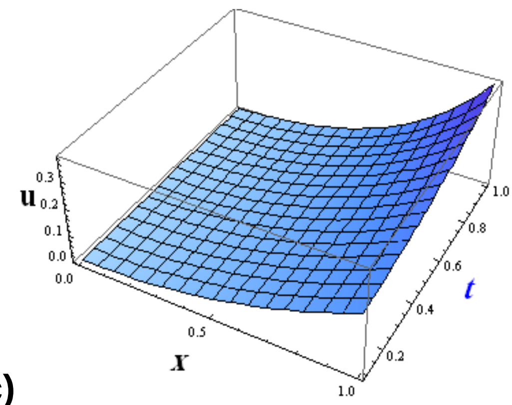


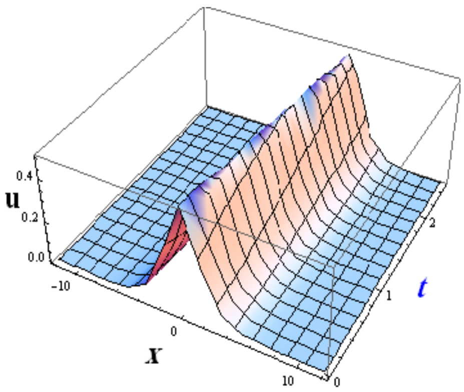
(a) **(b)**



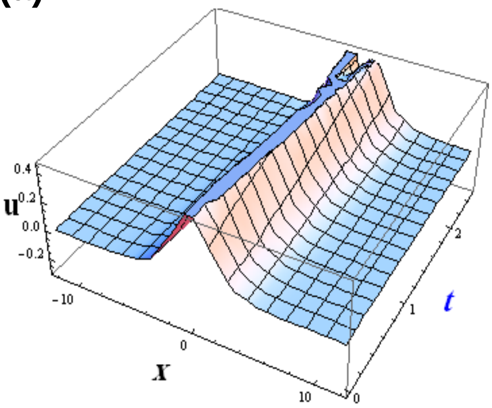


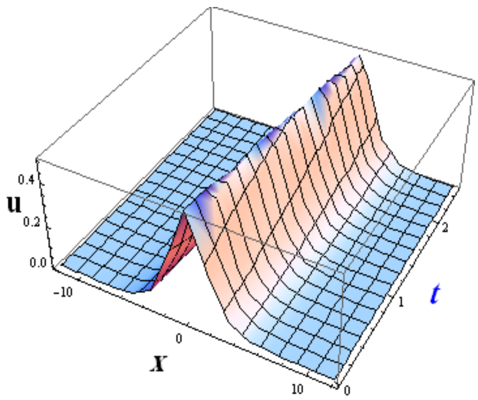
(c)

Fig.1: All graphs are plot the solutions of problem 1 with $0 \leq x \leq 1$and $0 < t \leq 1$ (a) Exact solution,(b)Approximate solution obtained by ADM,(c)Approximate solution obtained by MADM.



(a) (b)



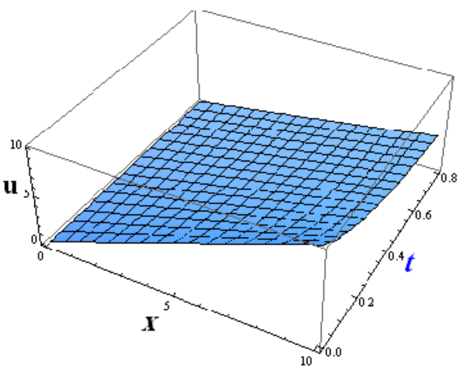


(c)

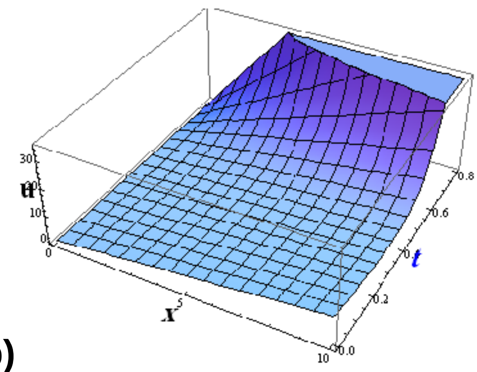
Fig.2: All graphs are plot the solutions of problem 3 with

$-12 \le x \le 12$ and $0 \le t \le 2.5$ (a)Exact solution,(b)Approximate solution

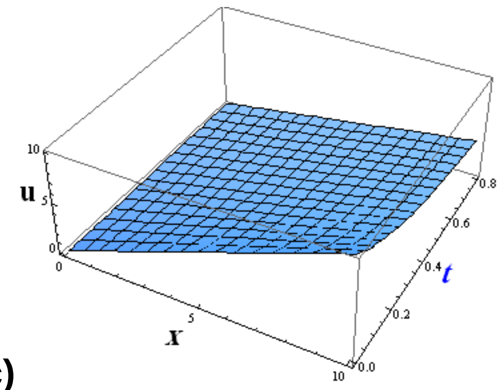
obtained by ADM,(c)Approximate solution obtained by MADM.



(a)



(b)



(c)

Fig.3: All graphs are plot the solutions of problem 3 with $0 \le x \le 10$ and $0 \le t \le 0.8$ (a) Exact solution,(b)Approximate solution obtained by ADM,(c)Approximate solution obtained by MADM.

حل معادلات الحل و Kdv و K(2,2) باستخدام طريقة تحليل أدومين المعدلة

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المستخلص

في هذا البحث قدمنا تطبيق جديد لطريقة تحليل أدومين المعدلة لحل معادلات الحمل بالاعتماد على التطوير الذي قدمه إبراهيم الكالة في المصدر [7] وأثبتنا بأن و Kdv وK(2,2)

هذه الطريقة متقاربة. النتائج التي حصلنا عليها من تطبيق هذه الطريقة قورنت مع نتائج طريقة

تحليل أدومين القياسية فأظهرت النتائج العددية لطريقة تحليل أدومين المعدلة بأنها تمتلك دقة عالية وأسرع في التقارب ومنطقة تقاربها أكبر من طريقة تحليل أدومين.