INSTANTANEOUS DEFLECTION OF PARTIALLY PRESTRESSED CONCRETE MEMBERS

Mustafa B. Dawood
Civil Engineering,
University of Babylon

Haider K. Amash
Civil Engineering,
University of Babylon

Abstract

A review of five different methods is presented to predict the instantaneous deflection of partially prestressed concrete members. A new simplified proposed method to predict instantaneous deflection of partially prestressed concrete members is presented. The results of the proposed method were lying between of those of the studies.

Introduction

In many prestressed concrete structures, it is unlikely that the full service load will be applied during the life of the structure. It is therefore reasonable to design prestressed structural members in such a way that cracking will assumed to occur under full service load. This can be achieved by the use of partial prestressing.

The advantages of partial prestressing are(13):

1- Reduction of prestressing force.
2- Reduction of initial camber, which is of importance for some types of precast members.
3- In some instances, a reduction in prestressing force may allow an increase in tendon eccentricity.
4- In post-tensioned members, cracking in the end zones may be reduced.
5- In pretension members, where partial prestressing achieved by a lower than allowable stress in the tendons, the transfer length of the cable is reduced.

One of the disadvantages of the partial prestressing is that the reduced stiffness of the member after cracking may result in an increase in deflection, which may be sufficient to exceed the acceptable serviceability limit. Hence, the calculation of the deflection becomes a matter of practical importance.

The reduction in the flexural stiffness of the member due to cracking can be taken into account in the deflection calculation by the well-known "Effective Moment of Inertia" I_e Method, originally developed by Branson(1) for non-prestressed members. At present, five methods are available in the literature to calculate deflections in cracked prestressed members. A new proposed method is presented to calculate deflection of partial prestressed concrete members. This paper presenting a new procedure to calculate effective moment (M_e). The aim of this paper is to evaluate the live load and service load deflection so that the most reliable method may be used into design of partially prestressed concrete members.

Modifications to the I_e method

In a partially prestressed member, cracks develop under full service load at several sections along the span. Theoretically, the cracked moment of inertia applies at the cracked section. In between cracks, the moment of inertia is close to that of the gross section. Branson(1) suggested a method to calculate a moment of inertia of the section (I_e) which takes into account both the cracked and uncracked stages. The value of (I_e) originally suggested by Branson(1), at a given section of the member, is:

$$I_e = \left(\frac{M_{cr}}{M_s}\right)^4 I_g + \left(1 - \left(\frac{M_{cr}}{M_s}\right)^4\right) I_{cr} \le I_g \qquad (1)$$

and, for the entire length of a simply supported member:

$$I_e = \left(\frac{M_{cr}}{M_s}\right)^3 I_g + \left(1 - \left(\frac{M_{cr}}{M_s}\right)^3\right) I_{cr} \le I_g \qquad (2)$$

where M_{cr} and M_s are the cracking moment and the full service load moment of the section, respectively. (I_g and I_{cr}) are the gross moment of inertia and the moment of inertia of the cracked section, respectively. (I_e) in equation (2) may be used in a standard deflection formula for simply supported beam, or between inflection points of a continuous beam. (I_e) is calculated at the support section of a cantilever beam. When a concrete flexural member is still uncracked (I_g), its deflection or camber can be computed on his basis of the uncracked moment of the Inertia. Once cracking occurs, computation should be done on the basis of the effective moment of Inertia (I_e) as a way to include the effect of tension stiffening and, therefor, also progressive cracking(13).

The upper limit for (I_e) is the uncracked value (I_g) while the lower limit is the fully cracked value (I_{cr}), the (I_e) effective method was originally proposed for non-prestressed members(2), and it is application to cracked prestressed members has been suggested by many researches and has proved to be accurate and convenient. However, there is quite number of different options on how to best apply the I_e-effective method to cracked prestressed members. The differences in options are related, mostly to the presence of prestressing. The term M_{cr}/M_s in the I_e-effective formula (equation 2) was derived without considering prestressing; thus, it may need to be modified. Equation (2) has been modified by several researches, to calculate short-term deflection of partially prestressed beams. Five methods and proposed method are summarized as follows.

Method 1

Chen(4), Bennett(3), Tadros(12), have suggested that the cracking moment and the total service load moment to be used in equation (2) is reduced by an amount equal to the decompression moment M_{dec}. Note that the decompression moment in partially prestressed members signifies the transition between the performance of the member complying with the principles of prestressed concrete and the principles of reinforcing concrete. The decompression moment is the moment leading to zero stress at the extreme fiber of the concrete section, at which the tensile stresses are caused by the applied loads. The decompression moment is given by:

$$M_{dec} = P_e e + \frac{P_e I_t}{A_t y_b} \qquad (3)$$

where P_e is the effective prestressing force after losses; e is the eccentricity, A_t and I_t are the area and the moment of inertia, respectively, of the uncracked transformed section. If the cracking moment M_{cr} and the service-load moment M_s are decreased by an amount equal to the decompression moment M_{dec}, equation (2) becomes:

$$I_e = \left(\frac{M_{cr} - M_{dec}}{M_s - M_{dec}}\right)^3 I_g + \left(1 - \left(\frac{M_{cr} - M_{dec}}{M_s - M_{dec}}\right)^3\right) I_{cr} \le I_g \qquad (4)$$

The service-load deflection is given by:

$$\Delta = k \frac{M_s L^2}{E_c I_e} \qquad (5)$$

where $\boldsymbol{k}$ is the coefficient depending on the support and loading condition, $\boldsymbol{L}$ is the effective length of the member, and $\boldsymbol{E_c}$ is the Young's modulus of concrete. The instantaneous deflection due to live load can be obtained by subtracting the deflection due to dead load and effective prestress from the total service-load deflection, respectively.

Method 2

Branson and Trost[2], have suggested a zero deflection point approach for partially prestressed beams, which defines the effective moment of inertia for the cracked beams as follows:

$$(I_e)_{L2} = \left(\frac{M'_{cr}}{M_{L2}}\right)^3 I_g + \left(1 - \left(\frac{M'_{cr}}{M_{L2}}\right)^3\right) I_{cr} \le I_g \quad \textbf{(6)}$$

where

$(I_e)_{L2}$ = effective moment of inertia for moment corresponding to downward (net positive) deflection.

$M'_{cr} = (f\, I_g / y_b) + (P_e I_g / A_g y_b)$ cracking moment or net positive moment to crack the section.

M_{L1} = part of the live load moment corresponding to zero deflection. $M_{L1} = P_e e - M_D$ for uniformly distributed dead load, live load, and equivalent upward prestressed load.

M_{L2} = part of the live load that corresponding to a downward deflection of prestressed member.

The instantaneous live-load deflection is given as:

$$\Delta_L = k\left[\frac{M_{L1}L^2}{E_c I_g} + \frac{M_{L2}L^2}{E_c (I_e)_{L2}}\right] \quad \textbf{(7)}$$

and the total deflection is given as:

$$\Delta = \Delta_{L2} = k\left[\frac{M_{L2}L^2}{E_c (I_e)_{L2}}\right] \quad \textbf{(8)}$$

Method 3

Branson[1] have proposed a simplified expression for $\boldsymbol{I_e}$ to calculate instantaneous live load deflection as:

$$(I_e)_L = \left(\frac{(M_L)_{cr}}{M_L}\right)^3 I_g + \left(1 - \left(\frac{(M_L)_{cr}}{M_L}\right)^3\right) I_{cr} \le I_g \quad \textbf{(9)}$$

Where $\boldsymbol{M_{Lcr}}$ is defined as the live load cracking moment calculated as:

$$(M_L)_{cr} = M'_{cr} - M_D \quad \textbf{(10)}$$

The instantaneous live load deflection is given by:

$$\Delta_L = k\left[\frac{M_L L^2}{E_c (I_e)_L}\right] \quad \textbf{(11)}$$

Method 4

Tadros, *et al.* [12] have observed that there is a shift in the centroid of the cross section upon cracking that results in a larger prestressing force eccentricity $\boldsymbol{e_{cr}}$ than the uncracked member eccentricity. For prestressed flanged members, ignoring the change of eccentricity while calculating the curvature of the member may result in a significant overestimation of the deflection. Hence, the following equation was suggested for calculating the moment of inertia of the section as:

$$I'_e = \left(\frac{M'_{cr} - M_{dec}}{M_s - M_{dec}}\right)^4 I_g + \left(1 - \left(\frac{M'_{cr} - M_{dec}}{M_s - M_{dec}}\right)^4\right) I_{cr} \le I_g \qquad (12)$$

To determine the change in the eccentricity after cracking, the effective centroid distance is modified as:

$$y_e = \left(\frac{M'_{cr} - M_{dec}}{M_s - M_{dec}}\right)^4 y_g + \left(1 - \left(\frac{M'_{cr} - M_{dec}}{M_s - M_{dec}}\right)^4\right) y_{cr} \qquad (13)$$

where y_g and y_{cr} are the centroidal distances of the gross section and the cracked section, respectively, measured from the extreme compression fiber, as shown in Figure (1). The curvature of the cracked section is calculated as:

$$\psi_e = \frac{M_s - P e_{cr}}{E_c I'_e} \qquad (14)$$

The curvatures at key sections along the span can be computed and integrated to obtain the deflection. For a simply supported beam with one-point depressed tendons, the service-load deflection can be given as:

$$\Delta = 5(\psi_c + \psi_{0.4L}) L^2 / 96 \qquad (15)$$

where ψ_c and $\psi_{0.4L}$ are the curvatures of the section at the center and at 0.4L, respectively. The instantaneous deflection due to the live load is obtained by subtracting the deflection due to dead load and the effective prestress from the total service load deflection.

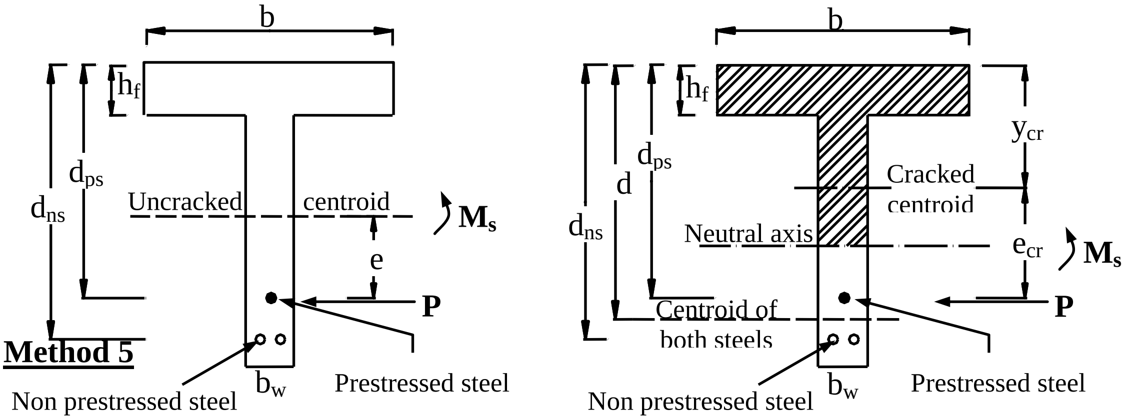


Figure (1): Uncracked and cracked partially prestressed section.

Scholz[6] have been suggested simple rules for the evaluation of deflection of partially prestressed members. The rigorous method was used as shown in Figure (2) as a first level assessment of the deflection. The simplest approach is to add the deflection components of the uncracked and cracked section together, using the corresponding stiffnesses and moments as follows:

$$y_{Rc} = kL^2 \left(\frac{M_{cr}}{EI_g} + \frac{M_L - M_{cr}}{EI_{cr}}\right) \qquad (16)$$

In equation (16) and Figure (2), the bending moment M_{cr} refers to the cracking moment, and the moment (M_L-M_{cr}) refers to the value above the cracking moment: both are measured against the moment corresponding to zero deflection and M_c is the resistance moment based on concrete tensile strength. The latter is often referred to as the balanced moment M_{bal}, for which only axial compression exists throughout the member. If the maximum service load moment M_{max}, of Figure (2) does not cause cracking, only the uncracked stiffness EI_g is used. The deflection coefficient k considers different loading and end conditions, and can be found in many standard textbooks.

Equation (16) can be used for non-prestressed members as well as for prestressed members. The equation can be applied to short and long term loads, including creep (by adjusting the stiffnesses EI accordingly).

The deflection of partially prestressed member y_{pp} can be predicted with reference to the deflection of the non-prestressed, reinforced concrete solution y_{Rc}, as follows:

$$y_{PP} = y_{Rc}\left(1 - \frac{M_{bal}}{M_{max}}\right) \qquad (17)$$

where M_{max} is the total bending moment due to loading at the determined section, and M_{bal} is the balanced moment, both as illustrated in Figure (2). Equation (17) is based on the simple premise that on the one hand the deflection of the reinforced concrete member applies when no prestressing exists, i.e. $M_{bal} = 0$. Furthermore, at the other hand of the spectrum, as the total load is balanced, i.e. ($M_{bal} = M_{max}$)zero deflection was obtained, since only axial stresses exist.

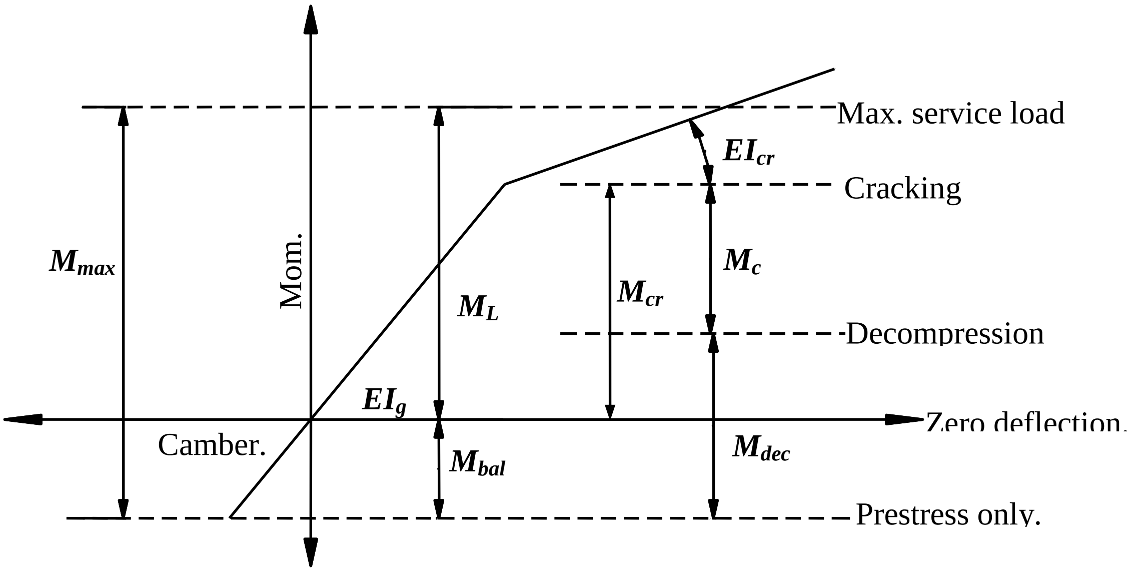


Figure (2): Moment-deflection diagram of method (5)[(6)].

Proposed simplified approach

An approach was suggested to predict the deflection of partially prestressed members by reducing the total service moment by an amount equal to the effective moment (M_e). The effective moment can be calculated as follows:

$$M_e = \left(\frac{M'_{cr}}{M_s}\right)^3 M_{dec} + \left(1 - \left(\frac{M'_{cr}}{M_s}\right)^3\right) M_{bal} \qquad (18)$$

This equation was assumed that the moment level must were between decompression moment level and balanced moment level, as shown in figure (2).

Thus, the moment of inertia of cross section can be calculated as follows:

$$I_e = \left(\frac{M'_{cr} - M_e}{M_s - M_e}\right)^3 I_g + \left(1 - \left(\frac{M'_{cr} - M_e}{M_s - M_e}\right)^3\right) I_{cr} \le I_g \qquad (19)$$

Thus, the service-load deflection is given, as follows:

$$\Delta = k \frac{M_s L^2}{E_c I_e} \qquad (20)$$

where the decompression moment is the moment leading to zero stress at the extreme fiber of the concrete section, at which the tensile stresses are caused by the applied loads. The decompression moment is given by:

$$M_{dec} = P_e e + \frac{P_e I_t}{A_t y_b} \qquad (21)$$

where P_e is the effective prestressing force after the losses; e is the eccentricity, A_t and I_t are the area and the moment of inertia, respectively, of the cracked transformed section. The cracking moment (M'_{cr}) is given as follows:

$$M'_{cr} = \left(f_r I_g / y_b\right) + \left(P_e I_g / A_g y_b\right) \qquad (22)$$

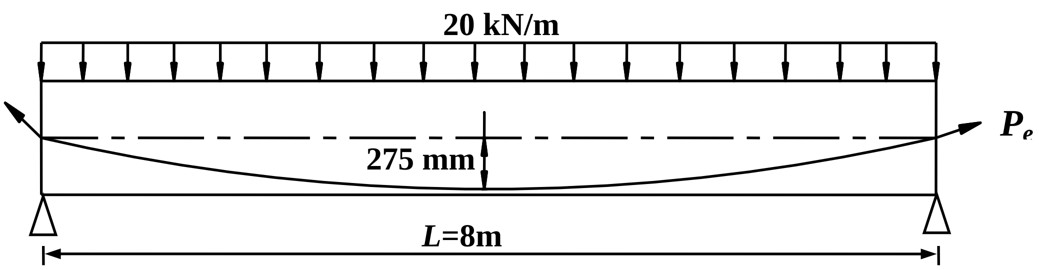
This equation can be used for non-prestressed and partial prestressed concrete members.

Comparisons with the other methods

Table (1) shows the data of simply supported beam, as shown in figure (3), and shows the variation of area of tendons that will cause variation in prestressing force and shows the reduction of area of reinforcing steel.

Table (1): Data of simply supported beam for calculating deflection.

Member	A_s mm^2	A_{ps} mm^2	P_e kN	M_{cr} kN.m	M_{dec} kN.m	M_{bal} kN.m	M_e kN.m
B1	1200	0	0	77.458	0.000	0.00	0.000
B2	1000	100	100	88.290	38.695	27.50	29.381
B3	800	200	150	99.123	57.999	41.25	45.232
B4	600	300	200	109.955	77.268	55.00	62.227
B5	400	400	250	120.787	96.507	68.75	80.692



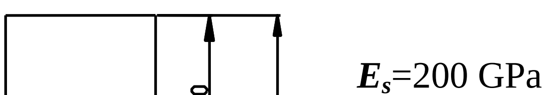


Table (2): Comparison of calculated deflections in (mm).

Member	Method 1	Method 2	Method 3	Method 4	Method 5	Proposed method
B1	13.788	13.788	13.788	10.893	11.543	13.788
B2	11.137	11.675	13.023	10.454	10.090	11.517
B3	10.624	10.654	11.989	9.587	9.644	10.070
B4	9.583	8.965	10.745	8.769	9.168	8.775
B5	8.091	7.187	9.412	8.04	8.664	7.700

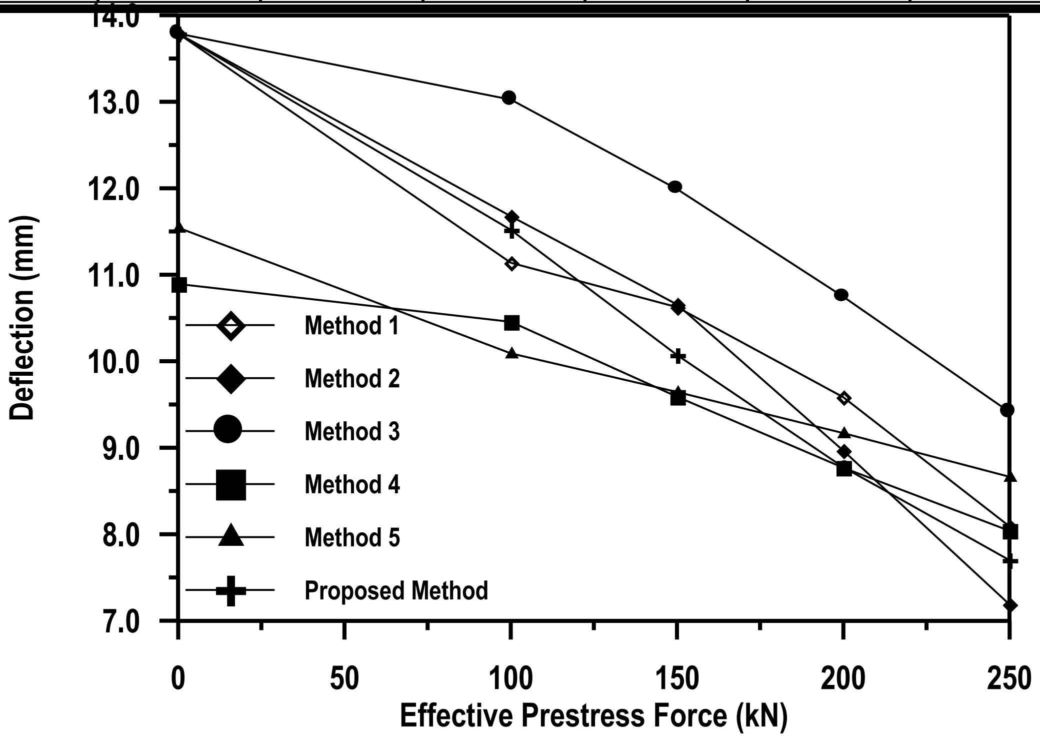


Figure (4): Deflection-effective prestress force curve.

From figure (4), can be noticed that for B1 method 1,2,3 and present study were giving rather equal results while method 4 was giving different result because this method used Branson method of fourth degree and taken the average deflection of two points at (0.5L, 0.4L) of beam. In the Method 5, the using of the rigorous method instead of using ***I***-effective method caused this difference. The present results of the proposed method for the other beams were lying in between those of the other methods.

Conclusion

Five different methods were reviewed with its results in predicting the instantaneous deflection of partially prestressed concrete members. A new simplified proposed method to predict instantaneous deflection of partially prestressed concrete member was presented. From this study, can be noticed that the proposed method gives in between those of the other methods. This method considered the level of moment between two to be levels (the decompression moment and the balance moment). Also this method can be used to reinforced concrete, partial prestressed concrete, and prestressed concrete.

References

1. **Branson**, **D. E.**, "Design Procedures for Computing Deflections." ACI Journal, Proc. V.65, Sept. 1968, pp. 730-742.
2. **Branson**, **D.E.**, and **Trost**, **H.**, "Application of I-Effective Method in calculating Deflection of Partially Prestressed Concrete Members." PCI journal, V.27, No.5, Sept.,1982, pp.62-77.
3. **Bennett**, **E.W.**, Discussion of "design of Partially Prestressed Concrete Flexural Members." By S.E. Moustafa, PCI journal, V.23, No.2, May June 1978, pp 92-94. Cited in [10].
4. **Chen**, **C. I.**, "Integrated Procedures for Predicting the Initial and Time-Dependent Deformation of Reinforced Concrete Sections." Ph.D. thesis, University of Lowa, 1973, 457 pp. Cited in [10].
5. **Nielsen**, **T.B.**, "Serviceability Limit State Analysis of Cracked, Partially Prestressed, or Reinforced Concrete Sections under Symmetric Bending." ACI journal, V.83, July, 1986, pp. 547-552.
6. **Nilson**, **A. H.**, "Design of Prestressed Concrete." 2nd Edition, John Wiley and Sons, Inc., 1987.
7. **Scholz**, **H.**, "Simple Deflection and Cracking Rules for Partially Prestressed Members." ACI journal, V.88, No. 2, March, 1991, pp. 199-203.
8. **Raju**, **N.**, **K.**, "Prestressed Concrete." 3rd Edition, Tata, McGraw-Hill Publishing Company Limited, New Delhi, 1997.
9. **Rao**, **A. S.**, "Selection and Design of Partially Prestressed Concrete Sections for Strength and Serviceability." ACI journal, V.88, No.3, May, 1991, pp.330-339.
10. **Rao**, **S.V.**, and **Dilger**, **W. H.**, "Evaluation of Short-Term Deflections of Partially Prestressed Concrete Members." ACI journal, V.89, No.1, Jan., 1992, pp.71-78.
11. **Tadros**, **M. K.**, "Expedient Service Load Analysis of Cracked Prestressed Concrete Sections." PCI journal, V.27, No.6, Nov., 1982, pp.86-111.
12. **Tadros**, **M. K.**, **Ghali**, **A.**, and **Meyer**, **A. W.**, "Prestress Loss and Deflection of Precast Concrete Members." PCI journal, V. 30, No.1, Jan., 1985, pp. 114-141.
13. **Teng**, **S.**, and **Branson, D**. **E**., "Initial and Time-dependent Deformation of Progressively cracking Nonprestressed and Partially Prestressed Concrete Beams." ACI journal, V.90, No. 5, Sept., 1993, pp. 480-488.

الخلاصة

قُدم عرض لخمسة طرق مختلفة لتقدير الهطول اللحظي للأعضاء الكونكريتية مسبقة الاجهاد الجزئي. قُدمت طريقة مقترحة بسيطة وجديدة لتقدير الهطول اللحظي للأعضاء الكونكريتية مسبقة الاجهاد الجزئي. نتائج الطريقة المقترحة كانت محافظة مع نتائج الدراسات الاخرى.