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Pre-test Shrinkage Estimation for Reliability Function of Burr XII Distribution Using Progressive Type II Censored Sample under Precautionary Loss Function (PLF)

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Abstract

This article deal with the proposal of suggest and study of the properties of pre-test shrinkage estimators of Reliability Function for the Burr XII distribution using Progressive Type II censored sample. Since some difficulties to derive equations of risk function for proposed shrinkage estimators of reliability function under Precautionary Loss Function (PLF), we to study properties by using Monte-Carlo simulation. The numerical and Monte-Carlo simulations show that the performance of the proposed estimators is better than classical estimators in terms of relative risk.

Keywords: Burr XII Distribution, Shrinkage Estimator, Precautionary Loss Function, Reliability Function, Risk Function, Relative Risk, Progressive Type II Censored Sample.

1. INTRODUCTION

The Shrinkage estimators were proposed by numerous scholars who were interested to look for estimators with a high relative risk with compared to the classical estimators. One of The first researchers to propose the shrinkage estimator was Thompson (1968)[8] when the initial information exists for an unknown parameter θ as a guess value θ_0 then we must use it. so Thompson(1968) proposed the shrinkage estimators moving the classical estimator $\hat{\theta}$ to guess θ_0 by using weighted shrinkage factor k .The shrinkage estimators defined as:

$$\tilde{\theta}_{sh} = k\hat{\theta} + (1-k)\theta_0 \qquad ,0 < k < 1 \qquad (1)$$

They researchers can n′t conform the real value of θ is closed to θ_0 Consequently, they proposed a preliminary test of hypothesis $H_0: \theta = \theta_0$ against hypothesis $H_1:\theta \neq \theta_0$ to ascertain how close θ_0 and θ, in order to ensure if the hypothesis $H_0:\theta = \theta_0$, Accept the estimator is $\tilde{\theta}_{sh} = k\hat{\theta} + (1-k)\theta_0$ otherwise the classical estimator.

The pre-test shrinkage estimator can be defined as:

$$\tilde{\theta}_{sh} = \begin{cases} k\hat{\theta} + (1-k)\theta_0 & If\ H_0: \theta = \theta_0\ is\ accepted \\ \hat{\theta} & otherwise \end{cases} \qquad (2)$$

The shrinkage estimator above studied by many Authors for example Prakash and Singh (2008)[5], Naghizadeh Qomi and Barmoodeh (2015)[9], Hossain and Howlader(2016)[17].

The Burr XII distribution, which was first suggested by Burr in (1942)[6]. Is a non-negative random variable′s continuous probability distribution. Sometimes referred to as the generalized log-logistic distribution (Burr,1942), and is one of several distributions with probability density function.

$$f(x,\theta,\beta) = \theta\beta \frac{x^{\beta-1}}{(1+x^{\beta})^{\theta+1}} \qquad ,x > 0\,,\theta,\beta > 0 \qquad (3)$$

and the accompanying cumulative distribution function

$$F(x,\theta,\beta) = 1-(1+x^{\beta})^{-\theta} \qquad ,x > 0\,,\theta,\beta > 0 \qquad (4)$$

The Burr XII distribution is a Computational model for failure times that is straightforward to apply and versatile. The Burr XII distributions characteristics are utilized in family income modeling, quality control, economics, and duration of failure time modeling It is comparable to the log-normal distribution as well. Additionally, due to its non-monotone failure rate, it bears resemblance to the log-normal distribution, a widely used model in life and reliability testing. To lower the probability of failure the Burr XII distribution is being used more and more in the areas of lifetime data analysis and actuarial science. Gomes et al.(2015)[1] proposed the McDonald Burr XII distribution Gunasekera (2018)[16] proposed the reliability function of Burr XII distribution by the concept of generalized variable method progressive type II right censored sample with random removals . Hassan et al. (2020)[11] developed a generalized Bayesian shrinkage estimator of Burr XII distribution parameters under various loss functions.

The progressive type II right censored samples is One of censoring technique that is widely used in clinical studies, product quality control, industrial experiments, reliability testing, and life testing. The progressive type II right censored sample is explained as follows in[Balakrishnan and Aggarwala (2000)][10]. Following the observation of, R_1 units are chosen at random and eliminated after the first failure ; similarly, R_2 units are chosen at random and eliminated following the observation of the second failure; and R_i units are chosen at random and eliminated following the observation of the ith failure.(i= 3, 4,...m). When the mth failure, is detected and the remaining $R_m = n - m - \sum_{i=1}^{m-1} R_i$ units are eliminated the experiment comes to an end.

Suppose $x_{1:m:n}, x_{2:m:n} \dots, x_{m:m:n}$ be a random progressive form the Burr XII distribution. The common function of the progressive censored sample $x_{1:m:n}, x_{2:m:n} \dots, x_{m:m:n}$ and expressed as:

$$f(x_{1:m:n}, \dots, x_{m:m:n}) = C \prod_{i=1}^{m} f(x_{i:m:n)} [1 - F(x_{i:m:n})]^{R_i} \quad (5)$$

where $C = n(n - R_1 - 1)(n - R_1 - R_2 - 2) \dots (n - R_1 - \cdots R_{m-1} - m + 1)$.

Many researchers studied progressive type II right censored samples Abu-Awwad et al.(2015) [14] , Al-Hussaini et al.(2015)[4], Qin and Gui (2020)[18], and Bantan et al. (2021)[12] .

the reliability function is the probability in which a device or system will operate up to determined time without failure. It is defined mathematically as

$$R(x;\theta) = 1 - F(x;\theta)$$

$$= P(X > x)$$

The reliability function of the Burr XII distribution is given by

$$= (1 + x^{\beta})^{-\theta} \qquad ; x > 0, \beta, \theta > 0 \qquad (6)$$

The Precautionary Loss Function is one Type of asymmetric Loss Function that was proposed by (N orstrom in (1996)[7]As a specific instance the general Loss of the Precautionary loss function was described. Norstrom (1996) introduce a class of precautionary loss functions of the form

$$L(\theta,\hat{\theta}) = w(\theta)\frac{(\hat{\theta}-\theta)^2}{\hat{\theta}^a} \qquad 0 \le a \le 2\,, w(\theta) > 0 \qquad (7)$$

where a is a precautionary index. For the case a = 1 and w(θ) = 1/θ in (7), we get the following asymmetric scale invariant loss function

$$L(\theta,\hat{\theta}) = \left(\sqrt{\frac{\hat{\theta}}{\theta}} - \sqrt{\frac{\theta}{\hat{\theta}}}\right)^2 = \frac{\hat{\theta}}{\theta} + \frac{\theta}{\hat{\theta}} - 2 \qquad (8)$$

Many researchers studied loss function are Karimnezhad et al. (2014)[3] , Chen and Liu (2019) [19] , Rao and Pandey (2021)[2]

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2. Proposed Pre-test Shrinkage Estimators

In this section, we use the guess value θ_0 as prior information about an unknown parameter θ, we will consider it existing, and depending on the density function, one can propose the following pre-test estimator. Where C is pre-test region for testing the null hypothesis $H_0: \theta = \theta_0$ against the alternative hypothesis $H_1:\theta \neq \theta_0$ with significance of level α.

The first proposed estimator $\tilde{R}_{sh1}$ is defined as:

$$\tilde{R}_{sh1} = \begin{cases} k_1\hat{R} + (1-k_1)R_0 & If\ H_0: \theta = \theta_0 \ is\ accepted \\ \hat{R} & otherwise \end{cases} \qquad (9)$$

Where the shrinkage factor k_1 is a constant such that $k_1 \epsilon [0,1]$, since $\hat{R}$ given by equation (6) and Let $\hat{R}(x) = (1 + x^{\beta})^{-\hat{\theta}}$, $R_0 = (1 + x^{\beta})^{-\theta_0}$. Let C be a pre-test region for test the null hypothesis H_0:θ=θ_0 against the alternative hypothesis H_1:θ≠ θ_0 at the significance of level α.

The second proposed estimator $\tilde{R}_{sh2}$ is defined as:

$$\tilde{R}_{sh2} = \begin{cases} k_2\hat{R} + (1-k_2)R_0 & If\ H_0: \theta = \theta_0 \ is\ accepted \\ \hat{R} & otherwise \end{cases} \qquad (10)$$

where $k_2 = (1 - p(H_0 \text{ Accepted}))^2$ thus

$$k_2 = (1 - [I(\frac{r_2}{2\lambda}, m) - (I\left(\frac{r_1}{2\lambda}, m\right))^2$$

where $I(t, m) = \frac{\int_0^t x^{m-1} \exp(-x) dx}{\Gamma m}$

The third proposed estimator $\tilde{R}_{sh3}$ is defined as:

$$\tilde{R}_{sh3} = \begin{cases} k_3\hat{R} + (1-k_3)R_0 & \text{If } H_0: \theta = \theta_0 \text{ is accepted} \\ \hat{R} & \text{otherwise} \end{cases} \quad (11)$$

where $k_3 = \frac{2m\theta_0}{\hat{\theta}\,(r_1+r_2)}$

The fourth proposed estimator $\tilde{R}_{sh4}$ is defined as:

$$\tilde{R}_{sh4} = \begin{cases} k_4\hat{R} + (1-k_4)R_0 & \text{If } H_0: \theta = \theta_0 \text{ is accepted} \\ \hat{R} & \text{otherwise} \end{cases} \quad (12)$$

where $k_4 = (\frac{2m\theta_0}{\hat{\theta}\,(r_1+r_2)})^2$

2.1 Simulation Concept

Simulation method can be understood as a representation or imitation of real reality, using certain methods, and models. One of the most prominent features of the simulation is to obtain very useful information about the real reality that it imitates, as well as the ability to repeat the experiment. The inputs that are changed each time a sufficient and appropriate explanation the nature of the mathematical sciences that were used.

2.2 Monte Carlo Method

The Monte Carlo method, also known as Monte Carlo experiments, is a general class of computational methods that provide numerical results by repeatedly sampling a given population at random. The basic idea is to employ randomness to solve problems that, in theory, may be deterministic. They come in handy most of the time when other methods are impractical or impossible to apply, and they are frequently employed in mathematical and physical difficulties. Three issue classes optimization, numerical integration, and drawing from a probability distribution are the primary applications for Monte Carlo methods, Harrison (2010),[13] Rubinstein and Kroese (2016)[15].

2.3 Steps of a Simulation Experiment

We can now assuming that are able to generate pseudo-random Uniform(0,1) variables, efficiently generate a progressively Type II right censored sample from Burr XII distribution using the following simple algorithm:

Step 1 Generate m independent Uniform(0,1) observations $W_1, W_2, \ldots, W_m$.

Step 2 Set $V_i = W_i^{1/(i+\sum_{j=m-i+1}^{m} R_j)}$ for $i = 1,2, \ldots, m$.

Step 3 $U_{i:m:n} = 1 - V_m V_{m-1} \ldots V_{m-i+1}$ for $i = 1, 2, \ldots, m$. Then $U_{1:m:n}, U_{2:m:n}, \ldots, U_{m:m:n}$ is the required progressively Type II right censored sample form the Uniform(0,1) distribution.

Step 4 Finally, we set $X_{i:m:n} = X_i = F^{-1}(U_i) = -((1-U_i)^{\frac{-1}{\theta}}) - 1)^{\frac{1}{\beta}}$, for $i = 1, 2, \ldots, m$, where $-((1-U_{i:m:n})^{\frac{-1}{\theta}})^{-1})^{\frac{1}{\beta}}$ is the inverse cumulative distribution function of the Burr XII distribution under consideration. Then $X_{1::m:n}, X_{2:m:n}, \ldots, X_{m:m:n}$ is the required progressively Type II right censored sample form the distribution F(.).

The following progressively type II right censored sample from the Burr XII was simulated using the above steps with n = 12, m=5 , R_i=(2,1,1,1,2) and with n=24, m=10, R_i=(2,1,1,1,2,2,1,1,1,2) and n=36, m=15 , R_i=(2,1,1,1,2,2,1,1,1,2.2,1,1,1,2) are considered.

The above simulational algorithm requires exactly m pseudo random uniform observations and does not require any sorting.

3. Relative Risk

To study the properties of estimators $\tilde{R}_{sh1}$, $\tilde{R}_{sh2}$, $\tilde{R}_{sh3}$ and $\tilde{R}_{sh4}$, we comparison were made with the relative risk under Precautionary Loss Function (PLF) of the estimators given above with respect to the classical estimator $\hat{R}$ for this purpose.

Therefore one can evaluate the relative risks with respect to the classical estimator $\hat{R}$ of proposed pre-test shrinkage estimator $\tilde{R}_{sh}$ denoted by R.R(.) of $\tilde{R}_{sh1}$, $\tilde{R}_{sh2}$, $\tilde{R}_{sh3}$ and $\tilde{R}_{sh4}$ under Precautionary Loss Function (PLF) Now, we define the relative risk for estimator $\tilde{R}_{sh1}$ under Precautionary Loss Function as :

We define the relative risk of the estimators $\tilde{R}_{sh1}$ is given by:

$$R_1.R(\tilde{R}_{sh1}|\,PLF) = \frac{R(\hat{R}|PLF)}{R(\tilde{R}_{sh1}|PLF)} \tag{13}$$

Similarly, we define the relative risk for estimators $\tilde{R}_{sh2}, \tilde{R}_{sh3}$ and $\tilde{R}_{sh4}$ as:

The relative risk of $\tilde{R}_{sh2}$ is given by

$$R_2.R(\tilde{R}_{sh2}|\,PLF) = \frac{R(\hat{R}|PLF)}{R(\tilde{R}_{sh2}|PLF)} \tag{14}$$

Further, The relative risk of $\tilde{R}_{sh3}$ is given by

$$R_3.R(\tilde{R}_{sh3}|\,PLF) = \frac{R(\hat{R}|PLF)}{R(\tilde{R}_{sh3}|PLF)} \tag{15}$$

Table 1. Relative Risk of the Estimator $\tilde{R}_{sh1}$ under Precautionary Loos Function at k_1

K_1	α	m	λ								
			0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8
0.1	0.01	5	0.3080506	0.6078613	1.4186341	3.844372	10.40956	5.35672	2.2875485	1.24566	0.8727968
		10	0.8493754	0.3858151	0.6904445	2.237275	7.690305	2.826587	0.9943714	0.5529311	0.3864114
		15	1	0.4225036	0.4807028	1.555065	8.723496	1.919751	0.6796732	0.3768683	0.3057808
	0.05	5	0.5859369	0.5931308	1.0191649	1.988382	3.385475	2.615965	1.8250075	1.2013097	0.9125095
		10	0.997551	0.5722351	0.6559814	1.397797	3.193781	1.998666	1.0106673	0.6383616	0.5090009
		15	1	0.7522824	0.5403125	1.13347	3.210026	1.562451	0.7023891	0.5032879	0.4566612
0.2	0.01	5	0.3429769	0.6933839	1.6218662	3.967327	8.148565	4.717342	2.3062487	1.3304938	0.9558425
		10	0.8681157	0.4310283	0.7890705	2.446153	6.404835	2.803301	1.0942316	0.6238375	0.4399109
		15	1	0.4636484	0.5479645	1.732081	7.078799	2.020006	0.7639717	0.4304629	0.3483673
	0.05	5	0.6225975	0.6447379	1.0935407	2.024395	3.1553	2.479652	1.8174908	1.2453545	0.9678542
		10	0.9978741	0.6112308	0.712517	1.468783	2.993619	1.985172	1.0700979	0.6942885	0.5575297
		15	1	0.7793024	0.5885951	1.208463	3.007887	1.602159	0.7628212	0.552673	0.4997931

Finally, The relative risk of $\tilde{R}_{sh4}$ is given by

$$R_4.R(\tilde{R}_{sh4}|\,PLF) = \frac{R(\hat{R}|PLF)}{R(\tilde{R}_{sh4}|PLF)} \tag{16}$$

We observe that equations of the relative risk of our proposed estimators with respect to the classical estimator $\hat{\theta}$ and the equations of the risk function depend on $k_1, m\ and\ \alpha$. To study these Equations numerically we assume the following values in Equations (13),(14),(15) and (16)

$k = 0.1, 0.2, m = 5,10,15, \alpha = 0.01, 0.05, \lambda = 0.2,0.4,0.6,0.8,1,1.2,1.4,1.6,1.8$

The results are shown in figures(1)-(10).

Table 2. Relative Risk of the Estimator $\widetilde{R}_{sh2}$ under Precautionary Loos Function at k_2

α	m	λ								
		0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8
0.01	5	0.4298314	0.5473914	1.2258797	3.470135	11.50022	5.630083	2.1892873	1.1574952	0.8141401
	10	0.997038	0.4327067	0.6113448	1.970078	8.25232	2.71633	0.9014377	0.5141693	0.3904422
	15	1	0.6488403	0.4413872	1.365697	9.463593	1.766584	0.616591	0.3717762	0.3604758
0.05	5	0.907585	0.6165803	0.9561709	1.903565	3.479126	2.678465	1.8036083	1.1780556	0.9180465
	10	0.9999981	0.7803975	0.644905	1.313676	3.270463	1.967161	0.9794595	0.6618811	0.5960619
	15	1	0.9633518	0.5805169	1.060372	3.285968	1.505605	0.6965246	0.5784762	0.6279701

Table 3 . Relative Risk of the Estimator $\widetilde{R}_{sh3}$ under Precautionary Loos Function at k_3

α	m	λ								
		0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8
0.01	5	0.3088971	0.6374202	1.5452509	3.667252	6.558198	4.612595	2.823345	1.8861623	1.51669
	10	0.862324	0.4291077	0.8220971	2.396524	4.280087	2.725952	1.539526	1.0549877	0.8334932
	15	1	0.4781374	0.5969808	1.816505	4.007911	2.155948	1.190096	0.8000706	0.6825733
0.05	5	0.6075575	0.6386572	1.0982315	1.934251	2.811288	2.427833	2.076044	1.5975535	1.3675346
	10	0.9979637	0.6326925	0.7576396	1.465951	2.413951	1.979857	1.368905	1.0221801	0.8690524
	15	1	0.8034884	0.6461089	1.252768	2.315017	1.70254	1.065962	0.8498354	0.7701218

Table 4 . Relative Risk of the Estimator $\widetilde{R}_{sh4}$ under Precautionary Loos Function at k_4

α	m	λ								
		0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8
0.01	5	0.2815836	0.5482513	1.2719207	3.414357	9.278812	6.250441	3.0793496	1.7978421	1.3493219
	10	0.8358918	0.3613687	0.6482075	2.063798	6.48448	3.440415	1.4512056	0.875558	0.6471692
	15	1	0.4053387	0.4623591	1.494871	6.795229	2.473532	1.0243536	0.611172	0.5073844
0.05	5	0.5604428	0.5625401	0.9733754	1.880588	3.249086	2.828476	2.2532373	1.5861943	1.2821677
	10	0.9973922	0.5589029	0.6430091	1.353055	2.966089	2.289867	1.3243967	0.8888427	0.7252517
	15	1	0.7478045	0.5383388	1.11858	2.939395	1.85517	0.9403461	0.7005144	0.6277306

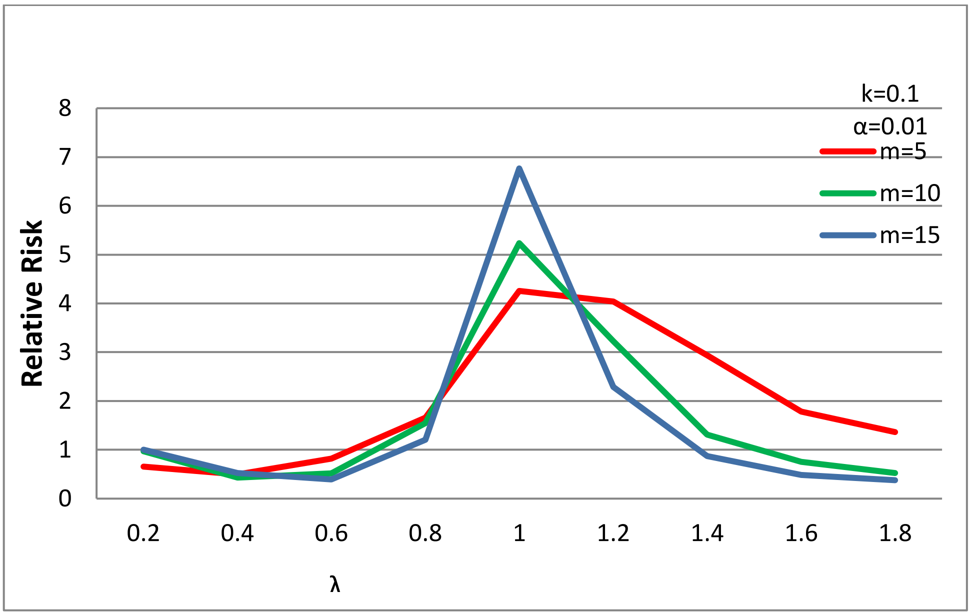

Figure 1. Relative Risk of the estimator $\widetilde{R}_{sh1}$ under (PLF) when, n=12, m=5, $R_i = (0,0,0,0,7)$

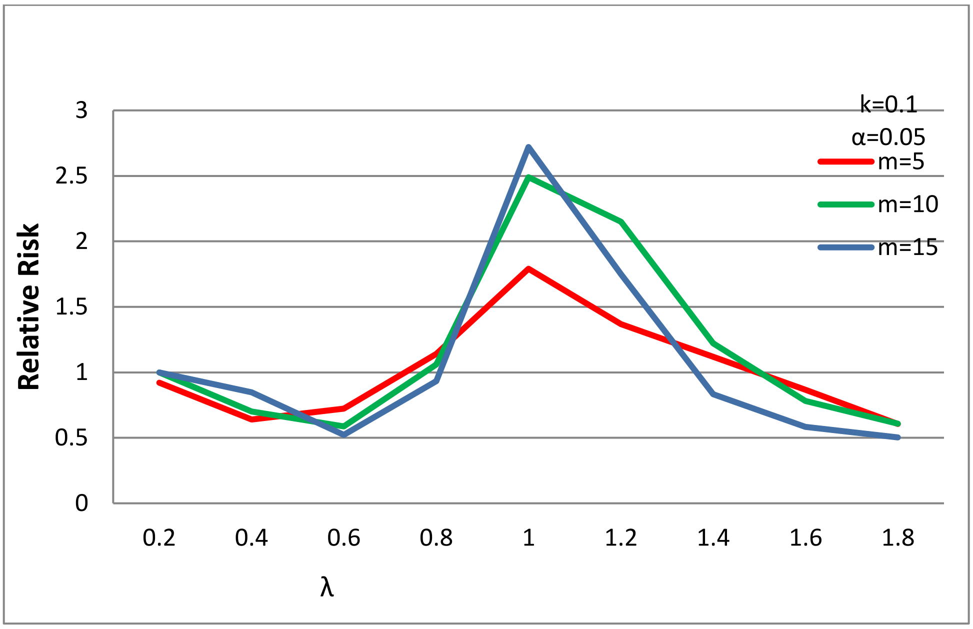

Figure 2. Relative Risk of the estimator $\tilde{R}_{sh1}$ under (PLF) when, n=12, m=5, $R_i = (0,0,0,0,7)$

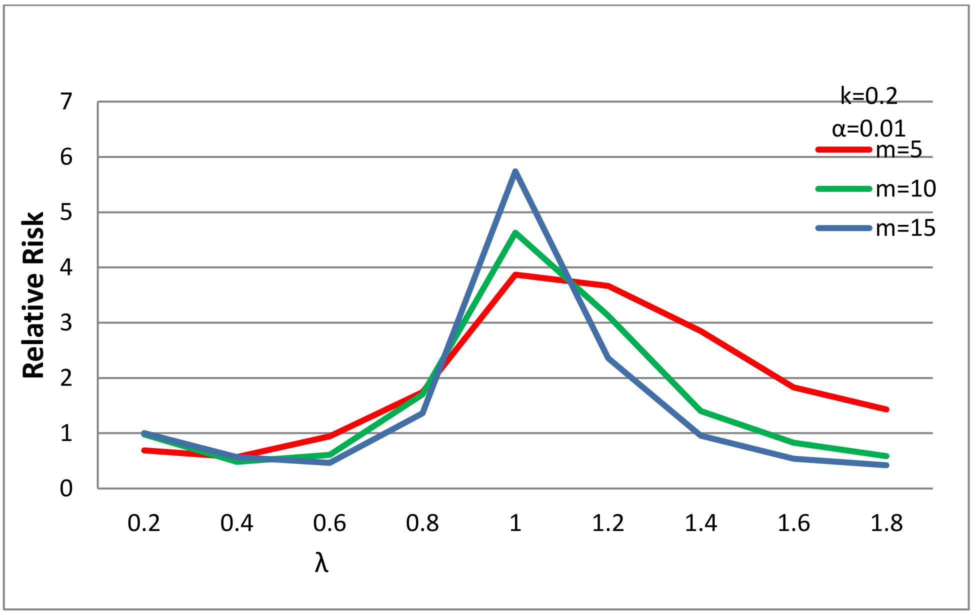

Figure 3. Relative Risk of the estimator $\tilde{R}_{sh1}$ under (PLF) when, n=12, m=5, $R_i = (0,0,0,0,7)$

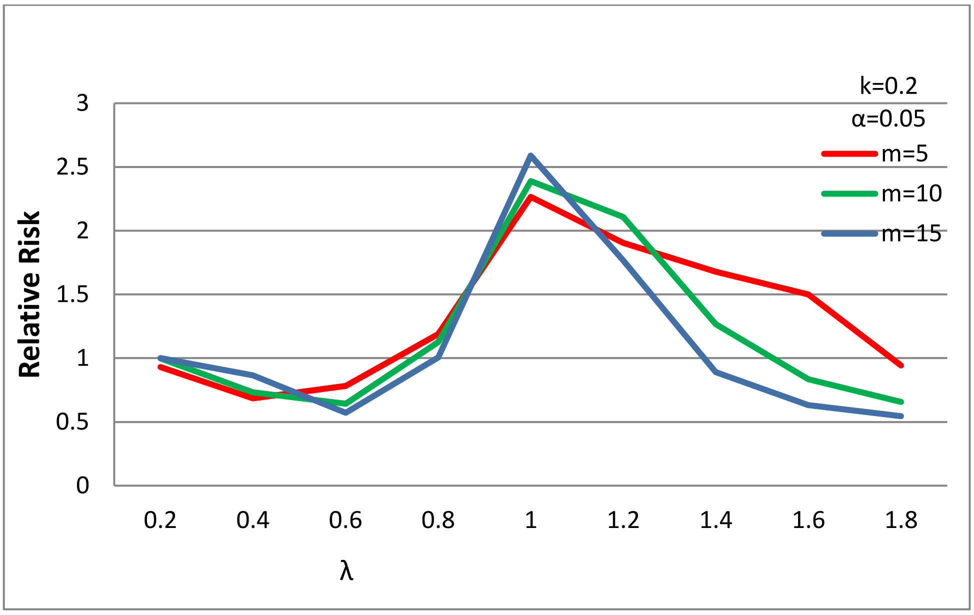

Figure 4. Relative Risk of the estimator $\tilde{R}_{sh1}$ under (PLF) when, n=12, m=5, $R_i = (0,0,0,0,7)$

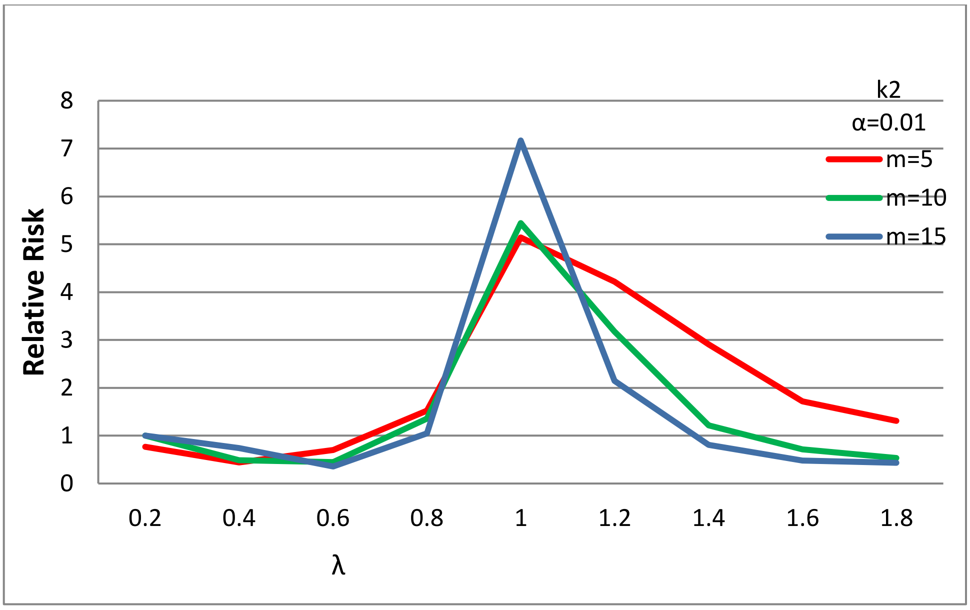

Figure 5. Relative Risk of the estimator $\tilde{R}_{sh2}$ under (PLF) when, n=12, m=5, $R_i = (0,0,0,0,7)$

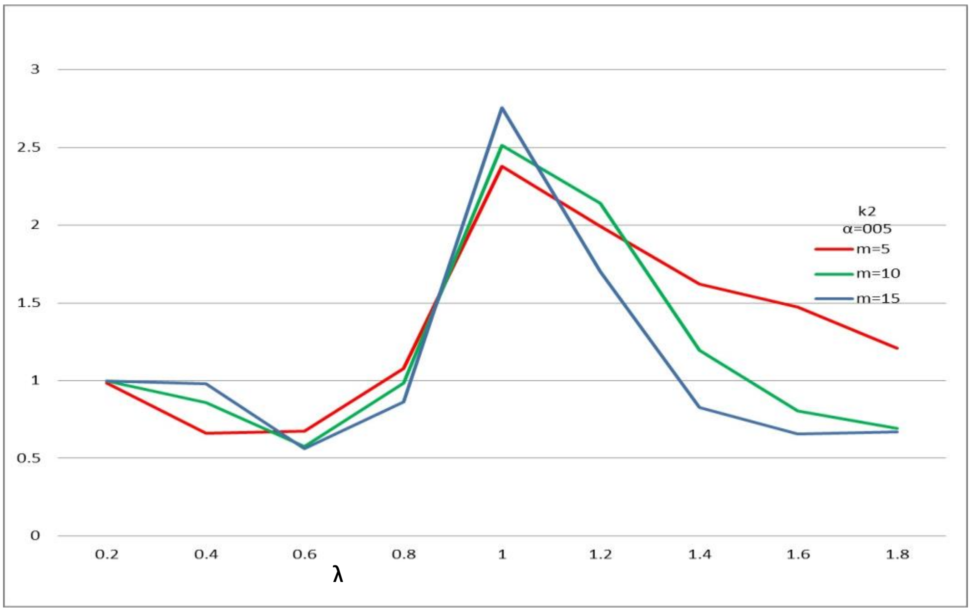

Figure 6. Relative Risk of the estimator $\tilde{R}_{sh2}$ under (PLF) when, n=12, m=5, $R_i = (0,0,0,0,7)$

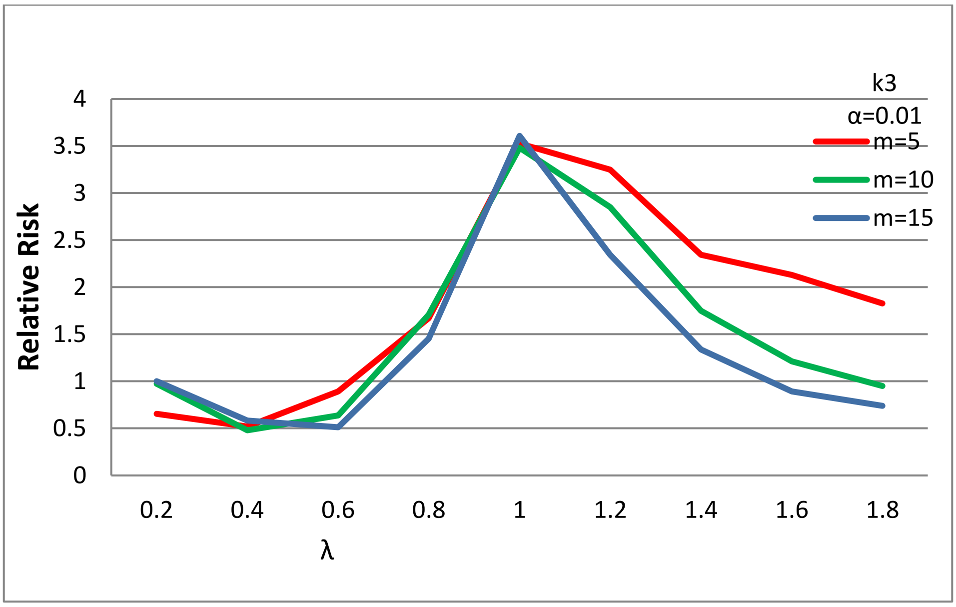

Figure 7. Relative Risk of the estimator $\tilde{R}_{sh3}$ under (PLF) when, n=12, m=5, $R_i = (0,0,0,0,7)$

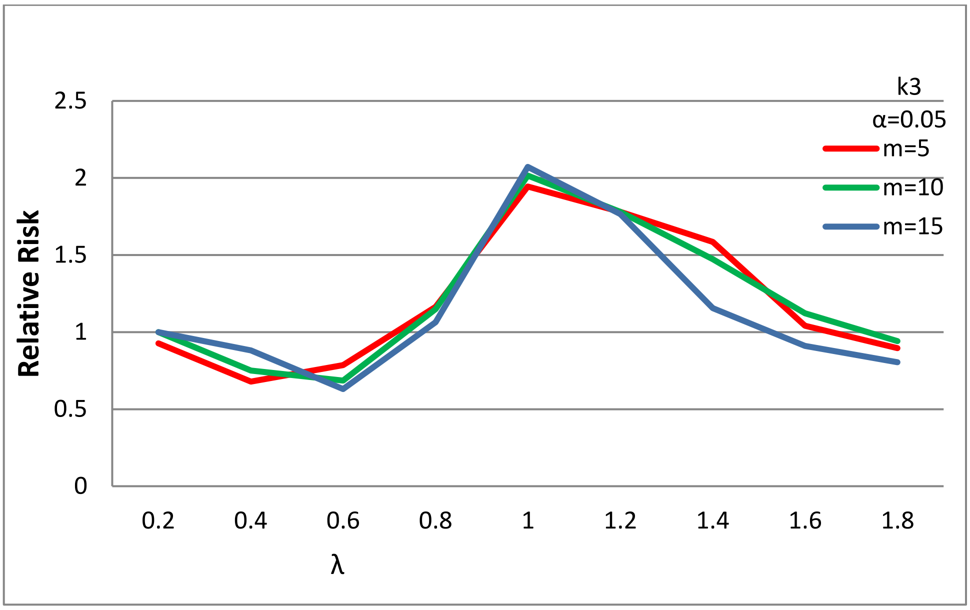

Figure 8. Relative Risk of the estimator $\tilde{R}_{sh3}$ under (PLF)) when, n=12, m=5, $R_i = (0,0,0,0,7)$

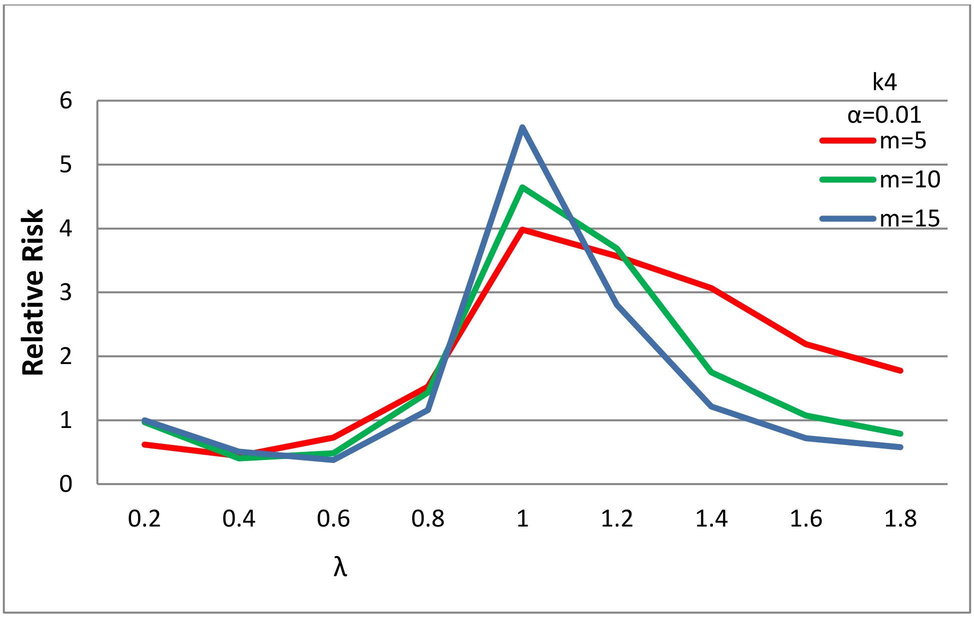

Figure 9 . Relative Risk of the estimator $\tilde{R}_{sh4}$ under (PLF) when, n=12, m=5, $R_i = (0,0,0,0,7)$

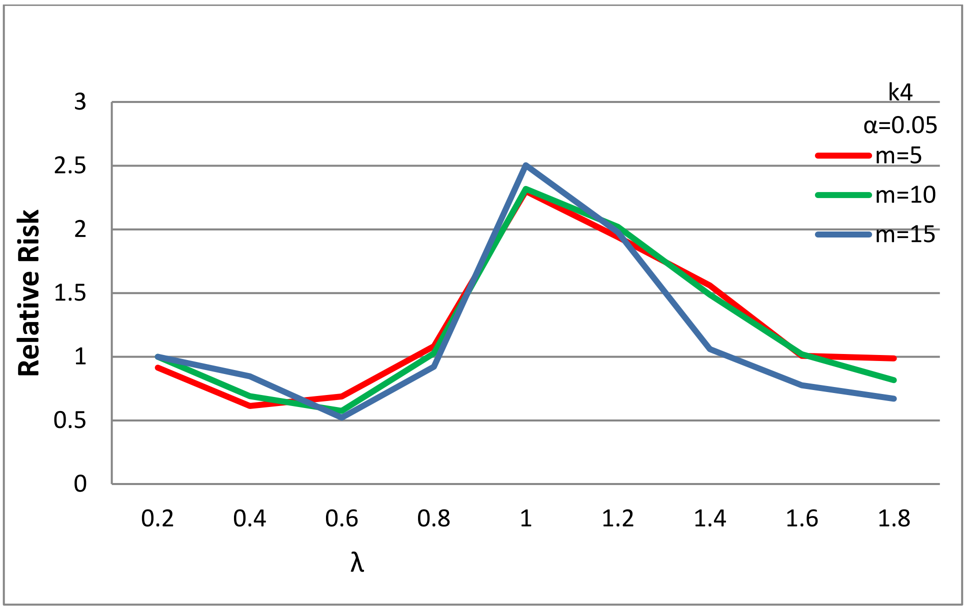

Figure 10 . Relative Risk of the estimator $\tilde{R}_{sh4}$ under (PLF) when, n=12, m=5, $R_i = (0,0,0,0,7)$

4. Conclusions

In our simulation study the process have been repeated 10000 time we generated samples of m= 5, 10, 15 from Burr XII distribution . the result were summarized tabulated in the following tables and figures for each estimator and for all sample.

i. The shrinkage proposed estimators $\tilde{R}_{sh1}$, $\tilde{R}_{sh2}$, $\tilde{R}_{sh3}$ and $\tilde{R}_{sh4}$ give high relative risk under PLF Concerning the classical estimator $\hat{R}$ in the neighborhood $\theta=\theta_0$ i.e. $\lambda \approx 1$ and it decreases when vaiues are away from λ=1. It can be noted that the suggested estimators perform better than classical estimator.

ii. We conclude that from figure (2), (3) and table (1) the relative risk of the estimator $\tilde{R}_{sh1}$, under PLF Concerning the classical estimator $\hat{R}$ is decreasing function of k when $(0.8 < \lambda < 1.4)$ also the relative risk of the estimator above under PLF is increasing function of k when $(\lambda < 0.6)$ and when $(1.4 < \lambda)$.

iii. the relative risk of the estimator $\tilde{R}_{sh1}$, under PLF Concerning the classical estimator $\hat{R}$ when $(0.6 < \lambda < 1)$ and when $(\lambda > 1.4)$ depend on figure(1) and table (1), and and for estimator $\tilde{R}_{sh2}$ under PLF Concerning the classical estimator $\hat{R}$ when $(0.6 < \lambda < 1)$ and $(1 < \lambda < 1.8)$depend on figure (5) and table (2) and for estimator $\tilde{R}_{sh3}$ under PLF Concerning the classical estimator $\hat{R}$ when ($\lambda >$ 0.6) depend on figure (7) and table (3) and for estimator $\tilde{R}_{sh4}$ under PLF Concerning the classical estimator $\hat{R}$ when $(0.6 < \lambda < 1)$ and$(1 < \lambda < 1.8)$ when depend on figure (9) and table (4) are decreasing function of m. but the relative risk of the estimator $\tilde{R}_{sh1}$ and for estimators $\tilde{R}_{sh2}$ under PLF Concerning the classical estimator $\hat{R}$ when $(\lambda < 0.6)$and$(\lambda \geq 1)$ depend on figure (1), (5) and table (1),(2) and for estimators $\tilde{R}_{sh3}$ under PLF Concerning the classical estimator $\hat{R}$ when(λ < 0.6) depend a on figures(7) and table (3) and for estimator $\tilde{R}_{sh4}$ under PLF Concerning the classical estimator $\hat{R}$ when (λ< 0.6) and $(\lambda \geq 1)$ depend on figure (9) and table (4) are increasing function of m.

iv. The estimators′ relative risk $\tilde{R}_{sh1}$, under PLF Concerning the classical estimator $\hat{R}$ when (0.8< $\lambda <$ 1.2) depend on figure(3),(4) and table (1) and for estimator $\tilde{R}_{sh2}$ under PLF Concerning the classical estimator $\hat{R}$ when $(0.6 < \lambda < 1)$ and $(1 < \lambda < 1.2)$depend on figure (5),(6) and table (2) and for estimator $\tilde{R}_{sh3}$ under Concerning the classical estimator $\hat{R}$ when (λ$0.6 < \lambda < 1) and (1 < \lambda < 1.8)$ depend on figure (7), (8) and table (3) and for estimator $\tilde{R}_{sh4}$ under PLF Concerning the classical estimator $\hat{R}$ when $(0.6 < \lambda < 1)$ and $(1 < \lambda < 1.4)$ depend on figure (10) and table (4) are decreasing function of α. but the relative risk of the estimator $\tilde{R}_{sh1}$ and for estimators $\tilde{R}_{sh2}$ under PLF Concerning the classical estimator $\hat{R}$ when $(\lambda < 0.6)$and$(\lambda \geq 1.4)$ depend on figure(3),(4), (6) and table (1), (2) and for estimators $\tilde{R}_{sh3}$ under PLF Concerning the classical estimator $\hat{R}$ when(λ < 0.6) and $(\lambda > 1.6)$ depend a on figures(8) and table (3) and for estimator $\tilde{R}_{sh4}$ under PLF Concerning the classical estimator $\hat{R}$ when (λ< 0.6) and $(\lambda \geq 1.6)$ depend on figure (10) and table (4) are increasing function of α.

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