MODULUS OF DISPLACEMENT THEORY FOR ANALYSIS BEHAVIOR OF REINFORCEMENT LAP SPLICES

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Abstract

In reinforced concrete construction, discontinuity of reinforcing bars is often encountered even within length of a single member. Lap splices are preferred means for providing continuity of reinforcing bars because of their practical and economical characteristics. Hence, extensive experiments and limited number of analytical studies have been implemented in this field to clear-up performance characteristics. In this paper, the modulus of displacement theory is adopted to analyze the local behavior of tensile reinforcement lap splices. Both compatibility and equilibrium conditions are utilized to determine the distribution of steel and bond stresses of the reinforcing bars along the lap region.

The equations derived herein and the general form for distribution of stresses, are quite consistent with the analytical and experimental observations of numerous researchers in this field. The analytical results confirmed that bond deterioration occur simultaneously from both ends of the lap splice toward its center. Also, the numerical examples implemented herein give reasonable agreement between the analytical and experimental results.

Notations

A_s=area of reinforcing bar, mm^2
E_s=steel modulus of elasticity, MPa
f_{cu}=cube compressive strength, MPa
K=tangent or secant modulus of bond stress-slip curve, N/mm^3
L_s=length of lap splice, mm
P=perimeter of reinforcing bar
ε_{s1}=tesile steel strain for reinforcing bar coming from negative (left) side.
ε_{s2}=tesile steel strain for reinforcing bar coming from positive (right) side.
σ_{s2}=tensile steel stress for reinforcement bar coming from positive (right) side, MPa.
ε_c=tensile strain of concrete.
τ=bond stress, MPa
τ_1=bond stress along reinforcing bar coming from negative (left) side, MPa
τ_2=bond stress along reinforcing bar coming from positive (right) side, MPa
δ=relative displacement between steel and concrete, mm.
α=angle of the bond forces resultant with bar axis.
σ_{so}=tensile steel stress at splice end, MPa.
σ_{s1}=tensile steel stress for reinforcement bar coming from negative (left) side, MPa.
σ_c=normal tensile stress of surrounding concrete , MPa.

Introduction

In order to insure reliable behavior of concrete members containing reinforcing bar discontinuity, lap splicing is used in this field, which is the most economical and practical type. The forces at any point along the lap splice are being transferred from each bar by bond to the surrounding concrete.

However, the behavior of lap spliced bar is further complicated and different from single anchored bars in several ways. In particular, the action of bond forces becomes more severe in lap splices and it can be explained by doubling of splitting forces against surrounding concrete, as shown in Fig.(l). As well as yielding of reinforcement and bond deterioration can occur at both ends of the splice simultaneously toward the interior.

As a result, the presence of lap splices is generally recognized to represent potential weakness in reinforced concrete components especially for earthquake resisting structures. Therefore; it is not surprising that most design codes(1,2) require splice length that are longer than the development length, as well as do not permit lap splicing within high stress regions or extensive limitations regarding their design are needed.

A large number of experimental programs(3,4,5) have been conducted at last two decades to enhance knowledge about the effect of the most notable parameters on behavior and strength of lap splices under static and cyclic loads.

Objective Of The Present Study

In addition to the experimental investigations, there is limited number of theoretical and analytical studies in this field. The modulus of displacement theory is adopted herein to determine the distribution of bond and steel stresses along reinforcement lap splices. In the present method, both compatibility and equilibrium conditions are utilized to derive the governing differential equations of the problem.

Background

The modulus of displacement theory states that the change in shear stress $d\tau$ between two materials for element dx is proportional to the difference in the displacement

$$d\tau = K\, d\delta \quad \text{.......... (1)}$$

where K (N/mm is either the tangent or the secant modulus for bond stress-slip curve in a pull-out test, which can be evaluated as follow (6):

$$K = 2.4\, f_{cu} \quad \text{(Grade 40) .... (2a)}$$

$$K = 3.4\, f_{cu} \quad \text{(Grade 60) .... (2b)}$$

Where, f_{cu} is cubic compressive strength (MPa).

The concept of this theory was first mentioned by Bleih(7) (1924) to determine the individual rivet loads in long riveted joints which are replaced by a continuous medium. Granholm(8) (1949) transferred this theory so as to be useful for evaluating the distribution of stress for nailed wooden beams and pillar constructions. Then he presented(9) (1958) this theory for bond problems between reinforcement and concrete. Perhaps the most important application of this theory is that by Tepfers(10) (1973) on behavior of tensile lap splices with and without contribution of surrounding concrete after elimination the cross-sectional area of concrete surrounding the reinforcing bar.

The Problem Of The Present Study

The problem herein is limited to lap splice regions in a beam where the moment is constant and no shear exist. This means that reinforcement at both ends of the splice has the same stress σ_{so} Flexural cracks in the concrete are also located and aggravated there because of the stiffness discontinuity caused by sudden change in the tensile reinforcement area at the splice ends, as shown in Fig.(2). The influence of the concrete in estimating the distribution of bond and steel stresses can be ignored, i.e.

normal tensile stress of concrete σ_c=0. Numerous researchers(10-13) in the study and idealization of bond-slip behavior, adopted concept of neglecting the contribution of surrounding concrete.

For the tensile reinforcement splices shown in Fig.(2), the reinforcement area, As, is symmetrically, with the origin placed in the middle of the splice and the x- axis is parallel to the spliced bars. The functions of the bar embedded from left (negative side) are given the index (1) and those of the bar embedded from right (positive side) the index(2).

Governing Differential Equations

Both compatibility and equilibrium conditions are utilized for the lapped splice bars, as shown in Fig.(3a). The compatibility includes that the change in shear stress for every element dx due to the displacement between the reinforcing bar and the concrete is,

$$d\tau = K(\varepsilon_s - \varepsilon_c)dx \quad \text{.....(3)}$$

and after neglecting the interaction of surrounding concrete becomes,

$$d\tau_1 = K(\varepsilon_{s_1})dx \quad \text{or} \quad \frac{d\tau_1}{dx} = K\frac{\sigma_{s_1}}{E_s} \quad \text{.....(4)}$$

$$d\tau_2 = K(\varepsilon_{s_2})dx \quad \text{or} \quad \frac{d\tau_2}{dx} = K\frac{\sigma_{s_2}}{E_s} \quad \text{.....(5)}$$

The equilibrium condition at any section of the whole splice is,

$$A_s\sigma_{s_1} + A_s\sigma_{s_2} = A_s\sigma_{s_o}$$

$$\text{or} \quad \sigma_{s_1} + \sigma_{s_2} = \sigma_{s_o} \quad \text{.....(6)}$$

The connection between bond stresses τ_1 and τ_2 and steel stresses σ_{s1} and σ_{s2} can be obtained from equilibrium of element dx, Fig,(3b) as follows,

$$(d\sigma_{s_2} - d\sigma_{s_1})A_s = (\tau_2 - \tau_1)P.dx$$

$$\text{or} \quad \left(\frac{d\sigma_{s_2}}{dx} - \frac{d\sigma_{s_1}}{dx}\right)\frac{A_s}{P} = \tau_2 - \tau_1 \quad \text{.....(7)}$$

By differentiating Equation (7) and substituting Equations (4) and (5), yields to:

$$\left(\frac{d^2\sigma_{s_2}}{dx^2} - \frac{d^2\sigma_{s_1}}{dx^2}\right)\frac{A_s}{P} = \frac{K}{E_s}(\sigma_{s_2} - \sigma_{s_1}) \quad \text{.....(8)}$$

Then, substitute Equation (6) into Equation (8), leads to:

$$\frac{d^2\sigma_{s_1}}{dx^2} - \lambda^2\sigma_{s_1} = -\frac{\lambda^2}{2}\sigma_{s_o} \quad \text{.....(9)}$$

where $\lambda^2 = \frac{PK}{A_sE_s}$

Equation (9) is a nonhomogenous second order differential equation, and the complete solution is,

$$\sigma_{s_1} = A\,\mathrm{Sinh}\lambda x + B\,\mathrm{Cosh}\lambda x + C \quad \text{.....(10)}$$

where the first two terms represent the complementary function and the third is the particular integral.
By substituting the particular integral C in the differential Equation (9) then,

$$C = \frac{\sigma_{s_o}}{2} \quad \text{...(11)}$$

The constants A and B are obtained by means of conditions which are,

$$\text{at} \quad x = -\frac{L_s}{2} \qquad \sigma_{s_1} = \sigma_{s_o} \quad \text{..................................(12a)}$$

$$\text{at} \quad x = +\frac{L_s}{2} \qquad \sigma_{s_1} = 0 \quad \text{..................................(12b)}$$

These give the constants as,

$$B = 0 \quad \text{and} \quad A = -\frac{\sigma_{s_o}}{2}\frac{1}{\text{Sinh}\frac{\lambda L_s}{2}} \quad \text{..............................(13)}$$

Now, substitute Equations (11) and (13) into Equation (10) to get the function of steel stress o

$$\sigma_{s_1} = \frac{\sigma_{s_o}}{2}\left(1 - \frac{\text{Sinh}\lambda x}{\text{Sinh}\frac{\lambda L_s}{2}}\right) \quad \text{..............................(14)}$$

and substitute Equation (14) into Equation (6) leads to:

$$\sigma_{s_2} = \frac{\sigma_{s_o}}{2}\left(1 + \frac{\text{Sinh}\lambda\ x}{\text{Sinh}\frac{\lambda\ L_s}{2}}\right) \quad \text{..............................(15)}$$

It is well known that the traditional relation between the steel stress and bond stress τ is,

$$\tau = \frac{A_s}{P}.\frac{d\sigma_s}{dx} \quad \text{..............................(16)}$$

Then, the bond stresses τ_1 and τ_2 can be determined as follows:

$$\tau_1 = -\frac{\sigma_{s_o}}{2}\frac{A_s}{P}\lambda\ .\frac{\text{Cosh}\lambda\ x}{\text{Sinh}\frac{\lambda\ L_s}{2}} \quad \text{..............................(17)}$$

$$\tau_2 = +\frac{\sigma_{s_o}}{2}\frac{A_s}{P}\lambda .\frac{\text{Cosh}\lambda\ x}{\text{Sinh}\frac{\lambda\ L_s}{2}} \quad \text{..............................(18)}$$

The general form for distribution of steel and bond stresses are shown in Fig.(4), for suitable constants mentioned in the same figure. From the scrutiny of the Equations (14),(15), (17), and (18) derived above and the curves of Fig.(4), many notable features can be emphasized:

a-The distribution of steel or bond stresses, for both bars of the same splice, is equal and has the same variation but in opposite directions.

b-The stress $\tau_1.\tan\alpha$ or $\tau_2.\tan\alpha$ which represents the splitting stress directed outwards from the bar, has a maximum value at both ends of the splice.

c-The distribution of the steel or bond stresses along spliced bars is quite consistent with the analytical and experimental observations of numerous workers in this field[6,10,14].

Numerical Examples

A simply supported reinforced concrete beam containing tensile lap splices and subjected to a constant moment was tested by Tepfers[10] (1973). The steel strains were measured at selected points of the spliced bars for the three levels of the applied loading. Also, the distribution of strains is determined for each clamping stress of splice end, σ_{so}, by using the equations derived above.

Fig.(5) include comparison of measured and calculated values of steel strains along spliced bars. Good agreement between the experimental and analytical results can be concluded.

Another reinforced concrete beam with two points loading was performed by Kluge and Tuma[15] (1945). The distribution of the tensile stresses along the spliced bar through lap region was determined experimentally by measuring the strains at selected points along lap region.

Fig. (6) shows comparison of the measured and calculated stresses for the spliced bar coming from negative side of lap region of steel stress at spliced end, σ=124.1 MPa, and acceptable agreement can be concluded.

Conclusions

From inspection of the equations derived in this study according to the modulus of displacement theory, and the general form for distribution of steel and bond stresses of tensile reinforcement lap splices, many notations can be deduced:

1- The equations developed herein by utilizing the compatibility and equilibrium conditions, are quite consistent with experimental and analytical evidences of numerous workers in this field.

2- The distribution of steel and bond stresses, for each bar of a lap splice, is same but in opposite directions.

3- The bond stresses and split stresses have the maximum values at both ends of the splice. This explains the fact that bond deterioration and yielding of reinforcement occur simultaneously from both ends of the lap region towards its center.

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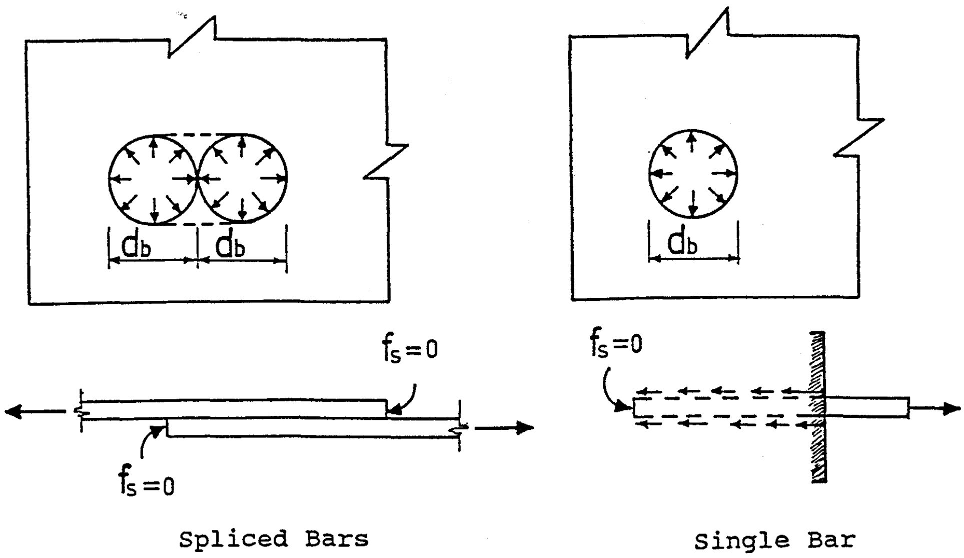


Fig(1): Single and lapped splice bars

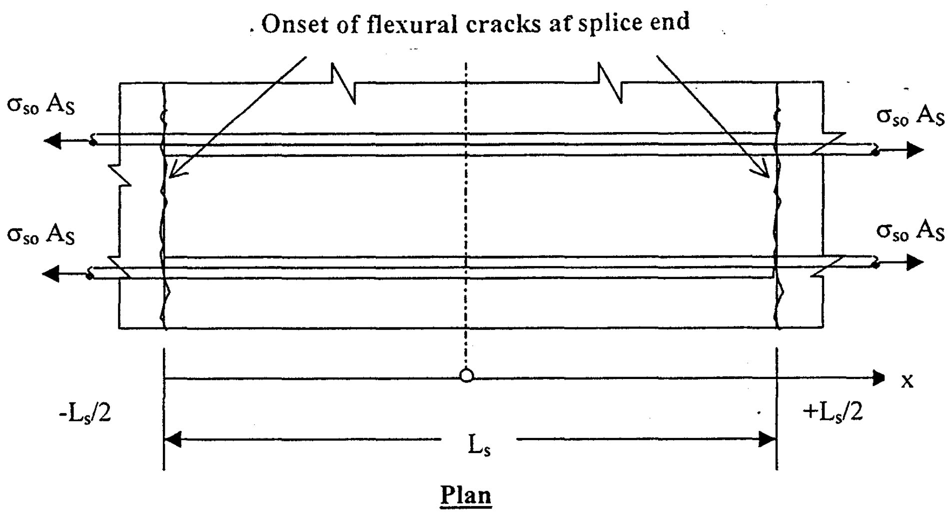
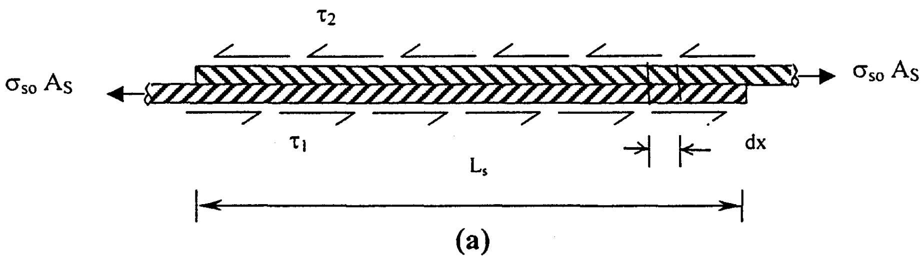


Fig.(2): Tensile zone of a beam with reinforcement lap splices



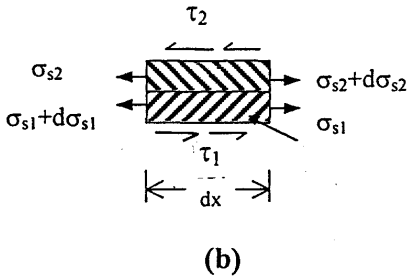
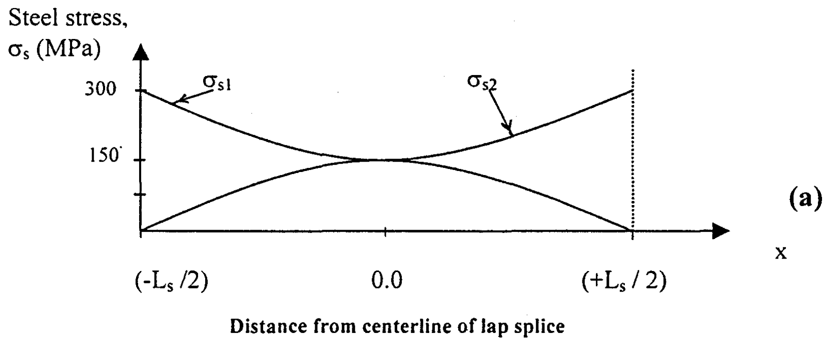
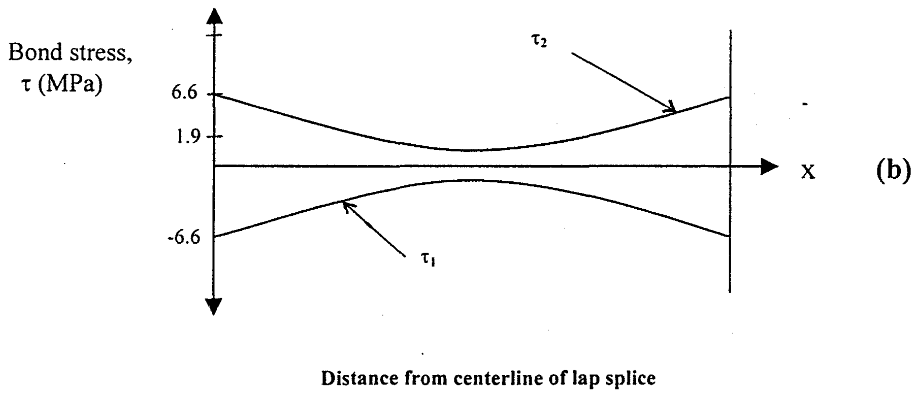


Fig.(3): Schematic representation of the bond and steel stresses acting on tensile reinforcement lap splices.

a. The whole lap splice

b. differential element (dx) of lap splice.





ϕ= bar diameter = 16 mm
$L_s = 30\ \phi = 480$ mm
$E_s = 210$ GPa
$K = N / mm^3$

Fig.(4): (a). Distribution of steel stresses for spliced bars
(b). Distribution of bond stresses along spliced bars

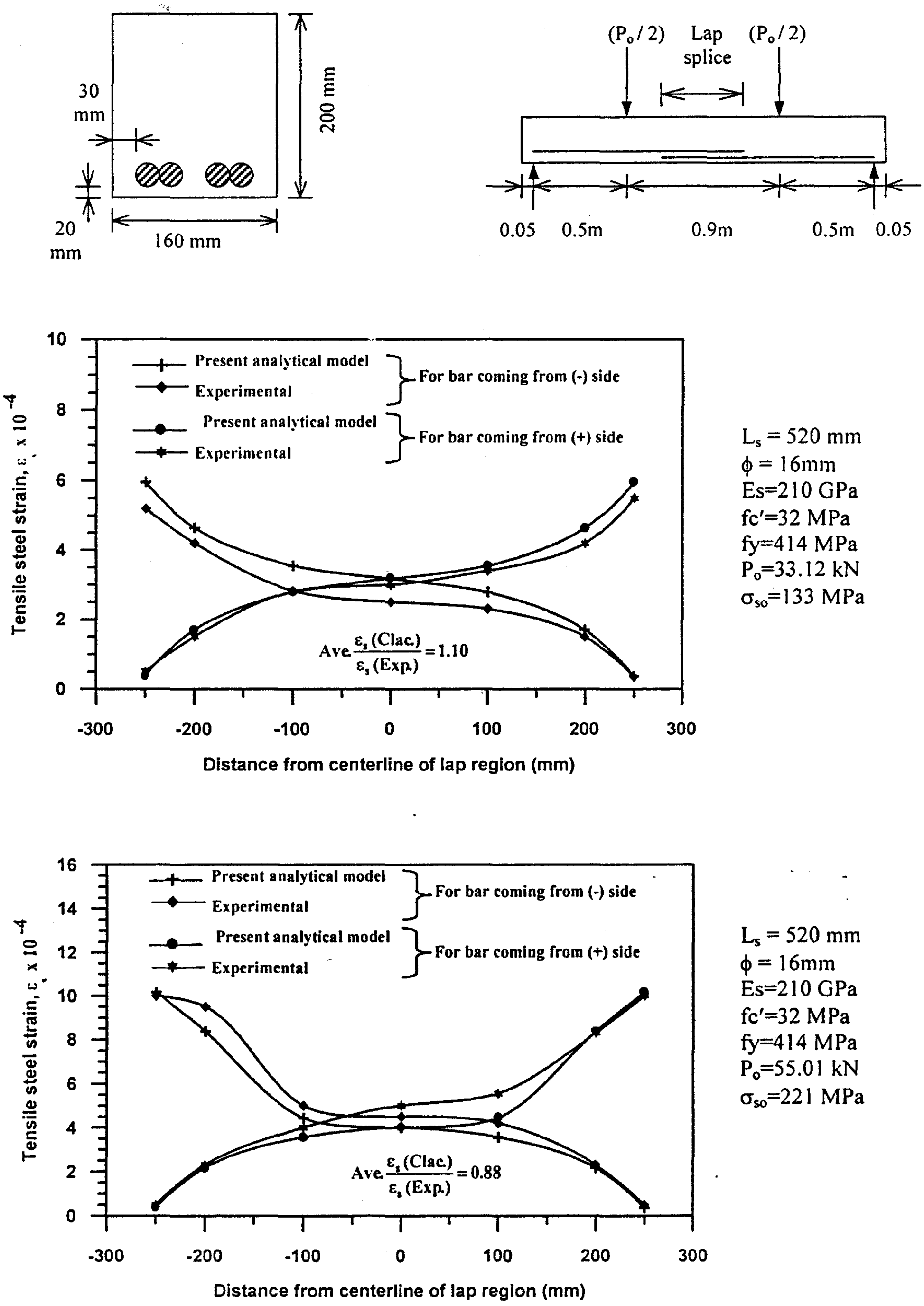


Fig.(5) Distribution of tensile strain along spliced bars for simply supported R.C. beams tested by Tepfers (10)

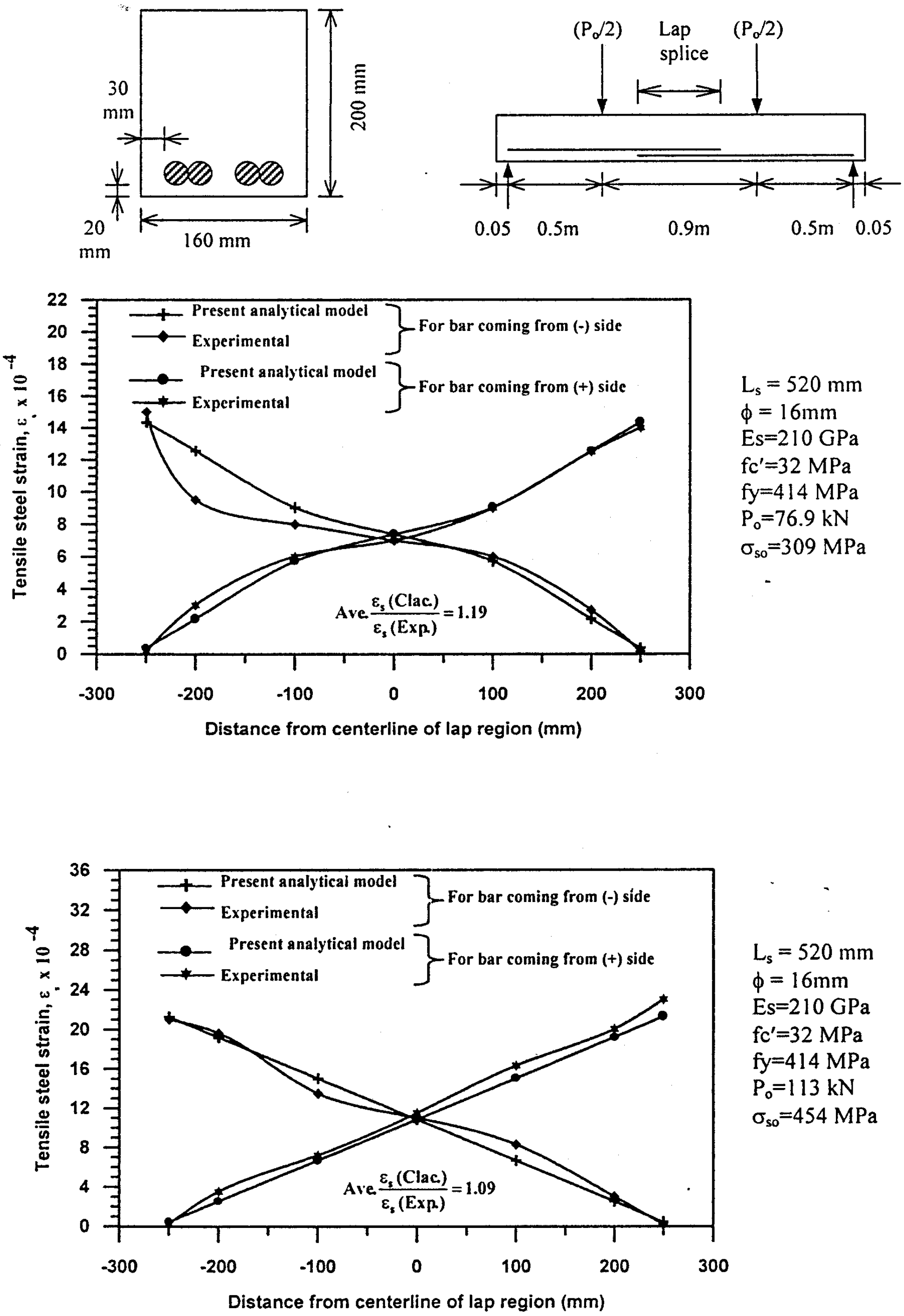


Fig.(5) Continue

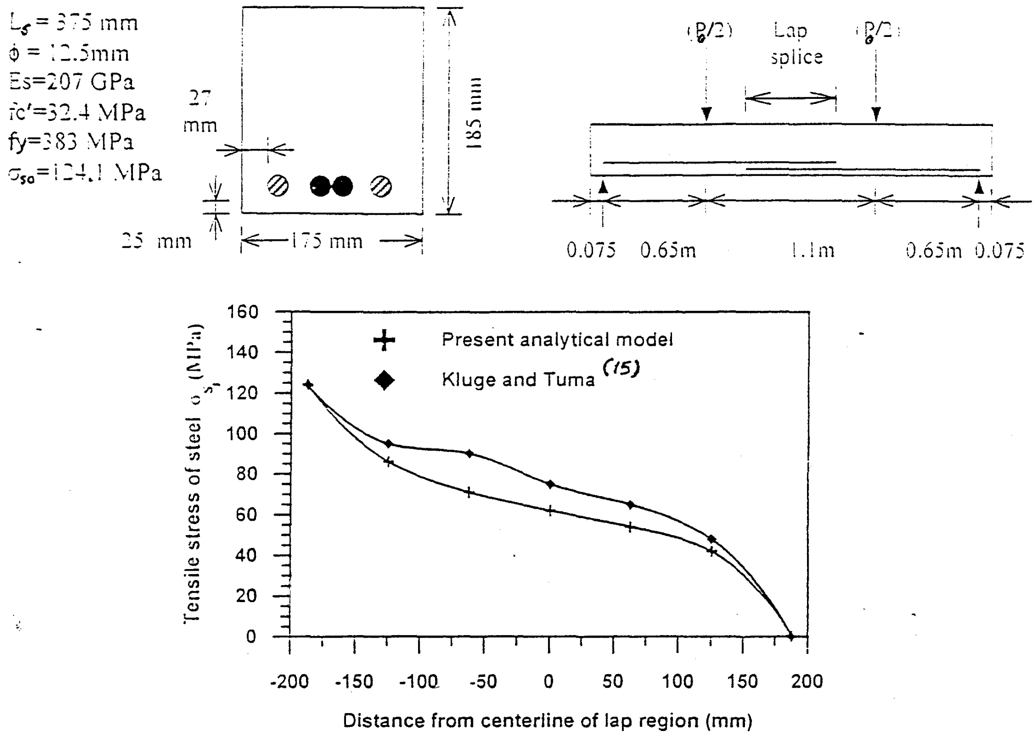


Fig.(6): Distribution of tensile stress along spliced bar coming from negative side of simply supported R.C. beam tested by Kluge and Tuma[15] (1945).

نظرية معايير الازاحة لتحليل سلوك وصل تعاقب التسليح

الخلاصة

في انشاء الخرسانة المسلحة، فأن عدم استمرارية قضبان التسليح كثيراً ما تواجه حتى ضمن فضاء الجزء الخرساني الواحد. ولهذا الغرض فإن وصلات التعاقب تعتبر من الوسائل المفضلة في هذا المجال لتحقيق الاستمرارية في حديد التسليح بسبب خصائصها العملية والاقتصادية. وبهدف تسليط الضوء على سلوك وصلات التعاقب في المنشآت الخرسانية، فأن العديد من البحوث العملية وعدد محدود من الدراسات النظرية انجزت في هذا المجال. الدراسة التي بين ايدينا تعتمد نظرية معايير الازاحة في دراسة وتحليل سلوك وصل التعاقب للتسليح، حيث يتم تطبيق شروط التوافق والتوازن لايجاد توزيع كل من اجهادات الشد في الحديد والربط على طول قضبان التسليح في منطقة التعاقب.

ان المعادلات المشتقة هنا لتوزيع اجهادات الشد في الحديد واجهادات الربط، هي متوافقة الى حد كبير مع الاستنتاجات العملية والنظرية لعدد من الباحثين في هذا المجال. كما ان النتائج التحليلية اعطت تفسيراً لظاهرة انحلال الربط وخضوع حديد التسليح الذي يحصل في نهايتي منطقة التعاقب الى داخلها. بالاضافة الى ذلك فأن الامثلة العددية المعتمدة في هذه الدراسة اعطت تطابق ملائم بين النتائج العملية والنظرية.